Neap:

Trial Examination 2016

VCE Further Mathematics Units 3&4

Written Examination 1

Suggested Solutions

SECTION A – CORE

Data analysis

Question 1 **A**

$$\text{mean} = \frac{3+3+5+6+7+7+12+12+12+18}{10}$$
$$= \frac{85}{10}$$
$$= 8.5$$

In finding the mean of the data set shown, we are able to find the descriptive statistic 8.5.

Question 2 **C**

$$\frac{84}{1.2} = 70$$

Question 3 **C**

A frequency table would best represent a discrete numerical set of data. The word ‘discrete’ rules out the use of a histogram as these are used for continuous data. A pie chart is best for categorical data and the other two options are for cumulative data.

Question 4 **D**

A negative value of r shows a decreasing association, so the gradient must be negative, but not necessarily –0.9. **B** is referring to the coefficient of determination, r^2. We do know that as one variable increases the other will decrease.

Question 5 **C**

Notice the question asks which is **not** true. The figures for the females are not symmetrical.

Question 6 **D**

Since the changed figure is inside the second quartile, the 5-figure summary is unaffected. The mean is affected by a change in individual values.

Question 7 **D**

$$\frac{58+54+55}{3} = 55.7$$

Question 8 **D**

Substituting $q = 10$ gives $s = 44 + 2.2 \times 10$.

Question 9 **D**

50 is 1 standard deviation below, so there is 34% of the data between it and the mean. 95 is 2 standard deviations above, so there is $\frac{95}{2} = 47.5\%$ between 65 and 95. The total is approximately 81%.

Question 10 E

Notice the question asks which is **inaccurate**. Even though the percentage of women who voted no is much higher than for men, the actual number of 20 is less than the 30 men who voted no.

Question 11 A

Deseasonalising is the only method that will achieve this purpose – deseasonalise the data and replot the figures.

Question 12 D

One method would be to enter the data into L_1 and L_2 on your calculator. Make $L_3 = L_1^2$ and then use L_3 and L_2 to find *r*.

Question 13 B

The number of hospital beds and the number of fast-food outlets both respond to a third variable: the population of the town.

Question 14 C

Calculate the fences. Find the IQR, Q1 and Q3 of the data.

5, 14, 16, 18, 20, 22, 25, 37

Median is 19, Q1 = 15 and Q3 = 23.5.

IQR is 8.5. Fences are at 15 – 1.5 × 8.5 = 2.25 and 23.5 + 1.5 × 8.5 = 36.25.

The 37 is outside the fence so is an outlier.

The range of the remaining is 25 – 5 = 20.

Question 15 B

The Q1 for data set A is 160 and Q2 for data set B is only 150, so at least 50% of the data in data set B is less than Q1 of data set A.

Question 16 C

By adding the frequency of each column, we see there are 29 pieces of data. The median is therefore the 15th piece of data, which will be the highest value in the 1–10 column. Since 7 and 9 are the only alternatives in that column, it must be 9.

Recursion and financial modelling

Question 17 C

$$P_1 = 5 \times 5 - 24$$
$$= 1$$
$$P_2 = 5 \times 1 - 24$$
$$= -19$$
$$P_3 = 5 \times 5 - 24$$
$$= -119$$

Question 18 A

Substitute into the financial section of your calculator. $PV = -230\ 000$, $FV = 0$, $r = 5\%$, $n = 120$ and there are 12 repayments per year. Solve to find the repayment.

Question 19 E

$$R = 1 - \frac{r}{100}$$
$$= 0.977$$

This rate is multiplied by the balance each year for 4 years.

Thus $M_4 = M_0 \times 0.977 \times 0.977 \times 0.977 \times 0.977$.

Question 20 D

The key here is simple interest. The simple interest for 4 years is $2000 \times 0.05 \times 4$. This must be added to the original principal to get the balance after 4 years.

Question 21 A

The y-intercept is 18 and therefore $P_0 = 18$. The value of each term reduces by 2, so $P_{n+1} = P_n - 2$.

