THE HEFFERNAN GROUP
P.O. Box 1180
Surrey Hills North VIC 3127
Phone 03 9836 5021
Fax 03 9836 5025
info@theheffernangroup.com.au
www.theheffernangroup.com.au

FURTHER MATHEMATICS
TRIAL EXAMINATION 1
SOLUTIONS
2017

SECTION A
(answers)

Core

1. E	**13.** B
2. C	**14.** D
3. D	**15.** B
4. E	**16.** A
5. D	**17.** B
6. E	**18.** A
7. B	**19.** D
8. E	**20.** E
9. C	**21.** C
10. B	**22.** A
11. A	**23.** C
12. D	**24.** C

SECTION B
(answers)

Module 1 Matrices	**Module 2 Networks & decision maths**	**Module 3 Geometry & measurement**	**Module 4 Graphs & relations**
1. A	**1.** B	**1.** B	**1.** B
2. C	**2.** D	**2.** E	**2.** B
3. B	**3.** C	**3.** D	**3.** E
4. E	**4.** A	**4.** C	**4.** D
5. D	**5.** E	**5.** C	**5.** D
6. B	**6.** E	**6.** D	**6.** C
7. E	**7.** D	**7.** A	**7.** E
8. D	**8.** B	**8.** E	**8.** A

SECTION A – Core - solutions

Data analysis

Question 1

The basic plus model of laptop is shown in the second bar from the left. The percentage of these laptops which are black is 44%.

The answer is E.

Question 2

The variable *model* is an ordinal variable because the various types of model suggest an order in what the model offers, i.e. from basic through to premium plus. The variable *colour* is a nominal variable because it just names the colour – there is no order implied.

The answer is C.

Question 3

Method 1 – use technology

Enter the female participation rate values into your CAS, i.e. 27, 29, 30, 35, 38,… and calculate 1-var stats to find Q_1 and Q_3.

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 63 - 40 \\ &= 23 \end{aligned}$$

The answer is D.

Method 2 – by hand

There are twenty data values. The middle of the lowest ten of these data values lies between 38 and 42. So $Q_1 = 40$. The middle of the highest ten of these data values lies between 61 and 65. So $Q_3 = 63$.

$$\begin{aligned} IQR &= 63 - 40 \\ &= 23 \end{aligned}$$

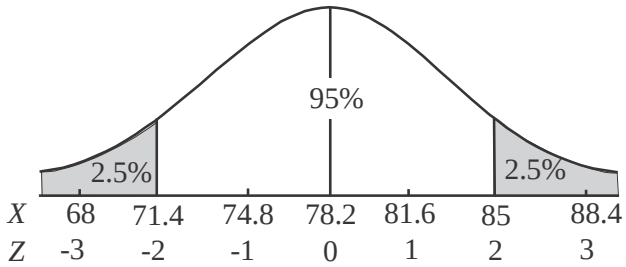
The answer is D.

Question 4

The median value of the distribution of female participation rates is 49 whereas it is 80 for males so the participation rates for females are lower.
The range of the distribution of female participation rates is $83-27=56$ whereas for males it is $93-63=30$ so the participation rate for females is more variable.
Option E includes both these conclusions.
The answer is E.

Question 5

The measurement 85 cm lies two standard deviations above the mean i.e. $78.2+2\times3.4=85$.
We know that 95% of the population lies between two standard deviations below and above the mean. Therefore, because of the symmetry of the normal curve, the remaining 5% lies above and below this, i.e. 2.5% lies above and 2.5% lies below. So 2.5% lies above 85 cm and therefore 97.5% lies below 85 cm



$$\text{So } 97.5\% \text{ of } 386$$
$$=\frac{97.5}{100}\times386$$
$$=376.35$$

The closest answer is 376.
The answer is D.

Question 6

Jane's standard waist measurement of $z=-2.1$ is negative which tells us that Jane's waist measurement is less than the mean (in fact 2.1 standard deviations less!). Option A is true.
To find Jane's waist measurement we use

$$z=\frac{x-\bar{x}}{s_x}\ \text{(formula sheet)}$$
$$-2.1=\frac{x-78.2}{3.4}$$
$$-2.1\times3.4=x-78.2$$
$$-7.14=x-78.2$$
$$-7.14+78.2=x$$
$$x=71.06$$

Jane's waist measurement is 71.06 cm which is greater than 70 cm so option B is true.
Marita's waist measurement is $71.06+10=81.06$ cm. Since 81.06 is greater than 78.2, option C is true.
Marita's standard waist measurement or *z* value, is given by

$$z=\frac{x-\bar{x}}{s_x}\ \text{(formula sheet)}$$
$$=\frac{(81.06-78.2)}{3.4}$$
$$=0.841176...$$

Since 0.841176… is greater than 0.8, option D is true.
Option E is incorrect by elimination but also, we know since Marita's *z* value is greater than 0, then more than 50% of all the women at the gym have a waist measurement less than hers.
The answer is E.

