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Student Name.....

FURTHER MATHEMATICS

TRIAL EXAMINATION 2

2017

Reading Time: 15 minutes
Writing time: 1 hour 30 minutes

Instructions to students

This exam consists of Section A and Section B.
Section A contains 7 short-answer and extended answer questions from the core.
Section A is compulsory and is worth 36 marks.
Section B begins on page 12 and consists of 4 modules. You should choose 2 of these modules and answer every question in each of your chosen modules. Each of the modules is worth 12 marks.
Section B is worth 24 marks.
There are a total of 60 marks available for this exam.
The marks allocated to each of the questions are indicated throughout.
Students may bring one bound reference into the exam.
Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.
Unless otherwise stated, the diagrams in this exam are not drawn to scale.
Formula sheets can be found on pages 30 and 31 of this exam.

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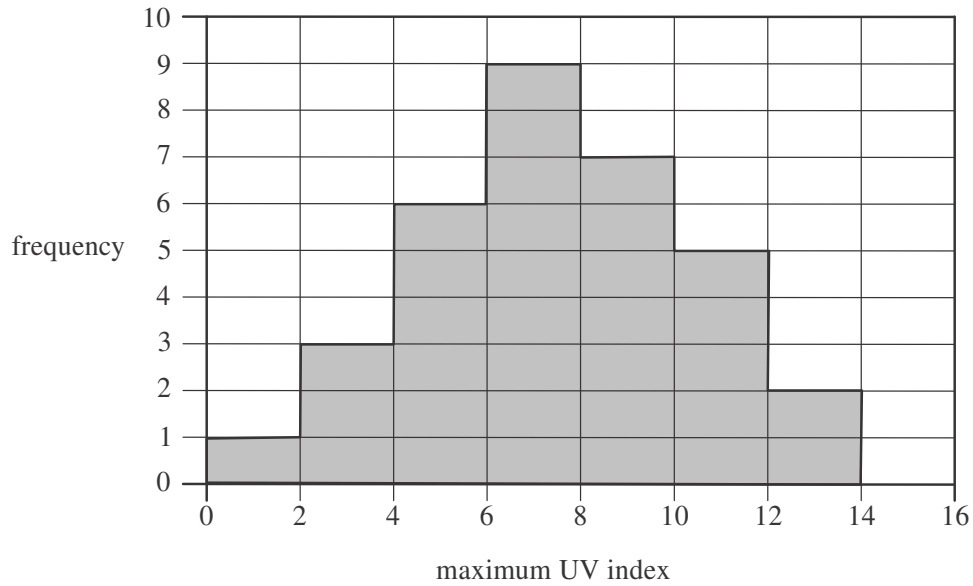
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SECTION A - Core**Data analysis**

This section is compulsory.

Question 1 (6 marks)

The histogram below shows the distribution of the maximum UV index recorded in March in 33 cities.

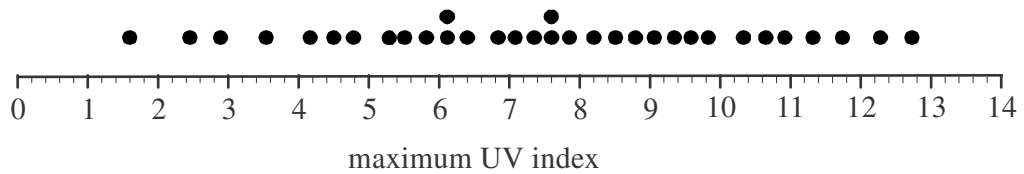


- a.** In how many of these cities was the maximum UV index greater than 10? 1 mark

- b.** What is the modal interval of this distribution? 1 mark

- c.** In what percentage of these cities was the maximum UV index less than 6? Round your answer to one decimal place. 1 mark

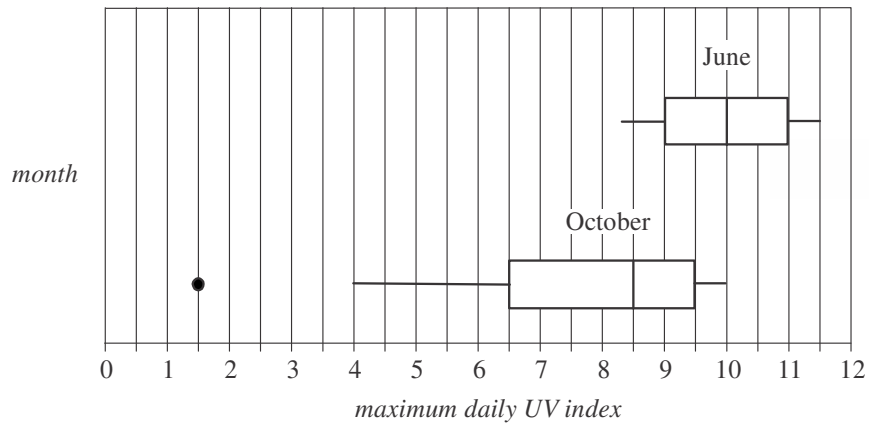
The dot plot below shows the same distribution.



- d.** Find the
- i.** median. 1 mark
- _____
- ii.** first quartile (Q_1). 1 mark
- _____
- e.** Explain why we could use the dot plot but **not** the histogram to find the exact mean of this distribution. (Do not calculate the mean). 1 mark
- _____
- _____
- _____

Question 2 (6 marks)

- a. The boxplots below show the distribution of the *maximum daily UV index* in a northern hemisphere city during June and October.



