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Year 12 *Trial Exam Paper*

2016

FURTHER MATHEMATICS

Written examination 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A – Core	9	9	36
B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students must write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question book of 35 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All responses must be written in English.

Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.

SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided. Write using blue or black pen.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (3 marks)

The following table shows data obtained by the Australian Bureau of Statistics in 2013, regarding the percentage of male and female salary earners in each age group.

	Age (years)					
Gender	**15–24**	**25–34**	**35–44**	**45–54**	**55–64**	**65 and over**
Male	51.0%	51.9%	51.9%	49.6%	52.2%	61.9%
Female	49.0%	48.1%	48.1%	50.4%	47.8%	38.1%

a. In the 55–64 years age group, 329 890 people were surveyed.

Find the number of males surveyed in this age group.

1 mark

_____

b. From the two-way table above, it appears that there is no association between *Gender* and *Age*.

Explain why, quoting appropriate percentages to support your explanation.

2 marks

_____

_____

_____

_____

Question 2 (2 marks)

In 2014, the age of the population of Australia was approximately normally distributed with a mean of 37.3 years and a standard deviation of 12.2 years.

The population of Australia in 2014 was estimated to be 22 507 617.

a. How many people are expected to be between the ages of 25.1 and 61.7 years?

1 mark

_____

b. Determine the standardised score (z-score) for an Australian person aged 27 years. Give your answer correct to one decimal place.

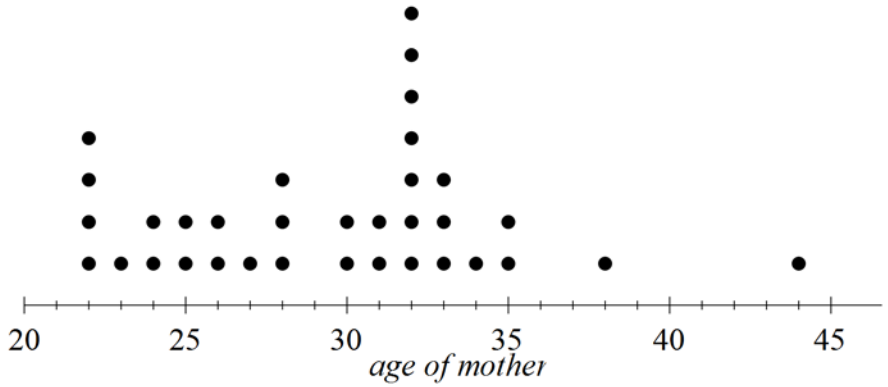
1 mark

_____

Question 3 (4 marks)

A sample of mothers from a local secondary school were surveyed regarding their age at the birth of their first child.

The dot plot below displays the values of the ages for the sample of 24 mothers.



a. The variable *age of mother* could best be described as what type of data?

1 mark

_____

b. Using the information in the dot plot, determine each of the following.

1 mark

The median ☐ The range ☐

c. Write down an appropriate calculation and explain why the mother who was aged 44 years at the birth of her first child is an outlier for this group.

2 marks

_____

_____

_____

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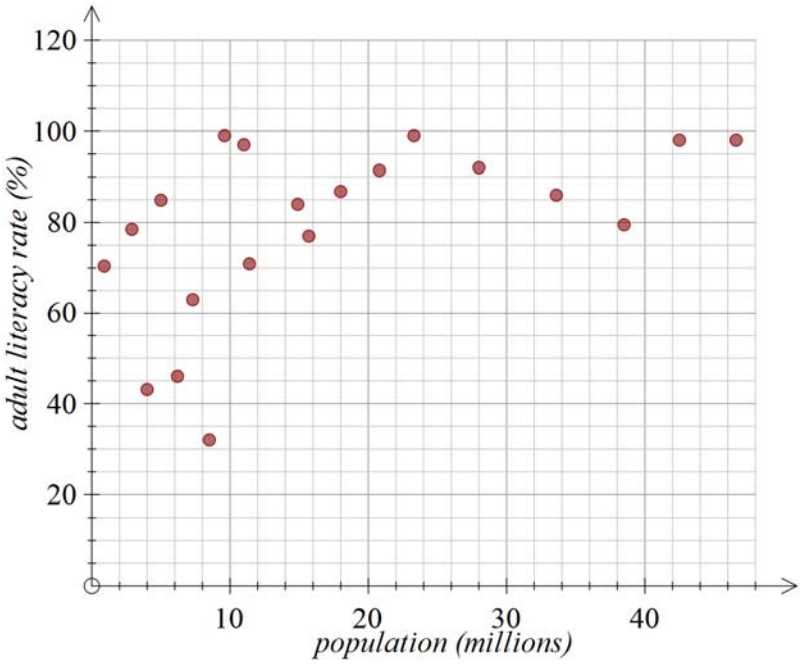
_____

Question 4 (6 marks)

The scatterplot and table below show the *population* (as measured in millions) and *adult literacy rate* (as a percentage) of a sample of countries from around the world.