Question 7

0.1 km can also be written as $\frac{1}{10}$ km or 10^{-1} km. The $\log_{10}$ (or index) of 0.1 is therefore – 1

i.e. $\log_{10}(0.1) = -1$ or $\log_{10}(10^{-1}) = -1$.

So the number of people who walked or ran less than 0.1 km equals $1 + 6 = 7$ because we want the $\log_{10}$ (*number of kilometers*) on the horizontal scale to be less than – 1.

The answer is B.

Question 8

Enter the data for 2016 into your CAS. Note that this is the column of figures on the right.

Calculate 1-Var stats.

The mean, $\bar{x}$, is \$11 018.18... . The closest value is 11 018.

The standard deviation s_x is \$2 426.03… . The closest value is 2 426.

The answer is E.

Question 9

You have already entered the data for the 2016 takings at the 11 stores. This was done to complete Question 8. Now enter the data for the 2015 takings at the 11 stores.

Calculate the linear regression $(a + bx)$. The statistics that come up include the value of *r* which equals 0.93586…

The closest answer is 0.9359.

Note that we are **not** asked to calculate the regression line equation so it is not important whether we put the 2015 or 2016 takings as the *x*-variable. The *r* value will be the same either way.

It WOULD however be crucial if we had been asked to find the equation of the least squares regression line. In such a case we would need to be given some information as to which variable, *2015 average weekly takings* or *2016 average weekly takings*, was the explanatory variable.

The answer is C.

Question 10

$$\textit{weight loss} = -40.12 + 0.62 \times \textit{initial weight}$$

When initial weight is 95 kg,

$$\begin{aligned}\textit{weight loss} &= -40.12 + 0.62 \times 95 \\ &= 18.78\end{aligned}$$

So the least squares regression line equation predicts that Jess will lose 18.78 kg.

$$\begin{aligned}\text{residual value} &= \text{actual value} - \text{predicted value (formula sheet)} \\ &= 14 - 18.78 \\ &= -4.78\end{aligned}$$

The closest answer is – 4.8.

The answer is B.

Question 11

The equation of the least squares regression line is $y = a + bx$, where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

(these formulas can be found on the formula sheet).

We know that $\bar{x} = 43.6$, $s_x = 11.6$, $\bar{y} = 75.2$, $s_y = 6.7$ and $r = 0.95$.

$$\begin{aligned}\text{So } b &= r\frac{s_y}{s_x} \\ &= 0.95 \times \frac{6.7}{11.6} \\ &= 0.548706...\end{aligned}$$

$$\begin{aligned}\text{And so } a &= \bar{y} - b\bar{x} \\ &= 75.2 - 0.548706... \times 43.6 \\ &= 51.276379...\end{aligned}$$

So $y = a + bx$ becomes

$$y = 51.276379... + 0.548706... \times x$$

The closest answer is $y = 51.3 + 0.55x$.

The answer is A.

Question 12

Substitute *distance* = 10 into the equation.

$$\frac{1}{annual\ rainfall} = 0.016 + 0.004 \times 10$$

$$\frac{1}{annual\ rainfall} = 0.056$$

$$1 = 0.056 \times annual\ rainfall$$

$$\frac{1}{0.056} = annual\ rainfall$$

So *annual rainfall* is predicted to be 17.8571….
The closest answer is 17.9.
The answer is D.

Question 13

The time series plot does not show seasonality because the peaks (or troughs) occur over different lengths of time, that is, the graph is not periodic (not repeating itself each year in this case).
The time series plot does show a general trend downwards.
Option B best describes the plot.
The answer is B.

Question 14

Quarter	turnover	4 – moving mean	4 – moving mean with centring
4 (2015)	24 000		
1 (2016)	20 000		
		$\frac{(24\,000 + 20\,000 + 14\,000 + 18\,000)}{4}$ $= 19\,000$	
2 (2016)	14 000		$\frac{(19\,000 + 17\,000)}{2}$ $= 18\,000$
		$\frac{(20\,000 + 14\,000 + 18\,000 + 16\,000)}{4}$ $= 17\,000$	
3 (2016)	18 000		
4 (2016)	16 000		

The answer is D.

