2018 VCE

Further Mathematics Trial Examination 2 Suggested Answers

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Data analysis

Question 1

a. (i) 154, 186, 192, 206, 300, 320, 450, 482, 580, 600, 600, 600 Median between 320 and 450 Median = $\dfrac{320+450}{2}=385$ (1 mark)	**a. (ii)** There were 8 months when there were at least 300 guest staying at the motel. Percentage = $\dfrac{8}{12}=66.67\%$ (1 mark)
b. (i) Underweight if < 500 g $Z=\dfrac{500-495}{5}=1$ % < 1 standard deviation from the mean = 100 − 16 = 84% (1 mark)	**b. (ii)** $z=\dfrac{505-495}{5}=2$ % > 2 standard deviations from the mean = 2.5% 2.5% of 600 = 15 boxes. (1 mark)

b. (iii) 0.15% < 500 g This is 3 standard deviations below the mean. $-3=\dfrac{500-x}{5}$ $x=515\text{g}$ (1 mark)

Data analysis

Question 2

a. This is the upper quartile, so 25% (1 mark)	b. (i) Use calculator to get $Q_1 = 2$, $Q_3 = 6$ $IQR = 6 - 2 = 4$ nights (1 mark)
b. (ii) Upper fence is 6 $6 + 1.5 \times IQR = 6 + 1.5 \times 4 = 12$ Anything greater than 12 is an outlier, so 14 is an outlier. (1 mark)	

b. (iii)

Number consecutive nights
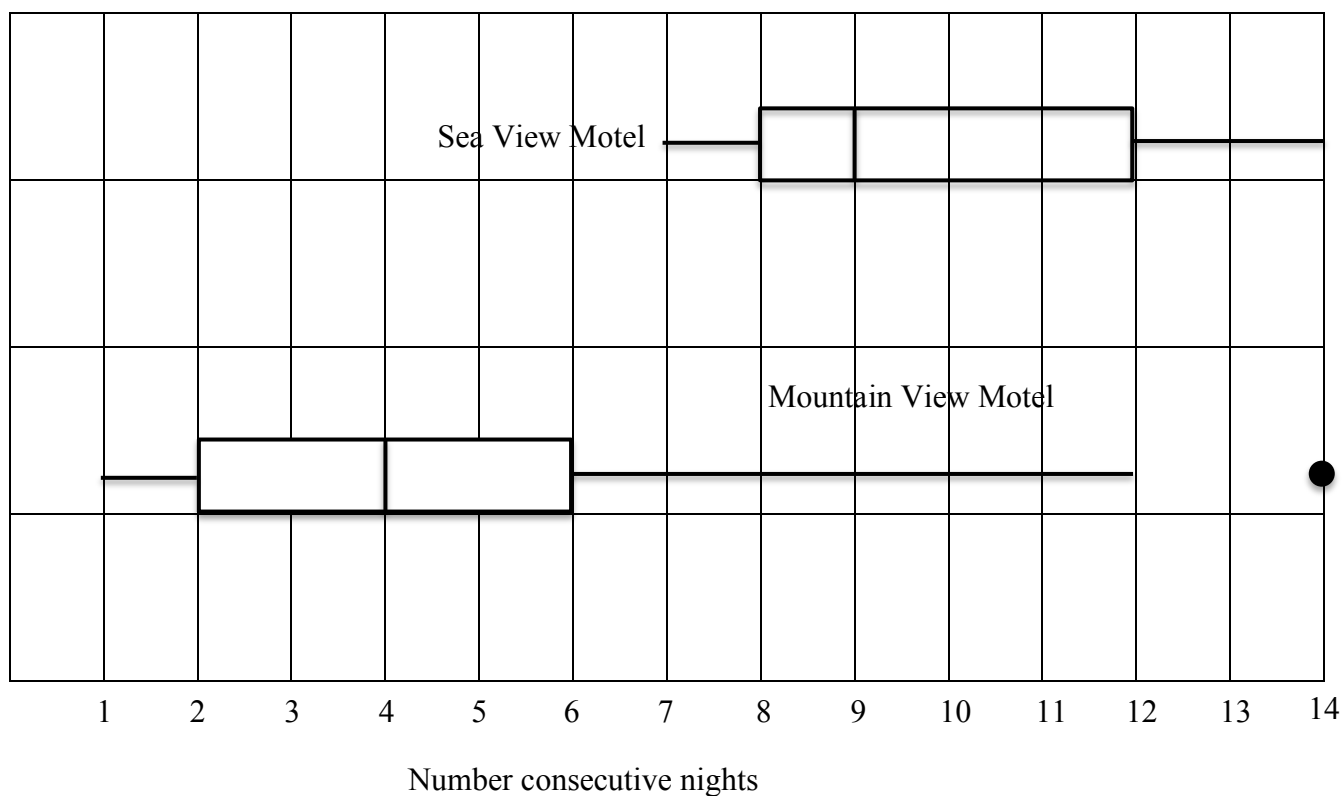
(2 marks)