Population **(millions)**	***Adult literacy rate* (%)**
42.5	98
8.5	32
14.9	84
11	97
15.7	77
33.6	86
5	84.9
7.3	63
6.2	46
46.6	98
38.5	79.5
11.4	70.9
0.9	70.4
20.8	91.4
28	92
9.6	99
2.9	78.5
18	86.8
23.3	99
4	43.1

The equation of the least squares regression line for the data is

$$\textit{adult literacy rate} = 65.7 + 0.8 \times \textit{population.}$$

The correlation coefficient, r, is equal to 0.531.

a. Write down the explanatory variable.

1 mark

_____

b. Comment on the strength of the relationship between *population* and *adult literacy rate*.

1 mark

_____

c. Interpret the slope of this least squares regression line in terms of the variables *population* and *adult literacy rate*.

2 marks

_____

_____

_____

d. Sierra Leone has a population of 6.2 million people and an adult literacy rate of 46%.

i. Calculate the residual value when the least squares regression line is used to predict the *adult literacy rate* for Sierra Leone from its population.

1 mark

_____

_____

_____

_____

ii. What percentage of the variation in the adult literacy rate of the countries sampled is explained by the variation in population?

Round your answer to one decimal place.

1 mark

_____

_____

Question 5 (4 marks)

The relationship between *population* and *adult literacy rate* was found to be non-linear.

A **squared** transformation can be applied to the variable *adult literacy rate* to linearise the scatterplot.

a. Apply the **squared** transformation to the data and determine the equation of the least squares regression line that allows the square of the *adult literacy rate* to be predicted from the *population*.

Write the slope and intercept of this least squares regression line in the boxes provided below.

Round your answers to three significant figures.

2 marks

$(\textit{adult literacy rate})^2 =$ ☐ $+$ ☐ $\textit{population}$

b. Use the equation of the least squares regression line in **part a.** to predict the adult literacy rate for a country with a population of 11.4 million.

Round your answer to the nearest percentage.

1 mark

_____

c. Is the prediction made in **part b.** an example of interpolation or extrapolation?

1 mark

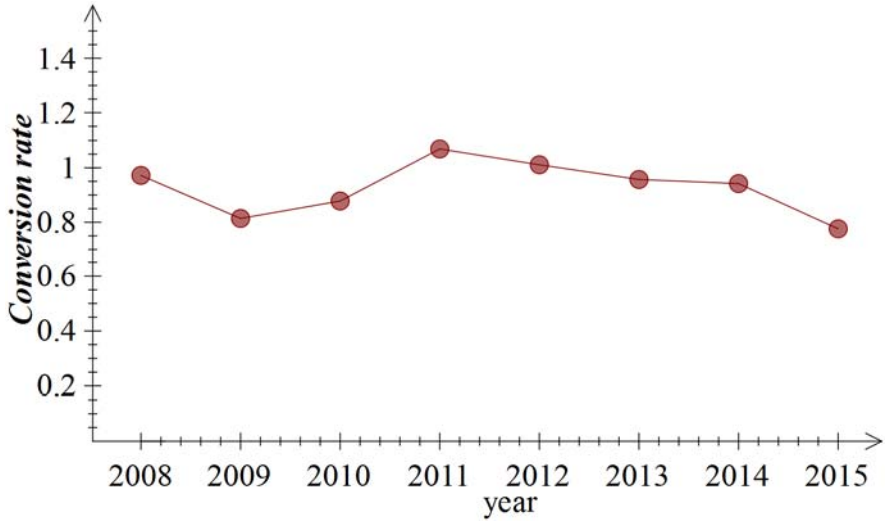
_____

Question 6 (5 marks)

The table below shows the conversion rate for Australian dollars (AUD) to United States dollars (USD) in June of each year, for the period 2008 to 2015.

Year	2008	2009	2010	2011	2012	2013	2014	2015
Conversion rate	0.972	0.814	0.878	1.069	1.011	0.957	0.942	0.776

This data is also displayed in the time series plot below.



a. Describe the general trend in the data.

1 mark

_____

b. Use the variables *year* and *conversion rate* to write down the equation of the least squares regression line that can be used to predict *conversion rate* from *time*.

Write the coefficients correct to three significant figures.

Re-label the axis for time, using 1 for 2008, 2 for 2009 etc.

2 marks

_____

c. Use the regression equation in **part b.** to predict the currency exchange rate from AUD to USD in 2017. Write your answer correct to two decimal places.