Question 15

Start by finding the 2015 quarterly seasonal index for June.

Average quarterly payment in 2015 equals

$$\frac{(8\,240+6\,920+10\,360+12\,180)}{4}$$
$$=9\,425$$

2015 quarterly seasonal index for June equals $\frac{6\,920}{9\,425}=0.7342175....$

Now find the 2016 quarterly seasonal index for June.

Average quarterly payment in 2016 equals

$$\frac{(9\,030+8\,450+9\,910+12\,490)}{4}$$
$$=9\,970$$

2016 quarterly seasonal index for June equals $\frac{8\,450}{9\,970}=0.8475426....$

Averaging the 2015 and 2016 quarterly seasonal indices for June gives

$$\frac{0.7342175...+0.8475426...}{2}$$
$$=0.79088...$$

The closest answer is 0.79.
The answer is B.

Question 16

Substitute 3 into the equation.

$$\textit{deseasonalised sales}=430\,600+59\,100\times 3$$
$$=607\,900$$

So deseasonalised sales in quarter 3 of 2016 are predicted to be \$607 900.
Since the quarterly seasonal indices should add to give 4, the missing quarterly seasonal index for quarter 3 is $4-(0.92+1.13+1.01)=0.94$.

$$\text{seasonal index}=\frac{\text{actual figure}}{\text{deseasonalised figure}}\quad\text{(formula sheet)}$$
$$0.94=\frac{\text{actual figure}}{607\,900}$$
$$\text{actual figure}=0.94\times 607\,900$$
$$=571\,426$$

The actual sales for quarter 3 in 2016 are predicted to be \$571 426.
The answer is A.

Recursion and financial modelling

Question 17

Generate the sequence on your CAS. It is $1, -1, -5, -13, -29, ...$
The answer is B.

Question 18

Because the deprecation is flat rate, the equipment will be depreciated by the same amount, i.e. \$800 each year.
The recurrence relation $V_0 = 4\,000,\ V_{n+1} = V_n - 800$ describes this.
The answer is A.

Question 19

The interest earned is $5.2\% \div 4 = 1.3\%$ (i.e. quarterly interest not annual interest).
So 1.3% of $\$11\,268.19 = \$146.49$.
So, Helena's principal increases by a total of $\$500 + \$146.49 = \$646.49$.
The balance required is
$\$11\,268.19 + \$646.49$
$= \$11\,914.68$
The answer is D.

Question 20

Option A is not correct because the 2 500 would need to be added not subtracted.
Option B is not correct because the rule would need to be for example, $V_{n+1} = 0.95V_n$ i.e. the coefficient of V_n should be less than one and no value added or subtracted.
Option C is not correct because the rule would not have a number added or subtracted.
Option D is not correct because the rule would need to be of the form $V_{n+1} = V_n + 200$ for example.
Option E is correct. To confirm this, generate the sequence 125 000, 125 000, 125 000, ….
This constant amount confirms we have a perpetuity.
The answer is E.

Question 21

The annual interest rate is 2.4% so the quarterly interest rate is $2.4\% \div 4 = 0.6\%$.

$$0.6\% = \frac{0.6}{100}$$
$$= 0.006$$

So V_n needs to be multiplied by 1.006.
The required recurrence relation is $V_0 = 6\,000,\ V_{n+1} = 1.006V_n + 450$.
The answer is C.

Question 22

Use finance solver.

N : 24 (2 years = 24 months)
$I\%$: 4.8
PV : –16 000 (Hong gave this to the bank)
Pmt : ?
FV : 25 000 (this is what the bank has to give Hong)
PpY : 12
CpY : 12
Pmt = –294.036...
Note it is negative because Hong is giving this amount to the bank each month.
The closest answer is \$294.04.
The answer is A.

Question 23

Use finance solver.

N : 24 (the difference in the number of months between three and five years)
$I\%$: ?
PV : 211 602.44 (positive because the bank gives this to Tony)
Pmt : –3 200 (negative because Tony gives this to the bank)
FV : –164 377.62 (negative because Tony still has to give this to the bank)
PpY : 12
CpY : 12
I = 7.79999...
The closest answer is 7.8.
The answer is C.

Question 24

Option A shows a constant decrease of \$10 000 per year in the value of the annuity. This cannot be the case because the amount the annuity decreases by, will grow each year. This is because the interest being earned each year is a little less from one year to the next because of the payment each year to Gerry. That is, the interest is being calculated each year on a smaller amount.
We reject option B for the same reason because it has the value of the decrease large to start with and then reducing.
Option D shows a perpetuity whereas this investment is for a finite time.
Option E starts with an investment value of \$0. So we reject it.
The answer is C.