Data analysis

Question 2 (continued)

c. (i)	c. (ii)
The median, which is a measure of centre is 9 for the Seaview motel. This is higher than the median of 4 for the Mountain view motel.	The interquartile range for both motels is 4. This measures the middle 50% of values. However, the Mountain View has a larger range of 13 than the Sea View, which has a range of 7. (1 mark)

d.
Number of consecutive nights spent at the Sea View is greater than 75% of the number of consecutive nights spent at the Mountain View. The median number of consecutive nights at the Sea View (9) is greater than the median number of consecutive nights at the Mountain View (4). The lower quartile at the Sea View (8) is greater than the lower quartile at the Mountain View (2). The upper quartile at the Sea View (12) is greater than the upper quartile at the Mountain View (6).

(1 mark)

Question 3

a. (i)	a. (ii)
Maximum temperature is the explanatory variable. (1 mark)	Use calculator to get the equation for the least squares line. It is *Number of ice creams sold* $= 27.4 + 10.5 \times maximum\ temperature$ (1 mark)

b. (i)
Enter coordinates of points in calculator and get least squares line. This is
Number of coffees sold
$= 445 - 8 \times maximum\ temperature$
According to this equation
When *Maximum temperature* = 12,
Number of coffees sold = 349
When *Maximum temperature* = 28,
Number of coffees sold = 221

Data analysis

Question 3

b. (i) (continued)

(1 mark)

(ii) For every one-degree rise in temperature there are 8 fewer coffees sold. (1 mark)	(iii) When maximum temperature = 20 Actual value = 300 Predicted value = 445 – 8 × 20 = 285 Residual value = 300 – 285 = 15 (1 mark)
(iv) r^2 = 55.96% = 56.0% to three significant figures. (1 mark)	

Data analysis

Question 4

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a.
May + June + July + August
$(620 + 490 + 400 + 320) \div 4 = 457.5$
June + July + August + September
$(490 + 400 + 320 + 780) \div 4 = 497.5$ 1 mark
Four-mean with centring
$= (457.5 + 497.5) \div 2$
$= 477.5$ kg   1 mark

(2 marks)