Question 22 D

$R = 1 + \frac{r}{100} = 1.04$, $t_0 = 25\ 000$

The multiplication by 1.04 must be followed by an addition of 10 000 in that order.

Question 23 E

Substitute into the formula for effective interest rate.

$$r_{effective} = \left[\left(1 + \frac{r}{100n}\right)^n - 1\right] \times 100\%$$
$$= \left[\left(1 + \frac{4.5}{100 \times 12}\right)^{12} - 1\right] \times 100\% = 4.59\%$$

Question 24 B

Calculate using the financial package on the calculator for $n = 5 \times 365$ with $r = \frac{5}{365}$ and $n = 60$ with $r = \frac{5}{12}$. The difference is \$96.88.

SECTION B – MODULES

Module 1 – Matrices

Question 1 D

$P_{1,\,2}$ refers to the element of the matrix in row 1 and column 2. Thus the answer is 2.

Question 2 A

To find the inverse of any matrix, we must first find the determinant.

$$\begin{aligned}\Delta &= 2 \times 2 - 4 \times 1 \\ &= 0\end{aligned}$$

Immediately we see a problem. The matrix is singular and no inverse exists.

Question 3 B

$$\begin{aligned} PQ + Q &= (P + I)Q \\ &= \begin{bmatrix} 2 & 2 & 4 \\ 3 & 1 & -1 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 0 \\ 4 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 26 & 10 \\ 5 & 1 \\ 8 & 7 \end{bmatrix}\end{aligned}$$

Question 4 B

A permutation matrix is a square matrix and so it can be multiplied by itself, thus P^2 is defined.

A permutation matrix can be represented by a series of arrows from source to destination. Each source has exactly 1 arrow departing and each destination has exactly 1 arrow arriving. P^2 carries this process out twice, so there will still be the same 1 arrow leaving from each source and 1 arrow arriving at each final destination. Thus it is a permutation matrix.

Question 5 C

We solve by premultiplying each side by the inverse of $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$.

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}^{-1} = \frac{1}{5}\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

$$\begin{aligned}\therefore Q &= \frac{1}{5}\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -6 & 4 \end{bmatrix} \\ &= \frac{1}{5}\begin{bmatrix} 34 & -4 \\ -16 & 6 \end{bmatrix}\end{aligned}$$

Question 6 **B**

First, we find the transformation matrix.

$$T = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix}$$

$$S_0 = \begin{bmatrix} 800 \\ 960 \end{bmatrix}$$

$$\begin{aligned} S_2 &= T^2 S_0 \\ &= \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix}^2 \begin{bmatrix} 800 \\ 960 \end{bmatrix} \\ &= \begin{bmatrix} 992 \\ 768 \end{bmatrix} \end{aligned}$$

Question 7 **E**

$$\begin{aligned} T^{50} &= T^{51} \\ &= \begin{bmatrix} 0.625 & 0.625 \\ 0.375 & 0.375 \end{bmatrix} \end{aligned}$$

Thus the steady state $S_\infty = \begin{bmatrix} 0.625 & 0.625 \\ 0.375 & 0.375 \end{bmatrix} \begin{bmatrix} 800 \\ 960 \end{bmatrix}$

$$= \begin{bmatrix} 1100 \\ 660 \end{bmatrix}$$

Question 8 **A**

Each month San Juan loses 4 books and Shaw gains 4 books when compared to the previous arrangement.

This change is noted after books are returned and thus the alteration can be made after the transformation matrix is applied.

Module 2 – Networks and decision mathematics

Question 1 E

Many of the options have the correct set of edges between different vertices. Options **B** and **E** are different in that they also have a loop at vertex *A*.

Option **E** is similar to option **B**, but treats vertex *A*'s loop as 2 edges. This is correct as both ends of the edge must be counted.

Question 2 C

To have an Eulerian circuit, all vertices must have an even degree.

Options **A**, **B** and **D** all have 2 odd vertices. Option **E** has 4 odd vertices. Only option **C** has all even vertices.

Question 3 D

Options **A** and **B** are not trees as they contain circuits. Option **C** is not spanning as 1 vertex is not connected. Option **E** contains edges not present in the original graph.