SECTION B - Modules

Module 1 - Matrices

Question 1

The matrix $\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ is a diagonal matrix because its elements are all zero except for those in the leading diagonal.
It is not a unit matrix because of the 3's in the leading diagonal.
It is not a triangular matrix because it has zeros above and below the leading diagonal.
It is not a binary matrix because it contains numbers other than 0 and 1.
It is not a permutation matrix because it is not a binary matrix.
The answer is A.

Question 2

Method 1
Using your CAS.

$$\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}^2 \times 4 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 80 \\ 32 \end{bmatrix}$$

The answer is C.

Method 2 – by hand

$$\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}^2 \times 4 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 20 \\ 5 & 8 \end{bmatrix} \times \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 80 \\ 32 \end{bmatrix}$$

The answer is C.

Question 3

The transpose of $\begin{bmatrix}1 & 2\end{bmatrix}$ is $\begin{bmatrix}1\\2\end{bmatrix}$ not $\begin{bmatrix}2\\1\end{bmatrix}$.

The transpose of $\begin{bmatrix}3 & 8\\1 & -2\end{bmatrix}$ is $\begin{bmatrix}3 & 1\\8 & -2\end{bmatrix}$ as shown.

The transpose of $\begin{bmatrix}2 & 1\\0 & 7\\5 & 4\end{bmatrix}$ is $\begin{bmatrix}2 & 0 & 5\\1 & 7 & 4\end{bmatrix}$ not $\begin{bmatrix}1 & 7 & 4\\2 & 0 & 5\end{bmatrix}$.

The transpose of $\begin{bmatrix}1\\3\\9\end{bmatrix}$ is $\begin{bmatrix}1 & 3 & 9\end{bmatrix}$ not $\begin{bmatrix}9\\3\\1\end{bmatrix}$.

So only one of the pairs show matrices which are the transpose of each other.
The answer is B.

Question 4

For the simultaneous linear equations to have no solution we require that the determinant of the 2×2 matrix equals zero.

$$\begin{aligned}4.2\times 2.5-2.1k&=0\\10.5-2.1k&=0\\k&=5\end{aligned}$$

The answer is E.

Question 5

The final matrix must contain 2 numbers, one which gives the total number of hours worked and one which gives the total amount paid to the employees.
We can therefore have a 1×2 matrix or a 2×1 matrix.
For option A, $(1\times 5)\times(5\times 1)$ gives us a 1×1 matrix.
For option B, same problem.
For option C, $(2\times 5)\times(5\times 1)$ gives us a 2×1 matrix.
The element in the first row will be the sum of all the hours worked but the element in the second row will be the sum of the hourly pay rates whereas we want the sum of the hourly pay rates multiplied by the hours worked.
For option D, $(2\times 5)\times(5\times 1)$ gives us a 2×1 matrix.
The element in the first row gives us the sum of all the hours worked and the element in the second row gives us the sum of the hourly pay rate multiplied by the number of hours worked.

$$\text{i.e. }\begin{bmatrix}1 & 1 & 1 & 1 & 1\\25 & 28 & 24 & 30 & 23\end{bmatrix}\begin{bmatrix}22\\31\\27\\15\\29\end{bmatrix}$$

$$=\begin{bmatrix}124\\3183\end{bmatrix}$$

The answer is D.

Question 6

The columns of a transition matrix must always add to give 1 so we can eliminate option A.
60% of ratepayers who pay at council offices one year do so again the next year so eliminate option D.
10% of ratepayers who pay online one year pay at council offices next year so eliminate option C.
15% of ratepayers who pay by post one year pay at council offices the next year so eliminate option E.
The answer is B.

Question 7

The elements in matrix *A* can be written as $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$.

Test each of these rules.

For option A, $a_{ij} = i - j + 3$

so $a_{11} = 1 - 1 + 3$
$= 3$
$a_{12} = 1 - 2 + 3$
$= 2$

but in matrix *A*, $a_{12} = 6$.
Reject option A.

For option B, $a_{ij} = i + j + 3$

so $a_{11} = 1 + 1 + 3$
$= 5$

but in matrix *A*, $a_{11} = 3$.
Reject option B.

For option C, $a_{ij} = 3i - j + 1$

so $a_{11} = 3 - 1 + 1$
$= 3$
$a_{12} = 3 - 2 + 1$
$= 2$

but in matrix *A*, $a_{12} = 6$.
Reject option C.