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b. (i)
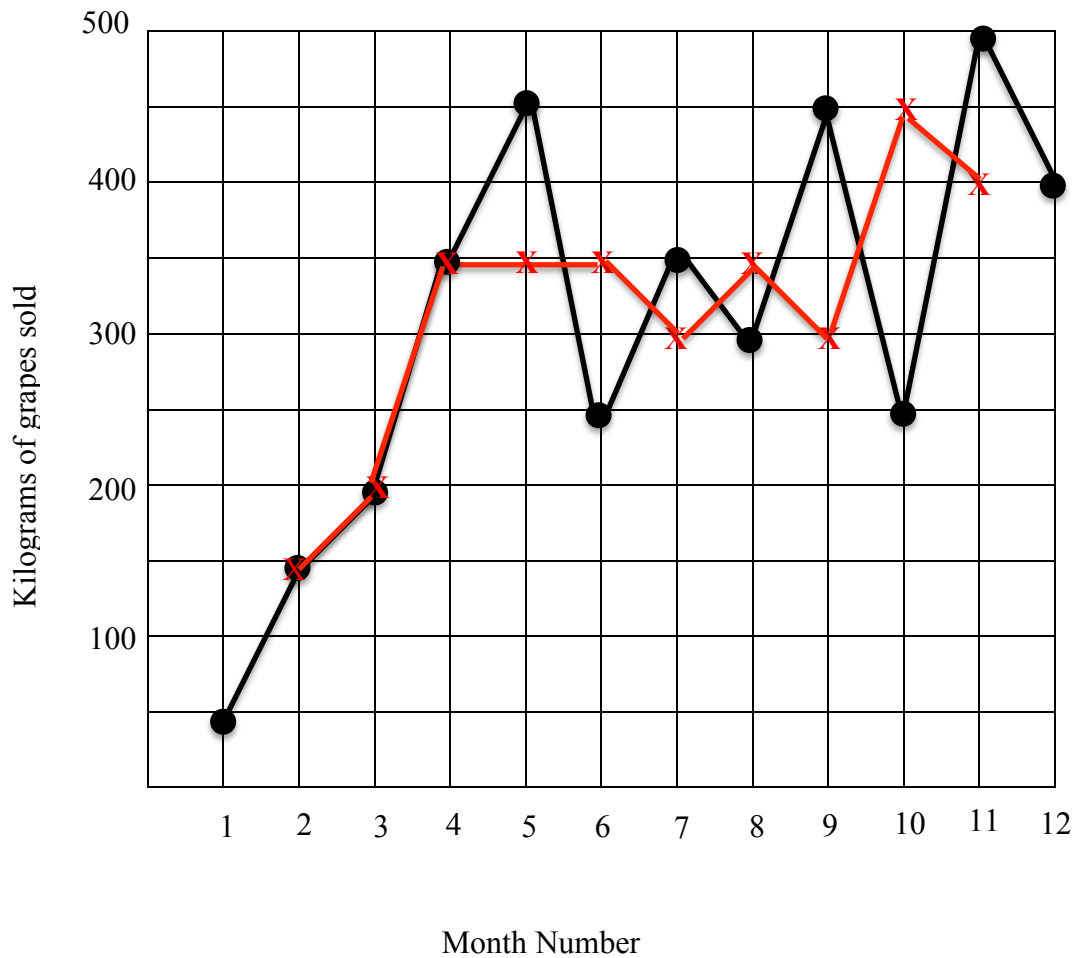
1 mark for points and 1 mark for line   (2 marks)

(ii)   The pattern increases till March but then becomes consistent over April, May and June. After this there is a decrease then an increase followed by a decrease. Overall, there is less fluctuation than in the original graph.

(1 mark)

Recursion and financial modelling

Question 5

a. (i) $A_4 = 84800$ $A_3 = 84800 - 1200 = 83600$ $A_2 = 83600 - 1200 = 82400$ $A_1 = 82400 - 1200 = 81200$ $A_0 = 81200 - 1200 = \$80000$ (1 mark)	**a (ii)** $A_0 = 80000 \qquad A_n = A_{n-1} + 1200$ (1 mark)
a. (iii) Her 7th year is A_6 Use calculator to get $A_6 = \$87200$ (1 mark)	**b. (i)** Her 10th year is A_9 Use calculator to get $A_9 = \$90800$ $V_0 = 90800 \quad V_{n+1} = 1.05V_n$ (1 mark)
b. (ii) $V_5 = 1.05^5 \times 90800 = \$115886.37$ (1 mark)	

Recursion and financial modelling

Question 6

a. (i) Quarterly interest rate = 4 ÷ 4 = 1% (1 mark)	a. (ii) Use solver on calculator $N = 4 \times 8 = 32$ $I = 4$ $PV = ¾ \times 52000 = 39000$ $PMT =$ $FV = 0$ $P/Y = 4$ $C/Y = 4$ This gives $PMT = \$1430.1645$ Total amount paid = $1430.1645 \times 32 = 45765.26$ Interest Paid = $45765.26 – 39000 = \$6765.26$ (1 mark)
b. (i) Use solver on calculator $N = 4 \times 3 = 12$ $I = 4$ $PV = ¾ \times 52000 = 39000$ $PMT = -1430.1645$ $FV =$ $P/Y = 4$ $C/Y = 4$ This gives $FV = \$25808.11$ (1 mark)	b. (ii) Use solver on calculator $FV = \$25808.1106$ becomes PV for next stage. $N =$ $I = 4$ $PV = 25808.1106$ $PMT = -2000$ $FV = 0$ $P/Y = 4$ $C/Y = 4$ This gives $N = 13.9$ quarters $= 13.9 \times 3 = 41.7$ months Total number of months of payments $= 36 + 41.7 = 77.7$ months Reduction in time = $96 – 77.7$ $= 18.3$ months (1 mark)

Recursion and financial modelling

Question 7

a.
Depreciation over 8 years
= 38000 – 6000 = \$32000
Number kilometres travelled in this time
= 50000 × 8 = 400000 km
Depreciation per km = 32000 ÷ 400000
= \$0.08
= 8 cents per km.