Question 4 B

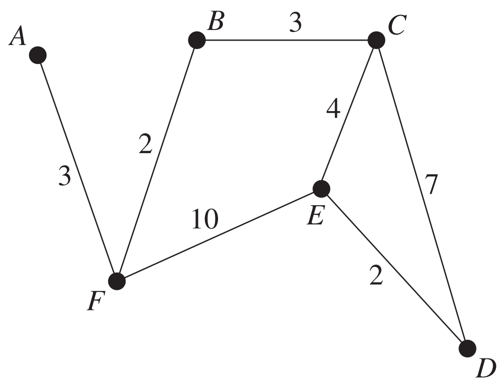
The sum of the vertices of any graph must be even. The answer is clear from this. The only other possible consideration is whether the graph has enough edges, but that is not a problem as the question does not state that the graph is connected.

Question 5 B

Edges can be removed when a shorter, equal-length path exists between the vertices at each end. Thus *FD* can be removed as travelling *FED* is the same.

Likewise, *AB* can be discarded as *AFB* is shorter. On the same basis, *BE* is removed in favour of *BCE*.

The graph below shows the result after this is done.



FE can now be removed as *FBC* is shorter, and *CD* is removed as *CED* is shorter.

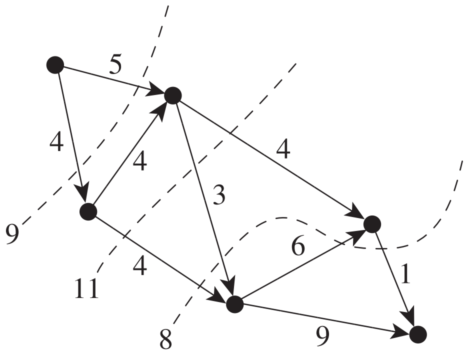
The best path is now clearly *AFBCED*, a length of 14.

Question 6 E

Use $V + f = e + 2$.

If V and f increase by a total of 5, then $e + 2$ must also increase. Thus e increases by 5.

Question 7 B



Three examples of cuts are shown above. We are seeking the smallest cut, so we choose the one that adds up to 8. At first glance it may appear that the value of this cut should be 14, as it cuts 4, 3, 6 and 1, but the '6' is an edge directed away from the source and cannot be counted.

Question 8 A

A logical process can be employed to allocate and eliminate.

Household *C* must choose foreign video, as nobody else gets it. Household *D* must choose netview as that is the only service they get. Household *A* must choose arnie TV as that is all they get. That leaves household *B* as getting cable TV.

Module 3 – Geometry and measurement

Question 1 **A**

$$l = \frac{60}{360} \times 2\pi \times 4$$
$$= \frac{4\pi}{3}$$
$$\cong 4.19$$

The other options focus mainly on common errors, such as finding whole circumference (option **D**).

Question 2 **C**

50 m is 5000 cm.

Since map lengths are $\frac{1}{2500}$ those in the real world, the map length of the road is $\frac{1}{2500} \times 5000 = 2$ cm.

Question 3 **B**

$$A = \frac{1}{2}ab\sin(C)$$
$$= \frac{1}{2} \times 8 \times 5\sin(60)$$
$$= 17.3 \text{ cm}^2$$

Question 4 **D**

The separation in longitude is 35° as one location is west and the other location is east.

The great circle that applies is not that of the entire globe either. The radius is 6400cos(12°), not 6400.

Thus $d = \frac{35}{180} \times 2\pi \times 6400\cos(12)$.

Question 5 **E**

Using cm as the base unit, first we find the length ratio. It is $\frac{360}{12} = 30$.

The volume ratio is thus $30^3 = 27\,000$.

Question 6 **E**

We apply the sine rule.

$$\frac{\sin(55)}{6} = \frac{\sin(\theta)}{7}$$
$$\sin(\theta) = 0.9557$$
$$\theta = 72.9,\ 180 - 72.9$$

Thus two possible angles exist. 72.9° is not listed and thus we choose 107.1°.