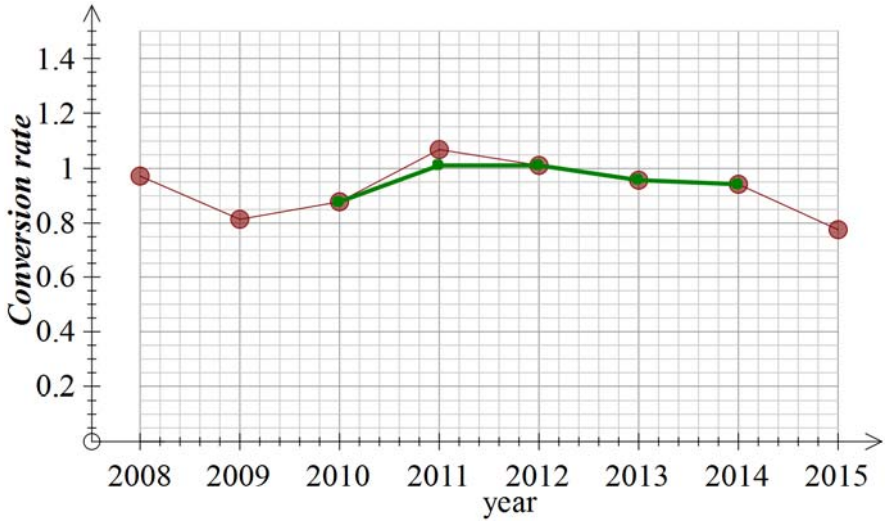
1 mark

_____

_____

_____

d. The time series plot below shows the three-median smoothed data for the information displayed in the table above.



Complete the graph above, by placing the three-median smoothed data value for 2009 onto the plot.

1 mark

(*Answer on the graph above*.)

Recursion and financial modelling

Question 7 (5 marks)

David is a small-business owner who has purchased an industrial printing machine for $56 000. He is investigating the best method to depreciate his machine to maintain the best resale value.

Initially, David's printing machine will be depreciated using the flat rate method.

The value of David's printing machine, V_n , in dollars, after *n* years can be modelled using the recurrence relation below.

$$V_0 = 56\ 000, \qquad V_{n+1} = V_n - 5880$$

a. Using the recurrence relation, write down calculations to show that the value of David's printing machine after three years is $38 360.

1 mark

_____

_____

_____

b. David will consider selling his printing machine when it reaches a value of $20 000 or less.

After how many years will David consider selling his printing machine?

1 mark

_____

_____

_____

_____

The unit cost method can also be used to depreciate the value of David's printing machine.

A rule for the value of the machine, in dollars, after printing n pages is

$$V_n = 56\ 000 - 0.15n$$

c. What is the value of the printing machine after 7 years if it prints 35 000 pages per year?

1 mark

_____

David has also considered using a reducing balance model of depreciation for his printing machine.

His friend has suggested an arrangement that depreciates the printing machine at a rate of 12.5% per annum, compounding quarterly.

d. Write down a recurrence relation, in terms of V_n, that models the value of David's printing machine after n months.

2 marks

_____

_____

_____

_____

_____

Question 8 (4 marks)

To expand his business, David must borrow $115 000 from the bank. He enters into a loan where interest is calculated monthly, as modelled by the recurrence relation below

$V_0 = 115\ 000$, $V_n = 1.008^n \times 115\ 000$

a. What is the annual interest rate for David's bank loan?

1 mark

b. If David does not make any payments, what would the value of the loan be after 18 months?

1 mark

c. David is able to make a repayment of $1250 per month, immediately after interest has been calculated. What is the value of his loan after 6 months, to the nearest cent?

1 mark

d. What is the effective annual interest rate, correct to two decimal places, for David's loan?

1 mark

Question 9 (3 marks)

Rita needs to buy a new car.

She invested $15 000 in a bank account to save enough money to buy the car, which is valued at $28 000. The bank account will pay interest monthly at a rate of 6.9% per annum.

a. How much additional money will Rita have to save each month to have enough money to buy her car after 12 months?

1 mark

_____

_____

_____

b. How much interest will Rita's investment earn over the 12 months?

1 mark

_____

_____

_____

c. After 6 months, Rita needs to reduce her monthly investment to $750.

How many months will it now take Rita to save $28 000 (including the first 6 months of additional investments calculated in **part a.**)?

1 mark

_____

_____

_____

END OF SECTION A
TURN OVER

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

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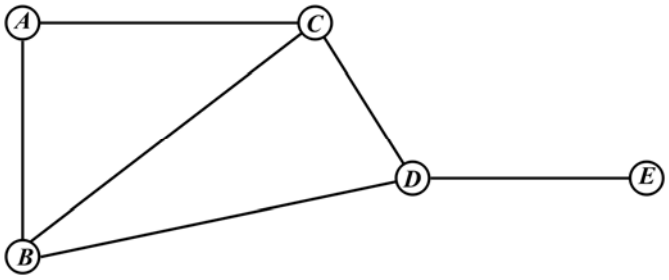
Module 1 – Matrices

Question 1 (2 marks)

A group of Year 9 girls (Abbey, Belinda, Carly, Diane and Erica) and their friendships with each other are represented in the diagram below.



The matrix F is used to represent the information in this diagram.

$$F = \begin{array}{c} \begin{matrix} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{matrix} \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{matrix}$$

In matrix F:

- the 1 in row 2, column 1, for example, indicates that Abbey and Belinda are friends.
- the 0 in row 6, column 2, for example, indicates that Belinda and Erica are not friends.

a. In terms of the friendships among the girls, what does the sum of the elements in row 3 of matrix F represent?