(1 mark)

b.
$V_n = 38000 - 0.08 \times 50000n$
$V_n = 38000 - 4000n$

(1 mark)

c.
2000 = 38000 – 4000n
Solve
n = 9 years
In 9 years the van would travel
9 × 50000 = 450000 km.

(1 mark)

Module 1 – Matrices

Question 1

a.	b.
This is the element in the second row, third column and is $4.10 (1 mark)	The cheapest price is $3.40. this is at the Care supermarket in Rosemount. (1 mark)
c. At Apex, the price will now be 95% of the original value, so $a = 0.95$. At Best, the price will now be 90% of the original value, so $b = 0.90$. At Care, the price will now be 80% of the original value, so $c = 0.80$. (1 mark)	d. $$\begin{bmatrix} 3.70 & 3.60 & 3.40 \\ 4.20 & 4.00 & 4.10 \\ 8.60 & 6.20 & 7.00 \end{bmatrix} \begin{bmatrix} 0.95 \\ 0.90 \\ 0.80 \end{bmatrix} = \begin{bmatrix} 9.475 \\ 10.87 \\ 19.35 \end{bmatrix}$$ The price for a kilo of tomatoes bought at each of the three stores in Temple is $19.35 (1 mark)

Question 2

a.	b.
From the diagram, it can be seen that the percentage going from B to C is 15%. Hence, percentage that do not go to C from B $= 100 - 15 = 85\%$. (1 mark)	A $\longrightarrow$ B $= 0.1 \times 16000$ B $\longrightarrow$ B $= 0.6 \times 14000$ C $\longrightarrow$ B $= 0.2 \times 10000$ $a = 0.1, b = 0.6, c = 0.2$ (1 mark)

c.

$$\begin{bmatrix} 0.8 & 0.25 & 0.1 \\ 0.1 & 0.6 & 0.2 \\ 0.1 & 0.15 & 0.7 \end{bmatrix}^3 \begin{bmatrix} 16000 \\ 14000 \\ 10000 \end{bmatrix} = \begin{bmatrix} 18197.5 \\ 10637 \\ 11165.5 \end{bmatrix}$$

Number who shop at either Apex or Care $= 18197.5 + 11165.5 = 29363$

(1 mark)

Module 1 – Matrices

Question 3

<table>
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a.
Staff not moved are
$0.5 \times 16 + 0.6 \times 12 + 0.7 \times 8 + 0.8 \times 5 = 25$
Total number of staff $= 16 + 12 + 8 + 5 = 41$
Staff moved from week one to week two
$= 41 - 25 = 16$

(1 mark)

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b.

$$S_2 = \begin{bmatrix} 0.5 & 0.2 & 0.1 & 0.05 \\ 0.2 & 0.6 & 0.15 & 0.1 \\ 0.25 & 0.1 & 0.7 & 0.05 \\ 0.05 & 0.1 & 0.05 & 0.8 \end{bmatrix} \begin{bmatrix} 16 \\ 12 \\ 8 \\ 5 \end{bmatrix} = \begin{bmatrix} 11.45 \\ 12.1 \\ 11.05 \\ 6.4 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 12 \\ 12 \\ 11 \\ 6 \end{bmatrix}$$

(1 mark)

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c.

$$S_3 = TS_2 = \begin{bmatrix} 0.5 & 0.2 & 0.1 & 0.05 \\ 0.2 & 0.6 & 0.15 & 0.1 \\ 0.25 & 0.1 & 0.7 & 0.05 \\ 0.05 & 0.1 & 0.05 & 0.8 \end{bmatrix} \begin{bmatrix} 11.45 \\ 12.1 \\ 11.05 \\ 6.4 \end{bmatrix}$$

$$= \begin{bmatrix} 9.57 \\ 11.85 \\ 12.18 \\ 7.5 \end{bmatrix}$$

Staff working in vegetables in week 3 = 12
Staff working in dairy in week 2 = 11
Of these 11, 15% go to vegetables.
15% of 11 = 1.65
Round this to 2 people
2 out of 12 = 16.7%

(2 marks)

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d.