For option D, $a_{ij} = 3i + j - 1$

so $a_{11} = 3 + 1 - 1$
$= 3$
$a_{12} = 3 + 2 - 1$
$= 4$

but in matrix *A*, $a_{12} = 6$.
Reject option D.
The answer is E.

Question 8

We have the transition matrix

$$T = \begin{matrix} \\ M \\ S \\ C \end{matrix} \begin{matrix} \begin{matrix} M & S & C \end{matrix} \\ \begin{bmatrix} 0.4 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \\ 0.5 & 0.3 & 0.2 \end{bmatrix} \end{matrix}$$

We are not given the initial state matrix but we know that there are 90 members choosing each month.
Try some different initial state matrices and see what happens over the long term.
For example:

$$\text{Try } S_0 = \begin{bmatrix} 30 \\ 30 \\ 30 \end{bmatrix} \begin{matrix} M \\ S \\ C \end{matrix}$$

$$\text{so } T^{100} \times S_0$$

$$= \begin{bmatrix} 32.59... \\ 26.92... \\ 30.47... \end{bmatrix}$$

$$\text{Try } S_0 = \begin{bmatrix} 5 \\ 60 \\ 25 \end{bmatrix} \begin{matrix} M \\ S \\ C \end{matrix}$$

$$\text{so } T^{100} \times S_0$$

$$= \begin{bmatrix} 32.59... \\ 26.92... \\ 30.47... \end{bmatrix}$$

$$\text{Try } S_0 = \begin{bmatrix} 70 \\ 10 \\ 10 \end{bmatrix} \begin{matrix} M \\ S \\ C \end{matrix}$$

$$\text{so } T^{100} \times S_0$$

$$= \begin{bmatrix} 32.59... \\ 26.92... \\ 30.47... \end{bmatrix}$$

So regardless of the initial state matrices, over the long term there is convergence to the

$$\text{state matrix} = \begin{bmatrix} 32.59... \\ 26.92... \\ 30.47... \end{bmatrix} \begin{matrix} M \\ S \\ C \end{matrix}$$

We can't say whether option A is true or not.
Option B is not true since over the long term 30.47… members will choose to sit and chat whereas 32.59… members will choose to go to the movies.
Option C is not true since 26.92… members will choose to listen to a speaker.
Option D is correct since the least number of members (26.92) will choose to listen to a speaker.
The answer is D.

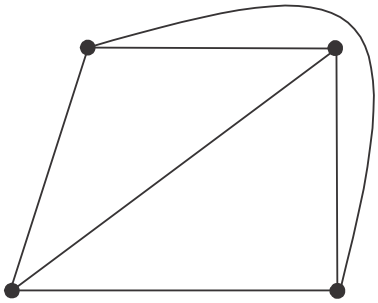
Module 2 - Networks and decision mathematics

Question 1

Jean starts at *B* and finishes at *A* so she cannot have completed a cycle or a circuit so reject options D and E.
A network is a weighted graph (i.e. a graph with numbers on the edges) so reject option C.
Jean visits every vertex but does not walk along every edge so she completes a Hamiltonian path.
The answer is B.

Question 2

The graph is complete because every vertex is connected to every other vertex by an edge.
The graph is planar because it can be redrawn so that no edges cross as shown in the example below.



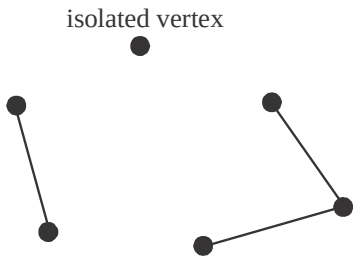
The answer is D.

Question 3

Using trial and error the shortest distance is along the route *A*, *C*, *F*, *I*, *K* which is 19 m.
The answer is C.

Question 4

An isolated vertex is one which is not connected to any other vertex. For the other five vertices **not** to be isolated there needs to be a minimum of 3 edges as shown in the example below.



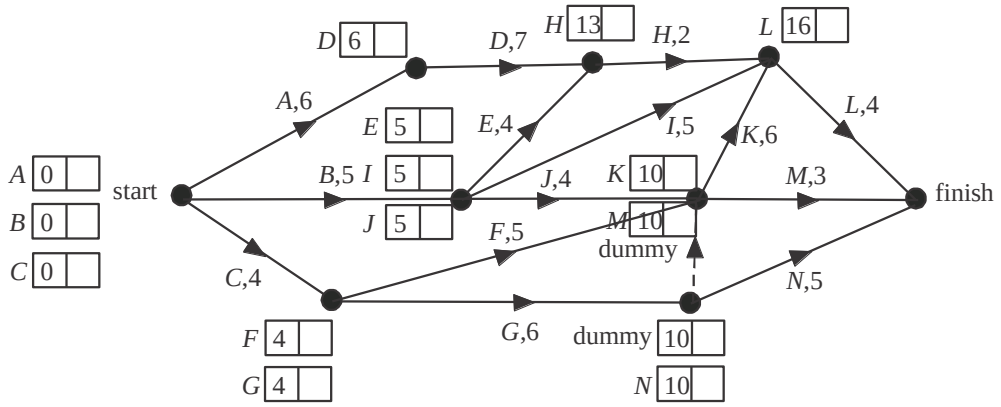
The answer is A.