1 mark

_____

_____

b. Explain the meaning of the diagonal of 0 elements in matrix F.

1 mark

_____

_____

Question 2 (2 marks)

Abbey, Carly and Diane go to the school canteen for lunch together.

Abbey bought one chocolate milk and two sausage rolls for \$7.00, Carly bought one chocolate milk and one pizza slice for \$5.00, and Diane bought one sausage roll and a slice of pizza for \$5.50.

This information is represented by the following equations, where m is chocolate milk, s is sausage roll and p is pizza

$$m + 2s = 7$$
$$m + p = 5$$
$$s + p = 5.5$$

a. The equations can be represented using matrices.

Complete the matrix equation below by entering the missing elements.

$$\begin{bmatrix} 1 & 2 & \square \\ 1 & 0 & 1 \\ 0 & \square & 1 \end{bmatrix} \begin{bmatrix} m \\ s \\ p \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ 5.5 \end{bmatrix}$$

1 mark

(*Answer in matrix above.*)

b. Find the cost of one slice of pizza.

1 mark

_____

Question 3 (8 marks)

Teachers at a local school will run four workshops once a week to assist with revision for end-of-year exams. The workshops will focus on Mathematics, Science, English and Humanities.

Students sign up for one workshop per week. Their choice of workshop, from week to week, can be represented by the transition matrix T, where

$$T = \begin{array}{c} \\ \text{M} \\ \text{S} \\ \text{E} \\ \text{H} \end{array} \begin{array}{c} \begin{array}{cccc} \text{M} & \text{S} & \text{E} & \text{H} \end{array} \\ \begin{bmatrix} 0.45 & 0.38 & 0.55 & 0.15 \\ 0.35 & 0.42 & 0.13 & 0.11 \\ 0.12 & 0.10 & 0.17 & 0.28 \\ 0.08 & 0.10 & 0.15 & 0.46 \end{bmatrix} \end{array} \begin{array}{l} \text{M} \\ \text{S} \\ \text{E} \\ \text{H} \end{array}$$

this week

next week

a. Use the information in the transition matrix T to

i. interpret the element in row 3, column 2 in the context of the students' selection of workshops

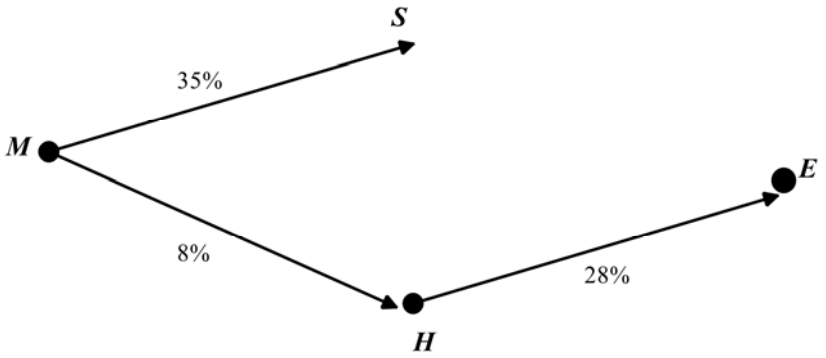
1 mark

_____

_____

ii. complete the transition diagram below, showing relevant percentages.

2 marks



The initial state matrix for the students' choice of workshop, S_0, can be written as

$$S_0 = \begin{bmatrix} 110 \\ 70 \\ 65 \\ 55 \end{bmatrix} \begin{matrix} M \\ S \\ E \\ H \end{matrix}$$

Let S_n represent the state matrix describing the students in each workshop after *n* weeks.

b. Using the rule $S_{n+1} = TS_n$, determine

i. S_1

1 mark

ii. the number of students predicted to choose the English workshop in the fifth week.

Round your answer to the nearest whole number of students.

1 mark

iii. the number of students predicted to choose the Humanities workshop in the long term.

1 mark

The rule $S_{n+1} = TS_n$ that was used to describe the students' selection of workshops does not take into account students who do not choose to continue to attend these optional workshops.

A matrix describing numbers in each workshop is found using the new transition matrix below

$$T_1 = \begin{array}{c} \\ \text{M} \\ \text{S} \\ \text{E} \\ \text{H} \end{array} \begin{array}{c} \text{this week} \\ \begin{bmatrix} 0.41 & 0.32 & 0.45 & 0.15 \\ 0.32 & 0.40 & 0.13 & 0.11 \\ 0.12 & 0.10 & 0.17 & 0.28 \\ 0.08 & 0.10 & 0.15 & 0.45 \end{bmatrix} \end{array} \begin{array}{l} \text{M} \\ \text{S} \\ \text{E} \\ \text{H} \end{array} \text{ next week}$$

c. Use this new transition matrix to determine S_1.

1 mark

_____

_____

_____

_____

_____

_____

Each week, the teachers running the workshops would like to maintain the same number of students in the study workshops as were enrolled in the first week.

d. In the matrix equation below, find the values of *a, b, c* and *d* required to ensure that the student numbers remain constant.