Week 1: 16 work in grocery.
Week 2: 12 work in grocery
Week 3: 10 work in grocery.
Week 4: T^3S gives 9 work in grocery.
Week 5: T^4S gives 8 work in grocery.
Week 6: T^5S gives 8 work in grocery.
The minimum number is 8

(1 mark)

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Module 2: Networks and decision mathematics

Question 1

a.

$$\begin{array}{c c} & \begin{array}{c c c c c c} F & A & B & C & D & E \end{array} \\ \begin{array}{c} F \\ A \\ B \\ C \\ D \\ E \end{array} & \left[\begin{array}{c c c c c c} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{array}\right] \end{array}$$

(1 mark)

b.
There are a total of 6 different ways.
He could go *FABCDEF* or *FEDCBAF* or
FBACDEF or *FBACEDF* or *FDECBAF* or
FDECABF.
1 mark for at least 2 correct answers. 2 marks
for all correct.

(2 marks)

c. (i)
No. The Council worker would have to start at
an odd vertex and finish at another odd vertex
since there are two odd vertices. He would
have to start and finish at either *Q* or *R*.
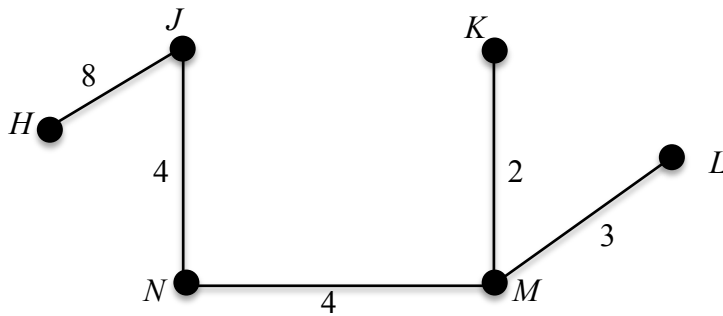
(1 mark)

c. (ii)
Planar graphs satisfy the formula
$v + f = e + 2$
Here, $v = 6, f = 5, e = 9$
$6 + 5 = 9 + 2$ is true.

(1 mark)

Module 2: Networks and decision mathematics

Question 2

a.

(1 mark)

b.
Minimum length = 8 + 4 + 4 + 2 + 3 = 21 km.

(1 mark)

Question 3

a.	**b.**
B, C and D must all be completed before activity G can begin. (1 mark)	The longest path is 31 days. If AEI is the critical path, then I = 11. If CDFI is the critical path, then I = 9. These are the only possibilities for the critical path so the minimum value for I is 9. (1 mark)

c.
Critical path is CDFI
If A or G are reduced in time, it will just make their path time shorter, so the total time for all activities will still be 31 days. There is no point in reducing the times for A or G.
If D is reduced by 3 days to 5 days CDFI will be 28 days, so there will be a new critical path CHJ, which will take 30 days. If F is reduced by 5 days to 2 days CDFI will be 26 days and again there will be a new critical path of 30 days. Hence no sense reducing either D or F by more than 1 day.
The shortest time that the cable can now be laid is 30 days.

(1 mark)

Module 2: Networks and decision mathematics

Question 3 (continued)

d.	**e.**
There is a saving of 1 day so could reduce either D or F by 1 day to reduce the overall time to 30 days. One new employee. (1 mark)	Better to employ this 1 extra worker for activity F as this only costs $1000. The company gets a bonus of $4000, so will be able to save 4000 – 1000 = $3000 (1 mark)

Module 3: Geometry and measurement

Question 1

a.

$V = L \times W \times H = 45 \times 22 \times 24 = 23760 \text{ cm}^3$

(1 mark)

b.

Area of triangular face = ½ × 6 × 8 = 24
Area of 2 triangular faces = 2 × 24 = 48
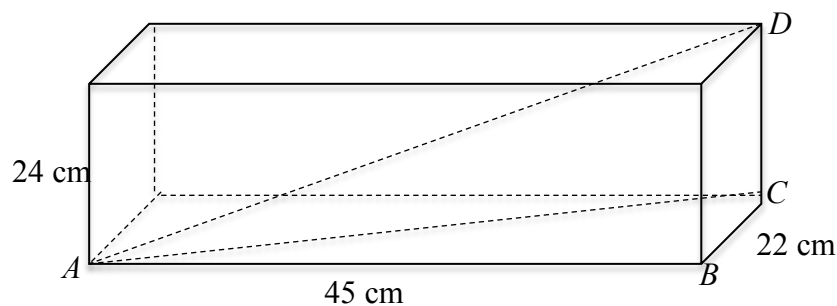Area of base rectangular face = 6 × 10 = 60
Area of upright rectangular face = 8 × 10 = 80
Using Pythagoras triads on right angle triangle, hypotenuse = 10 cm
Area of slant rectangular face = 10 × 10 = 100
Total surface area
= 48 + 60 + 80 + 100 = 288 cm^2

(1 mark)

c.