Question 5

There is a 1 in the leading diagonal of the matrix, i.e. row 2, column 2. This means that there must be a loop at vertex *N*.
We can therefore reject options A, B and D.
Option C has an edge from vertex *M* to *O* which is not indicated in the matrix i.e. there is a zero at row 3, column 1 and also at row 1, column 3, so reject option C.
The answer is E.

Question 6

Do a forward scan to find the EST for activity *L* as shown below.



The earliest start time for activity *L* is 16 weeks.
The answer is E.

Question 7

The immediate predecessors of activity *M* are activities *J*, *F*, and *G* only.
The answer is D.

Question 8

Using the Hungarian algorithm, set up the time matrix.

	V	*W*	*X*	*Y*	*Z*
A	4	5	4	4	5
B	7	8	9	5	6
C	10	11	9	8	12
D	9	7	7	8	9
E	6	5	5	7	6

Subtract the minimum entry in each row from each of the other entries in the row.

	V	*W*	*X*	*Y*	*Z*
A	0	1	0	0	1
B	2	3	4	0	1
C	2	3	1	0	4
D	2	0	0	1	2
E	1	0	0	2	1

Cover all of the zeros with as few lines as possible.

	V	*W*	*X*	*Y*	*Z*
A	0	1	0	0	1
B	2	3	4	0	1
C	2	3	1	0	4
D	2	0	0	1	2
E	1	0	0	2	1

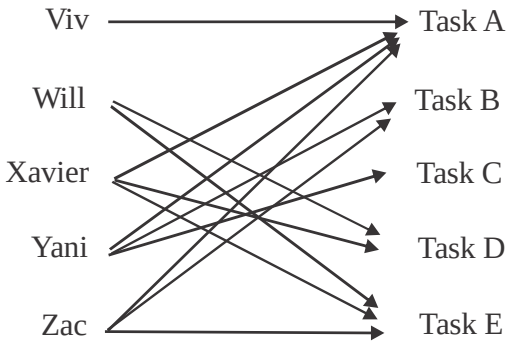
Column Z does not contain a zero so subtract the lowest number in that column, ie 1, from every other value in the column.

	V	*W*	*X*	*Y*	*Z*
A	0	1	0	0	0
B	2	3	4	0	0
C	2	3	1	0	3
D	2	0	0	1	1
E	1	0	0	2	0

Now cover all the zeros with as few lines as possible.

	V	*W*	*X*	*Y*	*Z*
A	0	1	0	0	0
B	2	3	4	0	0
C	2	3	1	0	3
D	2	0	0	1	1
E	1	0	0	2	0

We need five lines to cover all the zeros so we can now allocate.
We can use a bipartite graph to do this.



There are two possible allocations.

Consultant	Viv	Will	Xavier	Yani	Zac
Task allocated	A	D	E	C	B
Completion time	4	7	5	8	6

or

Consultant	Viv	Will	Xavier	Yani	Zac
Task allocated	A	E	D	C	B
Completion time	4	5	7	8	6

In both allocations Viv will finish first in four days. Second to finish will be either Xavier or Will in 5 days.

The answer is B.

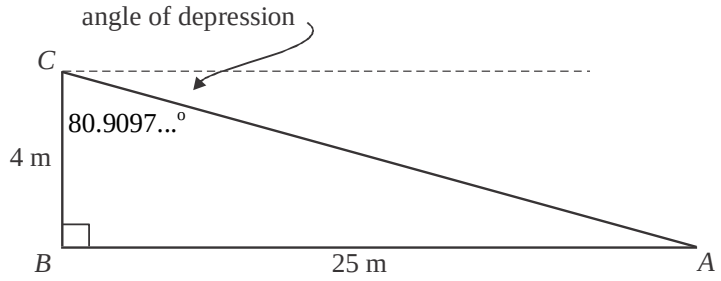
Module 3 - Geometry and measurement

Question 1

$$\text{In } \Delta ABC, \tan(\angle ACB) = \frac{25}{4}$$

$$\angle ACB = \tan^{-1}\left(\frac{25}{4}\right)$$

$$= 80.9097...$$



The angle of depression of A from C is $90 - 80.9097... = 9.0902...$.
The closest answer is 9.1 .
The answer is B.