- i. What is the range of the distribution of *maximum daily UV index* in October? 1 mark

- ii. Describe the shape of the distribution of *maximum daily UV index* in this city for:

1 mark

June _____

October _____

- iii. Find the value of the lower fence for the October boxplot and use it to explain why the *maximum daily UV index* of 1.5 is an outlier for the month of October. 2 marks

- iv. Explain why the *maximum daily UV index* is associated with the *month* of the year. Give values of appropriate statistics to support your explanation. 1 mark

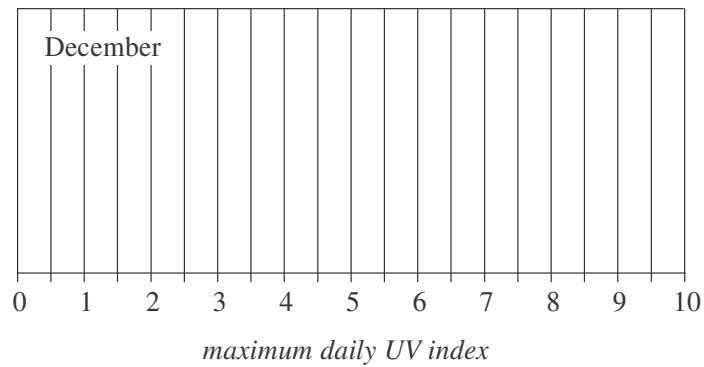
- b. The five-number summary for the distribution of the maximum daily UV index for December in this city is given in the table below.

Maximum daily UV index	
Minimum	1
Q_1	2.5
Median	3.5
Q_3	5
Maximum	6

The distribution contains no outliers.

Use this five-number summary for December to construct a boxplot on the grid below.

1 mark

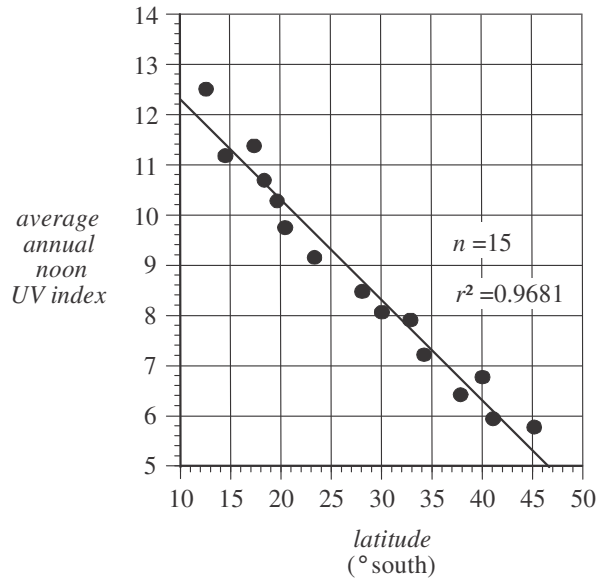


Question 3 (8 marks)

The table and scatterplot below shows the latitude, in degrees, and the average annual noon UV index for a sample of 15 cities located in the southern hemisphere. Data was collected each day at noon over a twelve year period to obtain these averages.

The association between the variables *average annual noon UV index* and *latitude* is to be investigated using this data.

Average annual noon UV index	Latitude (° south)
12.5	12.4
11.2	14.5
10.7	18.2
10.3	19.6
9.7	20.7
9.2	23.4
8.6	27.8
8.1	30.1
7.2	34.2
6.5	37.9
5.9	41.3
5.8	45.1
7.9	33.0
11.4	17.5
6.8	40.2



a. What is the explanatory variable?

1 mark

b. Describe the association between the *average annual noon UV index* and *latitude* in terms of strength, direction and form (use the scatterplot).

1 mark

c. i. Find the equation of the least squares regression line that can be used to predict the *average annual noon UV index* from the *latitude*. Write the values of the intercept and slope of this line in the boxes below rounded to two decimal places.

3 marks

$$\text{average annual noon UV index} = \boxed{} + \boxed{} \times \text{latitude}$$

ii. Interpret the slope of this least squares regression line in terms of the variables *average annual noon UV index* and *latitude*.