Round your answers to the nearest whole number.

$$S_1 = \begin{bmatrix} 0.41 & 0.32 & 0.45 & 0.15 \\ 0.32 & 0.40 & 0.13 & 0.11 \\ 0.12 & 0.10 & 0.17 & 0.28 \\ 0.08 & 0.10 & 0.15 & 0.45 \end{bmatrix} \times \begin{bmatrix} 110 \\ 70 \\ 65 \\ 55 \end{bmatrix} + \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 110 \\ 70 \\ 65 \\ 55 \end{bmatrix}$$

1 mark

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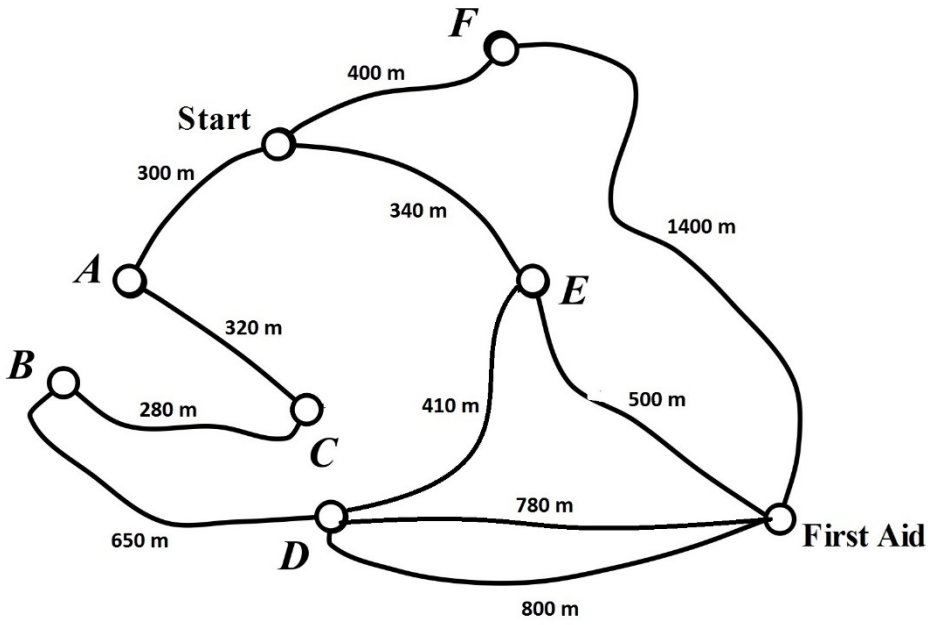
_____

Module 2 – Networks and decision mathematics

Question 1 (6 marks)

The following diagram represents a running route being considered for a charity fun run.



a. What is the degree of vertex *D*?

1 mark

_____

b. Write down an Eulerian trail that begins at Start and finishes at vertex *E*.

1 mark

_____

_____

c. The organiser of the event would like to have a dotted line marked in the middle of each track within the route to create lanes. When marking these lines, it is necessary to pass over each track exactly once and start and finish at the vertex marked Start.

i. Explain why this is not currently possible, by referring to the diagram opposite.

1 mark

_____

_____

_____

_____

ii. An additional track was added between Start and vertex *E*.

Write down the order in which the organiser should mark the tracks with her dotted line.

1 mark

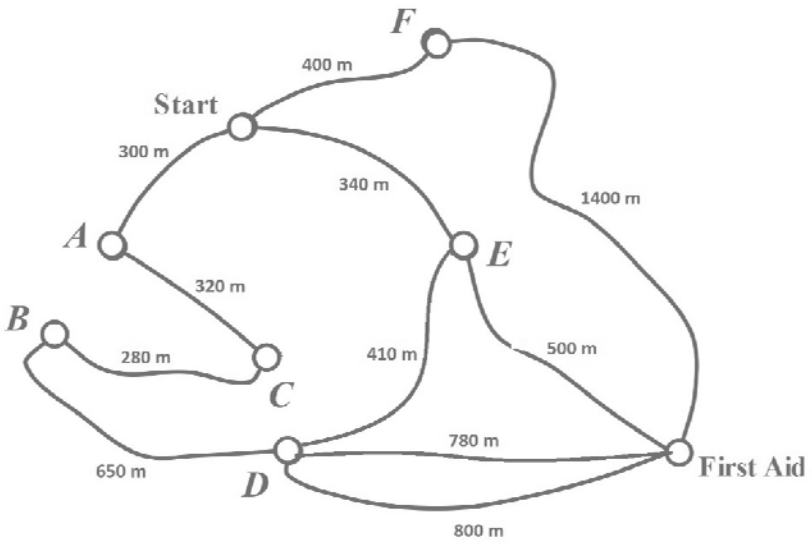
_____

_____

It is necessary to place lighting at each of the checkpoints because the running event will be held early in the morning. Wiring for the lights will be placed along the edges of the tracks.

d. **i.** On the diagram below, mark where the wiring should be placed to use the minimum amount of wiring to provide lighting for all checkpoints.