Question 2

$$\text{area of sector} = \pi r^2 \times \frac{\theta^o}{360} \text{ (formula sheet)}$$

$$= \pi \times 3.5^2 \times \frac{318}{360}$$

$$= 33.9946...$$

The closest answer is 34 cm^2.
The answer is E.

Question 3

$$\text{total volume} = \text{volume of pyramid} + \text{volume of cylinder}$$

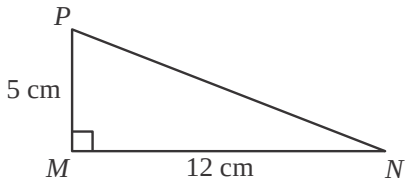
$$= \frac{1}{3} \times \text{area of base} \times \text{height} + \pi r^2 \times \text{height} \qquad \text{(formula sheet)}$$

$$= \frac{1}{3} \times 6 \times 6 \times 8 + \pi \times 7^2 \times 10$$

$$= 1635.3804...cm^3$$

The closest answer is 1 635 cm^3.
The answer is D.

Question 4



In ΔMNP,

$(NP)^2 = 5^2 + 12^2$

$NP = \sqrt{169}$

$NP = 13\,\text{cm}$

In ΔNPT

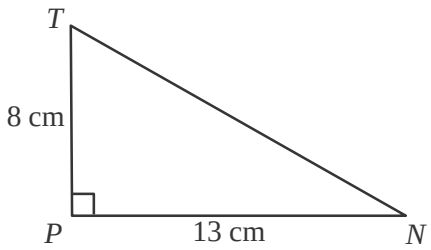
$(NT)^2 = 8^2 + 13^2$

$NT = \sqrt{233}$

$= 15.2643..$

The closest answer is 15.3 cm.

The answer is C.



Question 5

The ratio of the heights of the cones is given by

$h_S : h_L$

$3:6$

which simplifies to $1:2$

Therefore the ratio of the surface area (in our case the bases) of the cones is given by

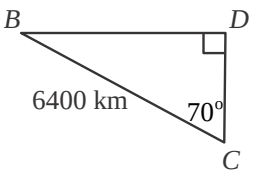
$1:2^2$

which simplifies to $1:4$

Since the area of the base of the smaller cone is 38.5 cm^2, the area of the base of the larger cone is $38.5 \times 4 = 154$ cm^2.

The answer is C.

Question 6



$BC = 6\,400$ km (radius of the Earth)

Point C lies vertically below point D.

Since $\angle ACB = 20$, then $\angle BCD = 70$.

So, in ΔBCD, $\sin(70) = \dfrac{BD}{6\,400}$

$BD = 6\,400 \times \sin(70)$

$= 6\,014.0327...$

The closest answer is 6 014 km.

Note: make sure your answer is reasonable, i.e. the distance we have found is the radius of a small circle of the Earth. The radius of a great circle of the Earth (i.e. the equator or a circle following a line of longitude) is 6 400 km so our answer must be less than that.

The answer is D.

Question 7

Using Heron's formula (from the formula sheet) we have

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)},$$

$$= \sqrt{440(440-275)(440-305)(440-300)}$$

$$= 37042.408...$$

$$s = \frac{1}{2}(a+b+c)$$

$$= \frac{1}{2}(275+305+300)$$

$$= 440$$

The closest answer is 37 042 m^2.
The answer is A.

Question 8

In ΔRST,

$$275^2 = 305^2 + 300^2 - 2 \times 305 \times 300 \times \cos(\angle RTS) \quad \text{(cosine rule - formula sheet)}$$

$$\cos(\angle RTS) = \frac{(275^2 - 305^2 - 300^2)}{(-2 \times 305 \times 300)} \quad \text{(include the brackets)}$$

$$= 0.5868...$$

$$\angle RTS = \cos^{-1}(0.5868...)$$

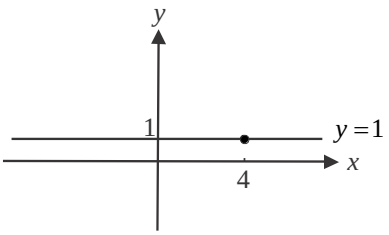
$$= 54.0637...$$

Since point *R* is due west of point *T*, the bearing of point *S* from point *T* is
$270 + 54.0637... = 324.0637...$.
The closest answer is 324°.
The answer is E.