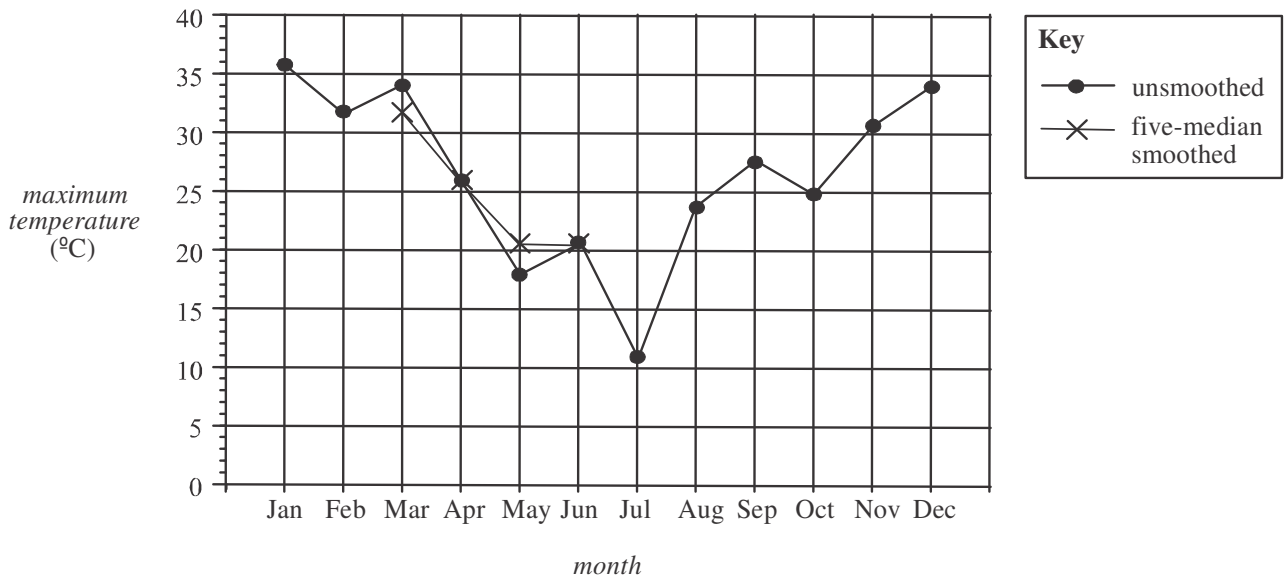
1 mark

- d.** The coefficient of determination for the association between the variables *annual average noon UV index* and *latitude* is 0.9681.
- i.** What is the value of Pearson's correlation coefficient for this data? Round your answer to two significant figures. 1 mark

- ii.** Interpret the coefficient of determination in terms of the two variables. 1 mark

Question 4 (4 marks)

The time series plot below shows the *maximum temperature*, in degrees Celsius, in a southern hemisphere city plotted against the *month* in which it was recorded during 2015.



- a. Five median smoothing is to be used to smooth the time series plot above and the first four smoothed points are indicated above as crosses (X). Complete the five median smoothing by placing crosses on the time series plot above to mark the smoothed values. 2 marks

The monthly seasonal indices for the maximum temperature in this city are shown in the table below.

monthly seasonal indices											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1.16	1.09	1.06	1.01	0.91	0.87	0.85	0.96	0.98	1.01	1.03	1.07

- b. In 2015 the maximum temperature recorded in March in this city was 34°C. Using the table above, find the deseasonalised value for the maximum temperature in March 2015 for this city. Round your answer to the nearest degree. 1 mark

- c. Explain what a monthly seasonal index of 0.85 tells us in terms of this city's maximum temperatures in July. 1 mark

Recursion and financial modelling

Question 5 (5 marks)

Courtney opens a savings account and deposits a sum of money. The value of her savings in this account after n years, V_n can be modelled by the recurrence relation

$$V_0 = 8\,000, \quad V_{n+1} = 1.035 \times V_n$$

- a. What was the sum of money Courtney deposited in her account? 1 mark

- b. What is the annual interest rate, compounding yearly, for Courtney's savings account? 1 mark

- c. Using recursion, write down calculations to find the amount of money in Courtney's account at the end of three years. 1 mark

A rule which also gives the value of Courtney's savings in her account after n years, V_n , is

$$V_n = a^n \times b$$

- d. Write down the values of a and b . 1 mark

- e. After how many years will the value of Courtney's savings first exceed \$10 000? 1 mark

Question 6 (3 marks)

Courtney is saving money to replace her car which she purchased for \$14 000 ten years ago.

- a.** Courtney's car has travelled 39 000 kilometres during the ten years. If the car is depreciated using the **unit cost** method of depreciation at the rate of \$0.32 per kilometre, find its current value. 1 mark

- b.** It is decided instead that Courtney's car will be depreciated using the **reducing balance** method of depreciation at the rate of 15% per annum.

Let C_n represent the value of Courtney's car n years after she purchased it.

- i.** Write down a recurrence relation, in terms of C_{n+1} and C_n , that models the value of the car. 1 mark

- ii.** What is the current value of the car? 1 mark

Question 7 (4 marks)

Courtney borrows \$30 000 to buy her new car. Interest is charged at the rate of 6.2% per annum compounding monthly.

Courtney will make monthly repayments of \$1 200 until the loan is fully repaid.

- a.** How much does Courtney still owe on the loan after six months? 1 mark

- b.** How much interest does Courtney pay during the first six months of the loan? 1 mark

- c.** How much interest will Courtney be charged in total over the life of the loan? 2 marks

SECTION B - Modules

Module 1 - Matrices

If you choose this module all questions must be answered.

Question 1 (3 marks)

A dog breeder sells four different breeds of puppies, cavoodles (C), groodles (G), poodles (P) and spoodles (S).

Matrix N below shows the number of each type of puppy the breeder sold last year.

$$N = \begin{bmatrix} & C & G & P & S \\ 25 & 40 & 10 & 55 \end{bmatrix}$$

- a. Write down the order of matrix N . 1 mark

The profit, in dollars, made on the sale of each type of puppy is shown in matrix M below

$$M = \begin{bmatrix} 1800 \\ 1350 \\ 2100 \\ 1950 \end{bmatrix} \begin{matrix} C \\ G \\ P \\ S \end{matrix}$$

- b. Find the matrix product NM . 1 mark

- c. Explain what the information in the matrix product NM represents. 1 mark

Question 2 (2 marks)

A litter of five puppies named Abby (*A*), Barney (*B*), Charlie (*C*), Drover (*D*) and Ebert (*E*) are observed by the breeder.