1 mark



ii. Give the length of wiring required, in metres.

1 mark

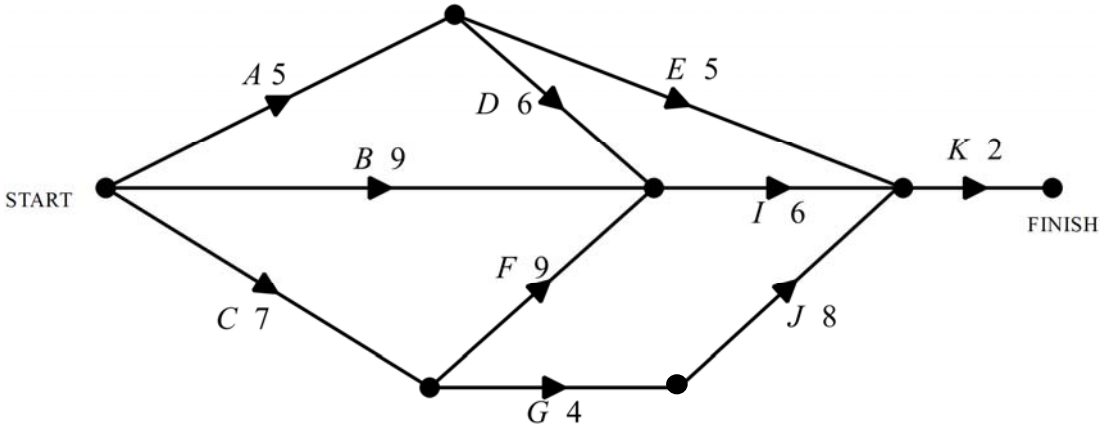
_____

Question 2 (6 marks)

A project will be undertaken to upgrade the running tracks in preparation for the running event. The project involves the 11 activities shown in the table below. The duration in hours and the predecessor(s) of each activity are also included in the table.

Activity	Duration (weeks)	Predecessor(s)
A	5	–
B	9	–
C	7	–
D	6	A
E	5	A
F	9	C
G	4	C
H	5	B, D, F
I	6	B, D, F
J	8	G, H
K	2	E, J, I

Activity *H* is missing from the network diagram for this project, which is shown below.



a. Complete the network diagram above by inserting activity *H*.

1 mark

(*Answer on the network diagram above*.)

b. Determine the earliest start time for activity *J*.

1 mark

c. Write down the activities that are on the critical path in the order that they are completed.

1 mark

d. Write down the slack time for activity *B*.

1 mark

e. Consider the following statement:

'Delaying activity *D* will not affect the minimum completion time for this project.'

Explain in what circumstances this statement will be correct.

1 mark

f. Assume that activity *F* is delayed by 4 weeks.

What will be the minimum completion time for the project?

1 mark

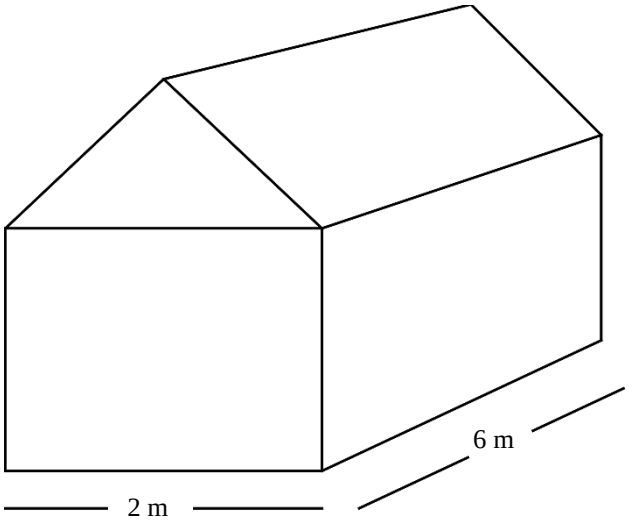
Module 3 – Geometry and measurement

Question 1 (4 marks)

Jason is renovating his backyard to host a family reunion.

As part of his backyard renovations, Jason will build a shed to store gardening materials.

A sketch for Jason's shed is shown below.



The shed will be 2 m wide and 6 m long.

Council regulations require that a concrete slab be created as the base of the shed, and that this must be 450 mm deep.

a. Calculate the amount of concrete required, in m^3.

1 mark

_____

_____

The shed is to be built with walls at a height of 2.2 m.

The tallest point of the shed roof reaches a height of 4 m.

b. Calculate the total amount of tin required to construct the roof of Jason's shed, to the nearest square metre.

1 mark

_____

_____

_____

_____

Jason would like to build a smaller shed, of a similar design, which he will use as a cubby house for his children. The total amount of tin required for this smaller shed is 29 m^2.

c. Show that the scale factor for these similar prisms is $\frac{2}{3}$.