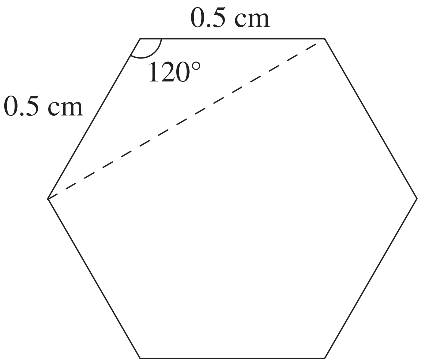
Question 7 C

Apply the cosine rule.

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{120^2 + 120^2 - 150^2}{2 \times 120 \times 120}$$

Question 8 D



The above diagram shows the length for a single hexagon.

Using the cosine rule:

$$l^2 = 0.5^2 + 0.5^2 - 2 \times 0.5 \times 0.5 \cos(120)$$

$$= 0.75$$

$$l = \sqrt{0.75}$$

$$\cong 0.866$$

Thus 6 hexagons is $6 \times 0.866 = 5.20$ cm.

Module 4 – Graphs and relations

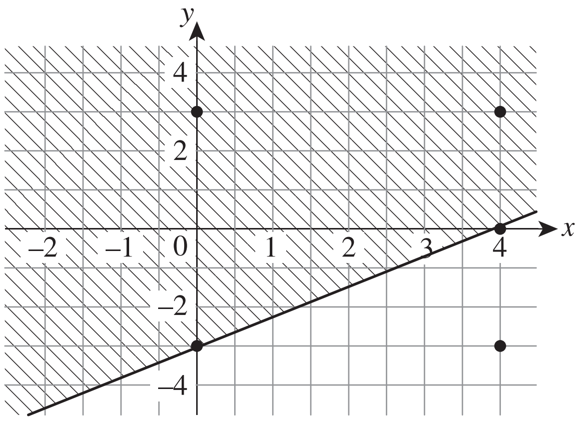
Question 1 C

We are looking for a section of the graph for which the gradient is 0. We want the line to be flat, indicating no change in position.

Between 1.25 and 1.50, the line is flat and we can say that she stopped for lunch between these times.

Question 2 D

Students can either try the points by substituting them into the inequation or can check the point on a graph.



As can be seen from the graph above, only one point of those given among options **A** to **E** lies outside the required region.

Question 3 D

Firstly, determine the gradient. Use intercepts.

$$m = \frac{5 - 0}{0 - (-2)}$$

$$= 2.5$$

This rules out options **A** and **B**, as neither have this gradient.

Options **C**, **D** and **E** can be checked by substituting the intercepts. Both **C** and **D** have the correct y-intercept, but only **D** has the correct x-intercept.

Question 4 C

The number of girls can be 10 more or 10 less than the number of boys. Those are the two constraints.

Options **A** and **B** have only one of those each. Option **C** has both constraints. Options **D** and **E** multiply and divide instead of adding and subtracting 10.

Question 5 E

We know that the equations are correct. We can also see that the graphs are exactly as they appear on the calculator screen. Thus we can rule out options **C** and **D**.

The problem with the graph is that it makes no allowance for the fact that passengers cannot travel within a non-integer number of regions. Thus the graphs should appear as a series of points, not straight lines.

Question 6 B

Students can either calculate the objective function at each of the three points or draw test lines across the graph.

At $V(15, 0)$, $C = 3 \times 15 + 4 \times 0$
$= 45$

At $W(8, 5)$, $C = 3 \times 8 + 4 \times 5$
$= 44$

At $Z(4, 11)$, $C = 3 \times 4 + 4 \times 11$
$= 56$

Thus we see that W gives the minimum.

Question 7 D

The number of hours spent assembling standard mowers is $8x$, while $13y$ hours are spent assembling deluxe mowers.

Thus a total of $8x + 13y$ hours are spent assembling mowers.

We have a total of $8 \times 10 = 80$ available to assemble all mowers and thus $8x + 13y \leq 80$.

Question 8 C

The objective is to maximise profit.

The profit is $420x$ on the standard mowers and $790y$ on the deluxe mowers, making a total of $420x + 790y$ profit.

$x + y$ would be the total number of mowers, not the total profit.

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