The dominance matrix below, indicates which puppies have a one-step dominance over others.

		non - dominant				
		<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
<i>A</i>	$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \end{bmatrix}$					
<i>B</i>	$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}$					
dominant <i>C</i>	$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \end{bmatrix}$					
<i>D</i>	$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \end{bmatrix}$					
<i>E</i>	$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \end{bmatrix}$					

The '1' in row *A*, column *B* indicates that Abby is dominant over Barney.

The '0' in row *A*, column *C* indicates that Charlie is dominant over Abby.

- a.** Using one-step dominances, which is the most dominant dog in the litter? 1 mark

- b.** Find the matrix that shows the two-step dominances for this litter. 1 mark

Question 3 (7 marks)

The breeder keeps 80 female dogs in the breeding program. These dogs are placed in one of the following four stages of the program:

- not ready for breeding (N)
- producing a litter (L)
- rested from breeding (B)
- retired from breeding (R)

The matrix S_0 below, shows the number of dogs that the breeder had at each stage of the program at the start of 2015.

$$S_0 = \begin{bmatrix} 10 \\ 20 \\ 35 \\ 15 \end{bmatrix} \begin{matrix} N \\ L \\ B \\ R \end{matrix}$$

The transition matrix T below, models the movement of these female dogs through the different stages of the program from the start of one year to the next.

$$T = \begin{matrix} & \begin{matrix} \text{this year} \\ N & L & B & R \end{matrix} \\ \begin{matrix} N \\ L \\ B \\ R \end{matrix} & \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0.4 & 0 & 0.6 & 0 \\ 0 & 0.7 & 0.2 & 0 \\ 0.1 & 0.3 & 0.2 & 1 \end{bmatrix} \end{matrix} \begin{matrix} N \\ L \\ B \\ R \end{matrix} \text{ next year}$$

- a.** What proportion of dogs who have produced a litter one year would be retired the following year? 1 mark

Let S_n represent the number of these dogs at each stage of the breeding program n years after the start of 2015.

- b. i.** Evaluate the matrix product $S_1 = TS_0$. 1 mark

- ii.** Write down a calculation to show that there would be 21 dogs being rested from breeding at the start of 2016. 1 mark

- iii. If no dogs are added to or leave the breeding program, at the start of which year will the number of dogs producing a litter first drop below ten? 1 mark

At the start of 2015, the breeder considered an alternative breeding program model whereby a certain number of dogs would be added to and removed from the program each year. This alternative model would see the number of dogs D_n at each stage of the breeding program, n years after the start of 2015 given by

$$D_{n+1} = TD_n + A \text{ where}$$

$$T = \begin{array}{c} \text{this year} \\ \begin{array}{cccc} N & L & B & R \\ \left[\begin{array}{cccc} 0.5 & 0 & 0 & 0 \\ 0.4 & 0 & 0.6 & 0 \\ 0 & 0.7 & 0.2 & 0 \\ 0.1 & 0.3 & 0.2 & 1 \end{array} \right] \begin{array}{l} N \\ L \\ B \\ R \end{array} \end{array} \text{ next year} \quad D_0 = \begin{array}{l} \left[\begin{array}{l} 10 \\ 20 \\ 35 \\ 15 \end{array} \right] \begin{array}{l} N \\ L \\ B \\ R \end{array} \end{array} \quad \text{and} \quad A = \begin{array}{l} \left[\begin{array}{l} 4 \\ 6 \\ 0 \\ -5 \end{array} \right] \begin{array}{l} N \\ L \\ B \\ R \end{array} \end{array}$$

- c. i. According to this alternative model, what would be the net or overall increase in the number of dogs added to the breeding program each year? 1 mark

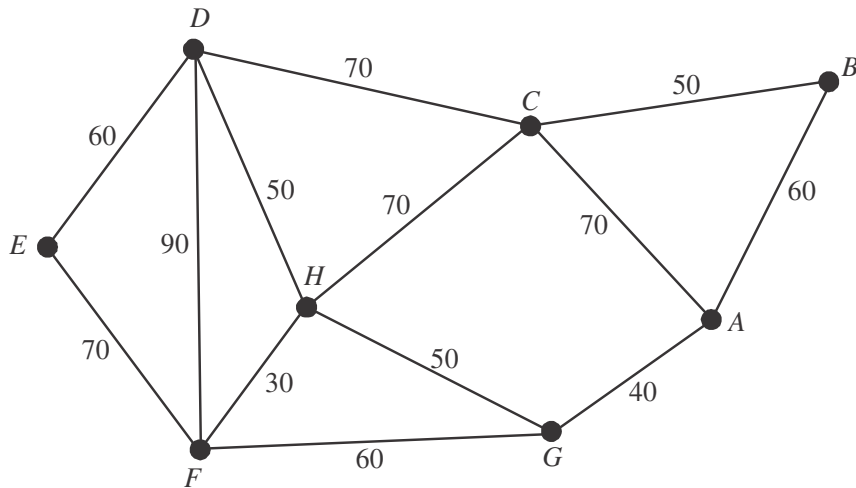
- ii. How many dogs would be producing a litter at the start of 2017? Round your answer to the nearest whole number. 2 marks

Module 2 - Networks and decision mathematics

If you choose this module all questions must be answered.

Question 1 (4 marks)

An industrial complex is comprised of eight buildings. These buildings A, B, C, D, E, F, G and H are shown as vertices on the graph below.



The edges of the graph indicate pedestrian walkways connecting the buildings. The numbers on the edges show the length of these walkways in metres.