2 marks

_____

_____

_____

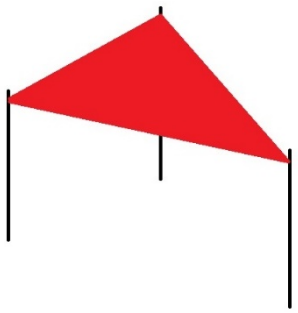
_____

_____

Question 2 (3 marks)

As part of his backyard renovations, Jason is also going to install a shade sail.

An example of the style of shade sail is shown below.



To build the shade sail, posts must be installed to hook the sail to, as shown above.

Jason has decided to install the posts first and then have a shade sail made to fit the distances between poles, as required.

Jason will place the first post at Point *A*.

The second pole will be placed at Point *B*, 2.6 m from Point *A* on a bearing of 137°.

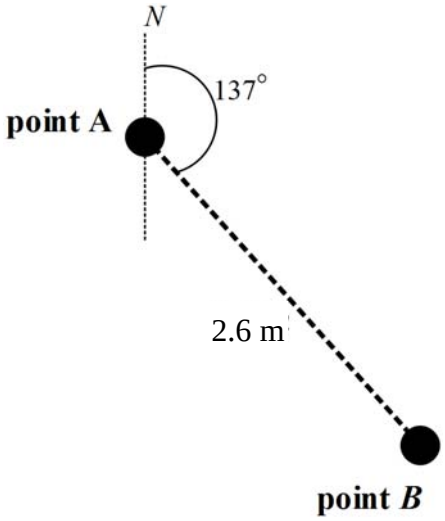
The third pole will be placed at Point *C*, 1.9 m from Point *B* on a bearing of 068°.

a. The following diagram represents the placement of poles for Jason's shade sail.

Complete the diagram by entering the missing information.

1 mark

(*Answer on the diagram below.*)



b. Calculate the bearing of the pole at Point *C* from Point *A*.
Give your answer to the nearest degree.

2 marks

Question 3 (5 marks)

Jason lives in Canberra (35° S, 149° E).
Jason's aunt is flying to Canberra from Adelaide (35° S, 138° E) for the family reunion.
Jason's cousin will fly to Canberra from London (52° N, 0.13° W).
For this question, assume that the radius of the Earth is 6400 km.

a. Find the shortest great circle distance to the South Pole from Canberra.

Round your answer to the nearest kilometre.

1 mark

_____

_____

_____

_____

b. The flight from Adelaide to Canberra travels along a small circle.

i. Find the radius of this small circle.

Round your answer to two decimal places.

1 mark

_____

_____

_____

ii. Find the distance the plane travels between Adelaide and Canberra.

Round your answer to the nearest kilometre.

1 mark

_____

_____

_____

c. Jason would like to call his cousin in London (52° N, 0.13° W) to arrange her travel to Canberra (35° S, 149° E).

If he calls London at 9.00 am Canberra time, what time will it be in London?

2 marks

_____

_____

_____

_____

Module 4 – Graphs and relations

Question 1 (8 marks)

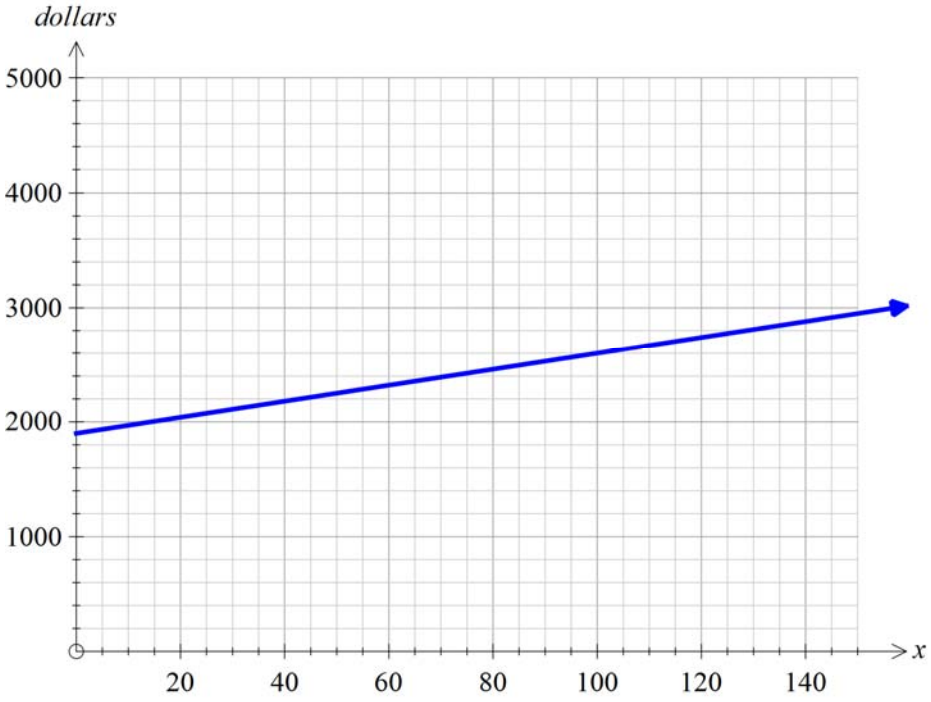
Laura is the designer and owner of a small fashion label that produces a limited range of women's dresses. Her business is very successful and she sells all of the dresses that she produces.