In triangle *ABC*
$AC = \sqrt{45^2 + 22^2} = \sqrt{2509}$
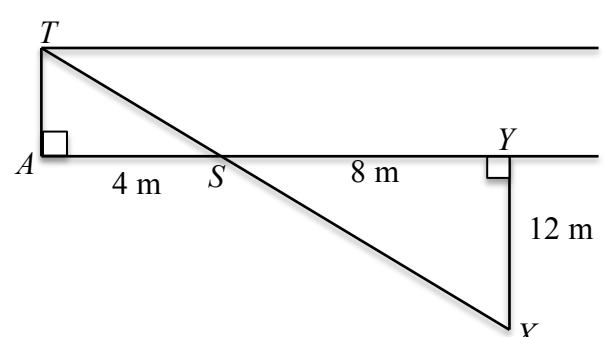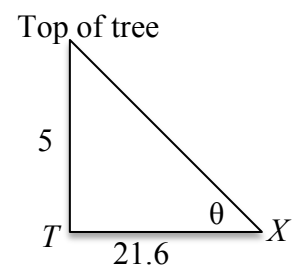In triangle *ACD*
$AD = \sqrt{2509 + 24^2} = 55.5 \text{ cm}$

(1 mark)

Module 3: Geometry and measurement

Question 2

a.

$$\frac{\sin \angle ABC}{1} = \frac{\sin\left(47^0\right)}{1.5}$$

$$\angle ABC = \sin^{-1}\left(\frac{\sin\left(47^0\right)}{1.5}\right) = 29^0$$

(1 mark)

b.

$$\angle CAB = 180 - (47 + 29) = 104^0$$

$$\frac{BC}{\sin(104^0)} = \frac{1.5}{\sin(47^0)}$$

$$BC = 2 \text{ km}$$

(1 mark)

c.

$$\text{Area} = \frac{1}{2} \times 1 \times 1.5 \sin(104^0) = 0.73 \text{ km}^2$$

(1 mark)

Module 3: Geometry and measurement

Question 3

a.

Triangles *TAS* and *SXY* are similar (AAA)

$$\frac{AT}{12} = \frac{4}{8}$$

$$AT = 6 \text{ m}$$

(1 mark)

b.

$$\vartheta = \tan^{-1}\left(\frac{6}{4}\right) = 56^0$$

(1 mark)

c.

$$TS = \sqrt{4^2 + 6^2} = 7.2 \text{ m}$$

(1 mark)

d.

Length of $SX = \sqrt{8^2 + 12^2} = 14.4$

Length of $TX = 14.4 + 7.2 = 21.6$

$$\theta = \tan^{-1}\left(\frac{5}{21.6}\right) = 13^0$$

(1 mark)

Module 3: Geometry and measurement

Question 4
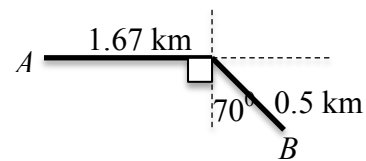
a.
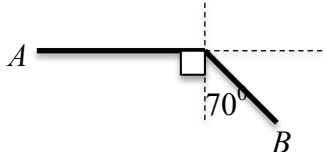
Angle = 90 + 70 = 160^0

(1 mark)

b.

A goes 10 km in 60 min

So $\dfrac{10}{6}$ km in 10 min.