- a. What is the length, in metres, of the walkway connecting building C and building H ? 1 mark

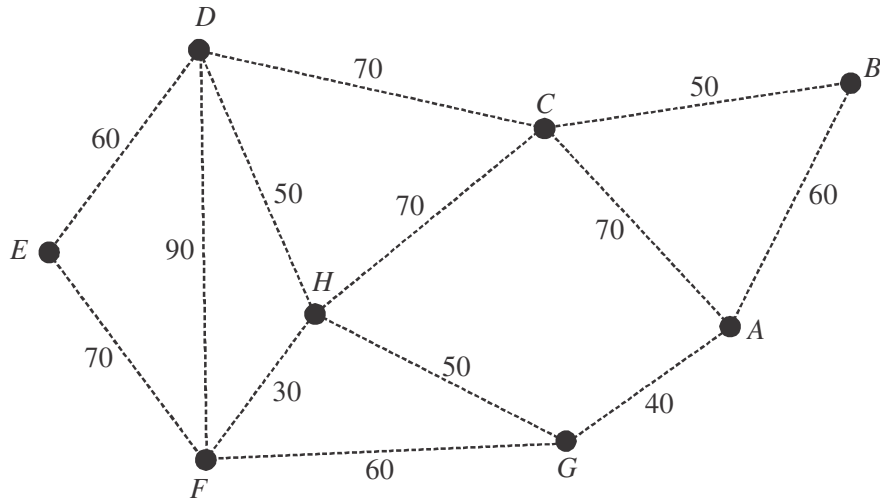
- b. Using the walkways, the manager moves around the complex completing a Hamiltonian cycle starting at building A . How many edges of the graph does this cycle involve? 1 mark

- c. A cleaning machine travels along each of the walkways just once. What is the distance between the starting point and the finishing point of this cleaning machine? 1 mark

Some of the walkways between the buildings are to be closed. Each building must be connected to at least one other building and the total length of walkways remaining open is to be a minimum.

- d. On the diagram below, draw the minimum length of walkways required to keep the eight buildings connected.

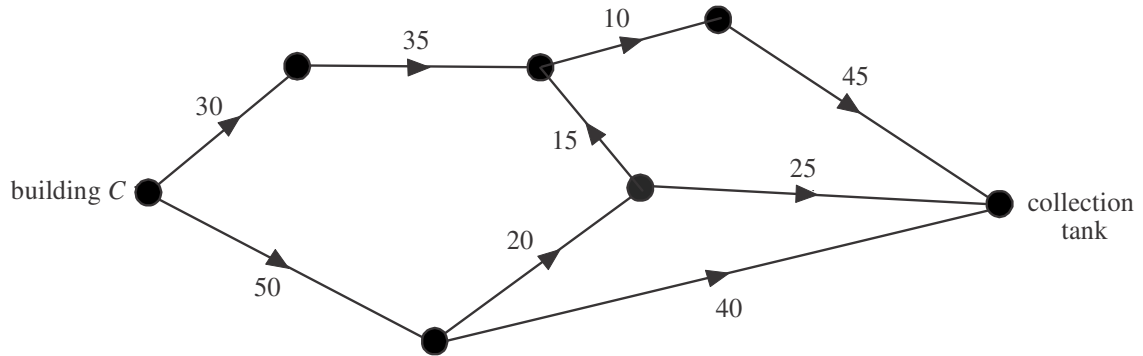
1 mark



Question 2 (2 marks)

Liquid waste produced in building *C* at the complex can flow through a system of pipes to a collection tank.

On the directed network diagram shown below, the numbers on the edges indicate the maximum number of litres of waste that can flow through each of the pipes per hour. The arrows indicate the direction of flow.



- a.** What is the maximum amount of liquid waste, in litres, that can flow from building *C* to the collection tank each hour? 1 mark

- b.** One of the pipes in the system can be removed without affecting the maximum amount of liquid waste that can flow each hour. Indicate clearly on the diagram above which pipe this is. 1 mark

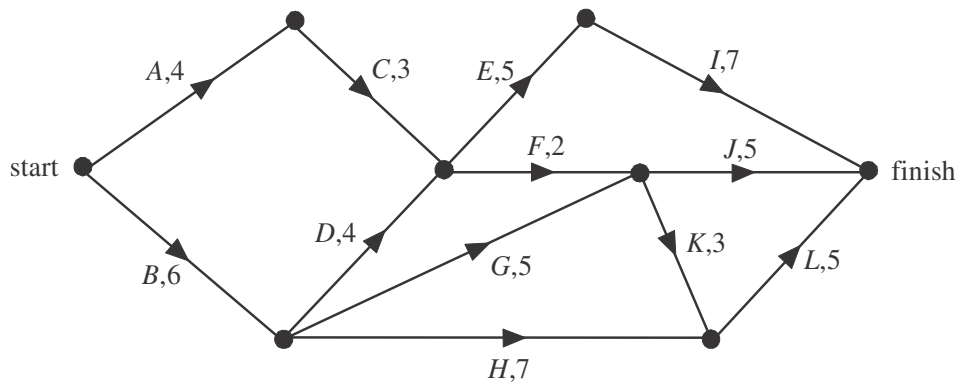
Question 3 (6 marks)

An additional building is to be constructed at the industrial complex. The construction project involves 12 activities *A – L*.

The duration, in weeks, earliest starting time (EST) and immediate predecessor(s) for these activities are shown in the table below.