Module 4 - Graphs and relations

Question 1
The patient's resting heart rate was less than 60 beats per minute between 4.5 and 6 minutes, between 6.5 and 8 minutes and between 9.5 and 10 minutes.
This is a total of 3.5 minutes.
The answer is B.

Question 2
The horizontal line that passes through the point $(4,1)$ is shown.
Its equation is $y=1$ because at every point along this line, the *y*-coordinate is 1.
The answer is B.



Question 3
Let *x* be the cost of a dim sim.
Let *y* be the cost of a hotdog.
For Hugh, $5x+3y=13.90$.
For Will, $2x+5y=16.20$.
Use your CAS to solve these equations $x=1.1, y=2.8$.
So a dim sim costs \$1.10 and a hotdog costs \$2.80.
The cost of one dim sim and one hotdog is therefore $\$1.10+\$2.80=\$3.90$.
The answer is E.

Question 4
We can instantly eliminate option C because the original graph is not linear.
For option A, the relationship is $y=\frac{1}{3}x^2$.
The point (3, 27) does not lie on the graph of this function i.e. $27\neq\frac{1}{3}\times 9$.
For option B, the relationship is $y=\frac{2}{1}x^3$.
The point (3, 27) does not lie on the graph of this function i.e. $27\neq 2\times 27$.
For option D, the relationship is $y=\frac{3}{1}x^2$.
The point (3, 27) **does** lie on the graph of this function i.e. $27=3\times 9$.
The answer is D.

Question 5
For option A, if 40 bottles are made $C=85+1.5\times 40=\$145$.
If 40 bottles are sold at \$3.60 each, the revenue will be $40\times 3.60=\$144$ so a loss will be made.
For option B, if 50 bottles are made $C=85+1.5\times 50=\$160$.
If 50 bottles are sold at \$3.20 each, the revenue will be $50\times\$3.20=\$160$ so the seller will break-even.
For option C, if 60 bottles are made, $C=85+1.5\times 60=\$175$.
If 60 bottles are sold at \$2.90 each, the revenue will be $60\times 2.90=\$174$ so a loss will be made.
For option D, if 80 bottles are made, $C=85+1.5\times 80=\$205$.
If 80 bottles are sold at \$2.60 each, the revenue will be \$208 so a profit will be made.
The answer is D.

Question 6

After 10 minutes, Joan has sprayed 100 litres of water on her garden.
So $k = 100$.
Between $t = 10$ and $t = p$ she stopped watering because the amount of water she had sprayed on the garden remained constant i.e. 100 litres.
She still had 100 litres to use up. In her final burst of watering she used up 5 litres per minute.
Now $\frac{100 \text{ litres}}{5 \text{ litres/minute}} = 20 \text{ minutes}$.
That is, she needed to water at this rate for 20 minutes to empty the tank.
So $p = 40 - 20 = 20$.
The answer is C.

Question 7

Choose any point that lies in the feasible region, for example (10, 10).
Substitute this into each of the options.

For option A, $y \le -4x + 20$

At (10,10) $10 \le -40 + 20$

$10 \le -20$ NOT TRUE

For option B, $x \ge 20$

At (10,10) $10 \ge 20$ NOT TRUE

For option C, $y \ge -\frac{1}{2}x + 25$

At (10,10) $10 \ge -\frac{1}{2} \times 10 + 25$

$10 \ge 20$ NOT TRUE

For option D, $y \le 5$

At (10,10), $10 \le 5$ NOT TRUE

The answer is E.

Question 8

The minimum cost can be found by first locating the corner points of the feasible region.
Three of the corner points can be read from the graph. They are (10, 20), (20, 15) and (20, 5).
Solve $y = x + 10$ and $y = -4x + 20$ simultaneously using CAS to find the fourth corner point. It is (2, 12).
Solve $y = -4x + 20$ and $y = 5$ simultaneously using CAS to find the fifth corner point. It is $\left(\frac{15}{4}, 5\right)$.
Substitute each of these points into the objective function.

At (10,20), $C = 4 \times 10 + 2 \times 20 = 80$

At (20,15), $C = 4 \times 20 + 2 \times 15 = 110$

At (20,5), $C = 4 \times 20 + 2 \times 5 = 90$

At (2,12), $C = 4 \times 2 + 2 \times 12 = 32$

At $\left(\frac{15}{4}, 5\right)$, $C = 4 \times \frac{15}{4} + 2 \times 5 = 25$

The minimum value is 25.
The answer is A.