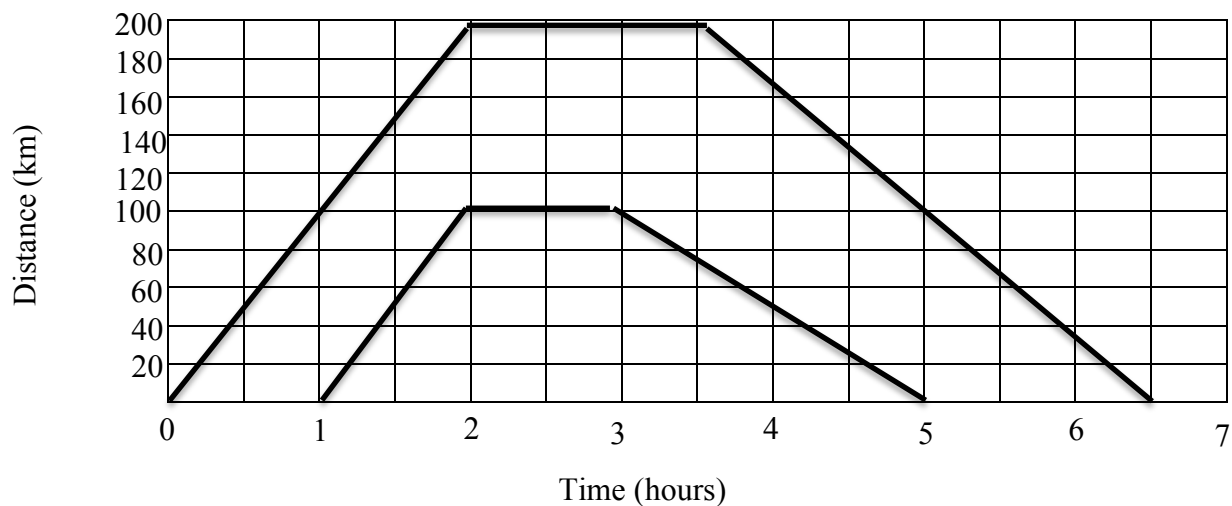B goes 3 km in 60 min

So $\dfrac{3}{6}$ km in 10 min.

$AB = \sqrt{1.67^2 + 0.5^2 - 2 \times 0.5 \times 1.67 \times \cos(160^0)}$
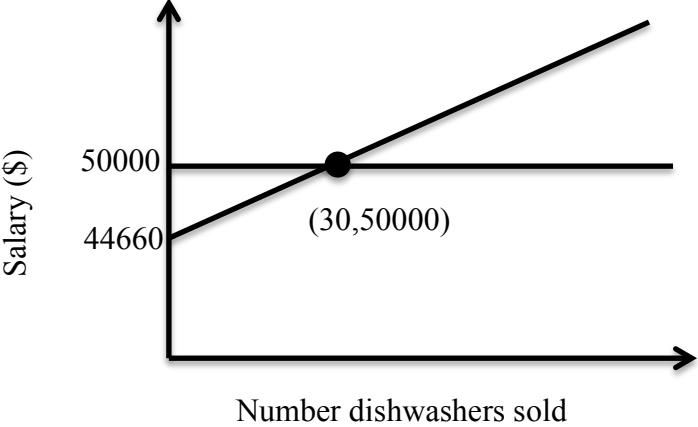
$AB = 2$ km

(1 mark)

Module 4: Graphs and relations

Question 1

a. Speed = $\dfrac{\text{Distance}}{\text{Time}} = \dfrac{200}{2} = 100$ km/hr (1 mark)	b. She drives for 1 hour She stops for 1 hour Her journey home is 100 km at 50 km/hr = 2 hours. Josie is away from the office for 1 + 1 + 2 = 4 hrs (1 mark)

c.

(1 mark)

Module 4: Graphs and relations

Question 2

a.	b.
$P = 44660 + 178\,n$ (1 mark)	$44660 + 178n = 50000$ $n = 30$ (1 mark)

c.

(1 mark)

Module 4: Graphs and relations

Question 3

a. $2x + 4y \le 320$ (1 mark)	b. There must be at least 80 standard dishwashers produced. (1 mark)

c.

$2x + 4y = 320$

$x + 2y = 160$

When $x = 0, y = 80$

When $y = 0, y = 160$

(1 mark)

d. Shading on graph

(1 mark)

Module 4: Graphs and relations

Question 3 (continued)

e. Profit = $130x + 280y$ Use calculator to find points of intersection by solving simultaneous equations. At A (80, 0) Profit = \$10400 At B (80, 30) Profit = \$18800 At C (120, 20) Profit = \$21200 At D (160, 0) Profit = \$20800 Should make 120 standard and 20 deluxe. (1 mark)	**f.** Maximum profit is \$21200 (1 mark)

End of Suggested Solutions 2018 VCE Further Mathematics Trial Examination 2

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