Activity	Duration	EST	Predecessor(s)
<i>A</i>	4	0	-
<i>B</i>	6	0	-
<i>C</i>	3	4	<i>A</i>
<i>D</i>	4	6	<i>B</i>
<i>E</i>	5	10	<i>C, D</i>
<i>F</i>	2	10	<i>C, D</i>
<i>G</i>	5	6	<i>B</i>
<i>H</i>	7	6	<i>B</i>
<i>I</i>	7	15	<i>E</i>
<i>J</i>	5	12	
<i>K</i>	3	12	<i>F, G</i>
<i>L</i>	5	15	<i>H, K</i>

The directed network below shows these activities.



- a. Write down the immediate predecessor(s) of activity *J*. 1 mark

- b. What is the latest starting time (LST) of activity *C*? 1 mark

- c. What is the minimum completion time in weeks for the construction project? 1 mark

- d. Which activities lie on the critical path? 1 mark

Activity *B* can be crashed by up to 4 weeks at a cost of \$500 per week.

- e. i.** What is the least cost of crashing activity *B* in order to obtain the greatest reduction in the minimum completion time for the project? 1 mark

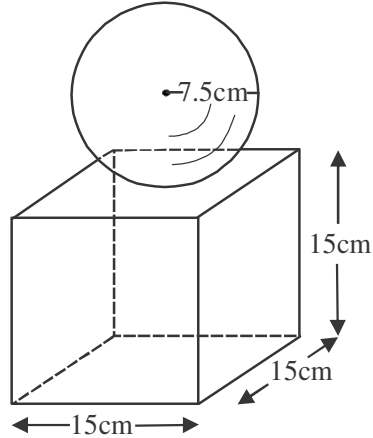
- ii.** Assuming the greatest reduction in the minimum completion time is achieved, which activities are now critical to the completion of the project in the minimum time? 1 mark

Module 3 - Geometry and measurement

If you choose this module all questions must be answered.

Question 1 (2 marks)

A horse racing trophy is in the shape of a sphere resting on a square prism as shown in the diagram below.



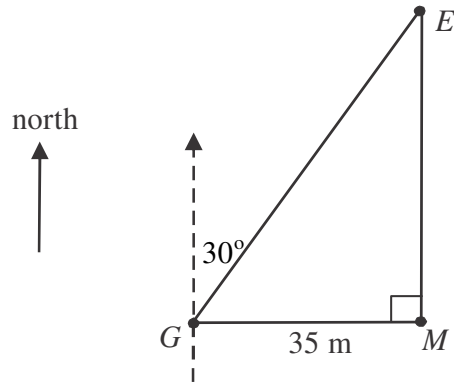
The radius of the sphere is 7.5 cm and the side lengths of the prism are 15 cm.

- a. What is the total height of the trophy? 1 mark

- b. What is the volume of the trophy? Round your answer to the nearest cubic centimetre. 1 mark

Question 2 (2 marks)

At a race track, the mounting yard at point M is 35 m due east of the corner of the grandstand at point G . The entrance to the track at point E is due north of point M . A fence extends from point E to point G and the bearing of point E from point G is 030° as shown in the diagram below.



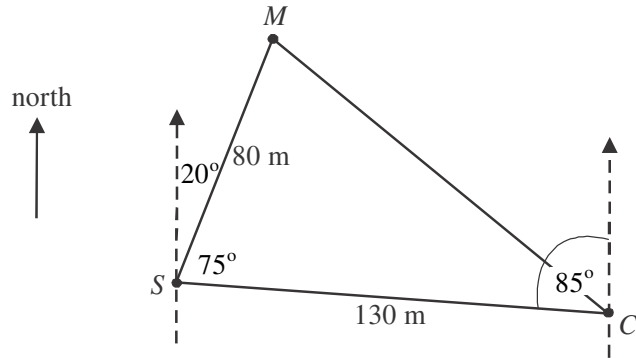
- a. What is the length, in metres, of the fence? 1 mark

- b. What is the bearing of point G from point E ? 1 mark

Question 3 (3 marks)

At the same race track, the mounting yard at point M is 80 m on a bearing of 020° from the stables at point S .

Point S is on a bearing of 275° from the carpark at point C and point C is 130 m on a bearing of 095° from point S as shown on the diagram below.



- a.** Find the distance from point C to point M using the cosine rule. Round your answer to the nearest metre.

2 marks

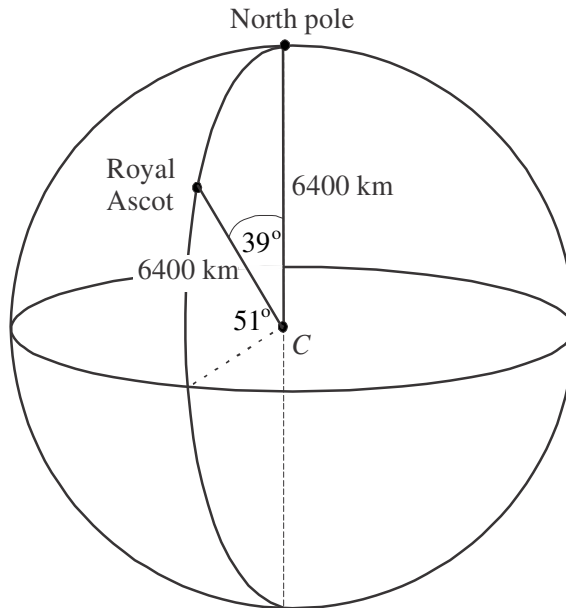
- b.** Find the bearing of point M from point C . Round your answer to the nearest degree.

1 mark

Question 4 (3 marks)

A horse racing carnival is held annually at Royal Ascot racecourse in England located at 51°N and 1°W .

- a. The town of Limburg, in the Netherlands, lies on the same line of latitude as Royal Ascot racecourse. What is the latitude of Limburg? 1 mark

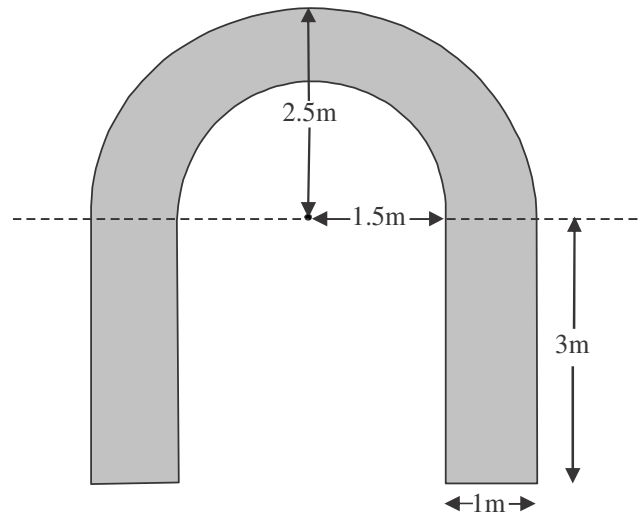


- b. Assuming that the radius of the earth is 6 400 km, find the shortest great circle distance between Royal Ascot racecourse and the north pole. Round your answer to the nearest kilometre. 1 mark

- c. The Queen of England arrives at Royal Ascot racecourse (51°N , 1°W) at 3:15pm one Saturday. A television viewer in Auckland, New Zealand (37°S , 175°E) watches her arrival live. The time difference between Auckland and Royal Ascot racecourse can be assumed to be 13 hours. What is the day and the time when the viewer in Auckland sees the arrival of the Queen? 1 mark

Question 5 (2 marks)

An advertising hoarding at a racetrack is shown below.



The hoarding has, to either side, two identical rectangles of height 3m and width 1m.

The upper curved edge of the hoarding forms a semi-circle of radius 2.5m.

The lower curved edge of the hoarding forms a semi-circle of radius 1.5m.

Find the total surface area of the front of the hoarding, which is shaded above.

2 marks

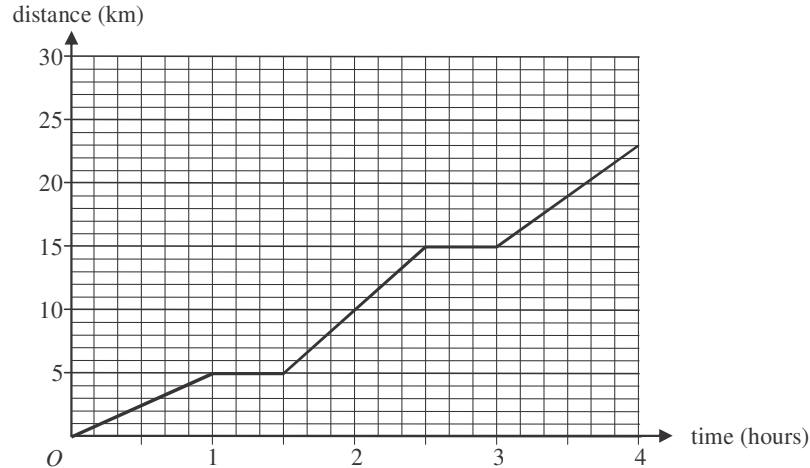
Round your answer to the nearest square metre.

Module 4 - Graphs and relations

If you choose this module all questions must be answered.

Question 1 (4 marks)

Leonie is training for a long-distance trek to raise funds for a charity. The graph below shows the *distance* in kilometres, Leonie travels against the *time*, in hours during one of her training sessions.



- a. What is the total distance, in kilometres, that Leonie travels during this training session? 1 mark

- b. What was the total amount of time, in hours, that Leonie stopped to rest during this training session? 1 mark

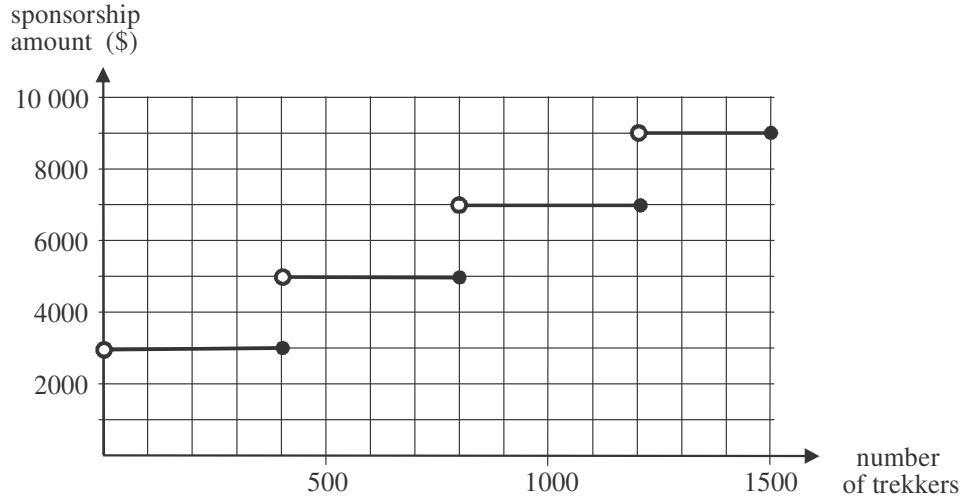
- c. Leonie travelled at three different speeds during her training session. What was the fastest speed Leonie travelled at during her training session. 1 mark

Wendy, who is also preparing for the trek, does a training session that lasts for four hours. During this time she travels at the same speed and takes no rests. Wendy travels six kilometers each hour.

- d. If Wendy and Leonie start their training session together, how many hours into the session would they meet up again? 1 mark

Question 2 (3 marks)

The charity trek has two major sponsors, an insurance company and a bank. The total sponsorship amount, in dollars, offered by the insurance company, is determined by the number of trekkers who participate as shown on the graph below.



- a. What is the least number of trekkers who need to participate in order for the charity to receive \$5000 from the insurance company? 1 mark

The sponsorship amount, in dollars, offered by the bank is given by the rule

$$\text{sponsorship amount} = 2000 + 5 \times \text{number of trekkers}.$$

- b. On the graph shown above, sketch the graph of this relationship between the *sponsorship amount* and the *number of trekkers* who participate. 1 mark

- c. On the day of the trek, the number of trekkers who actually participate is 1200. State whether the insurance company or the bank contributes the greater sponsorship amount and state, in dollars, how much greater the contribution is. 1 mark

Question 3 (5 marks)

Trekkers can opt to participate in a 30 kilometre trek or a 60 kilometre trek.

Let x be the number of trekkers participating in the 30 km trek.

Let y be the number of trekkers participating in the 60 km trek.

There can be a maximum of 1200 trekkers who participate in the 30 km trek.

There must be at least 200 trekkers who participate in the 60 km trek.

The total number of trekkers cannot exceed 1 500.

The inequalities below represent these constraints on the number of each type of trekker participating in the trek.

Constraint 1 $0 \leq x \leq 1\,200$

Constraint 2 $y \geq 200$

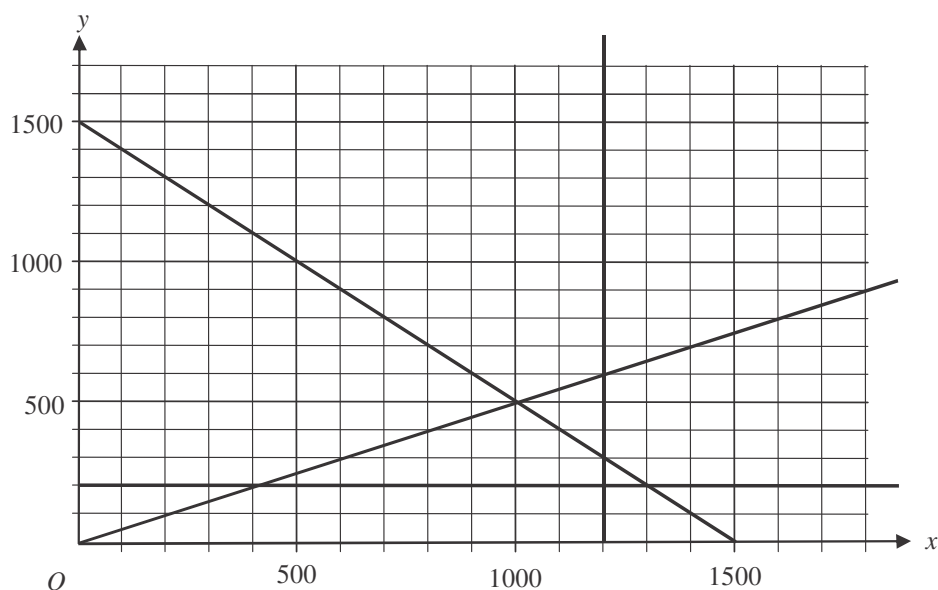
Constraint 3 $x + y \leq 1\,500$

Because of transportation limitations to the starting line, the number of 60 km trekkers participating can only be up to half the number of 30 km trekkers participating.

- a. Write down an inequality that represents this fourth constraint relating to transportation.

1 mark

The graph below shows the boundary lines of the inequalities representing the four constraints.



- b. On the graph above, shade the region that satisfies the four constraints.

1 mark

- c.** What is the maximum number of 60 km trekkers who can participate? 1 mark

On average, each trekker who participates raises \$500 for the charity.

The amount A , in dollars, raised for the charity by the trekkers is given by $A = 500x + 500y$.

- d. i.** Using the sliding-line method, find the maximum amount that can be raised by the trekkers for the charity. 1 mark

- ii.** If the amount raised by trekkers for the charity is a maximum, what is the minimum number of 30 km trekkers who participate in the walk? 1 mark

Further Mathematics formulas

Core - Data analysis

standardised score	$z = \frac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r \frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{\text{effective}} = \left[\left(1 + \frac{r}{100n} \right)^n - 1 \right] \times 100\%$

Module 1 - Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	$S_0 = \text{initial state}, \quad S_{n+1} = TS_n + B$

Module 2 - Networks and decision mathematics

Euler's formula	$v + f = e + 2$
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Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc \sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc \cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base \times height
volume of a pyramid	$\frac{1}{3} \times$ area of base \times height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET

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