Her total monthly revenue and costs are represented by the equations for R and C, respectively, as shown below

$$R = 45x$$
$$C = 7x + 1900$$

where x represents the number of dresses produced and sold.

The cost function is shown on the graph below.



a. Sketch the equation for the revenue on the graph above.

1 mark

(*Answer on the graph above.*)

b. Write down the gradient of the revenue function and interpret it in the context of Laura's small fashion label.

2 marks

_____

_____

_____

c. Determine the level of production for the break-even point.

1 mark

_____

_____

d. Determine the level of production at the break-even point if the total cost increases by 5%.

1 mark

_____

_____

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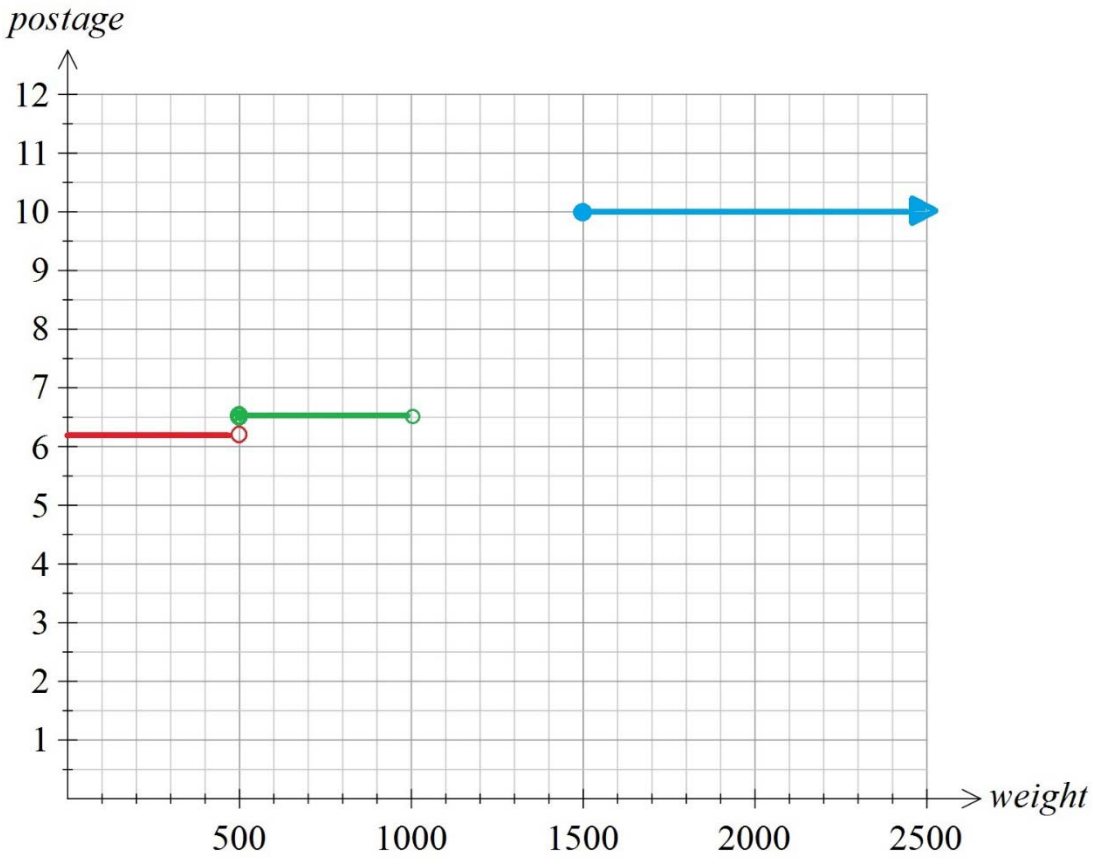
Laura offers the option to her customers of ordering dresses online and having them posted to home.

The following function, P, represents the postage charges advertised on Laura's website (postage is based on weight, in grams)

$$P_1 = \begin{cases} 6.20 & 0 < w < 500 \\ 6.50 & 500 \leq w < 1000 \\ 8.50 & 1000 \leq w < 1500 \\ 10.00 & w \geq 1500 \end{cases}$$

The graph below shows the sketch of the function P.



e. Complete the graph by sketching the cost of postage for a parcel ranging from 1000 g to 1500 g.

1 mark

(*Answer on the graph above.*)

f. What is the postage charge for ordering a parcel weighing 2.3 kg?

1 mark

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Another small-business owner recommends that Laura alters the way that she charges her customers for postage.

He suggests that she charge for postage according to the following function:

$P_2 = 2.50 + 0.35w$

g. Find the weight for which the second postage function, P_2, is cheaper than the original postage function, P_1, to the nearest gram.

1 mark

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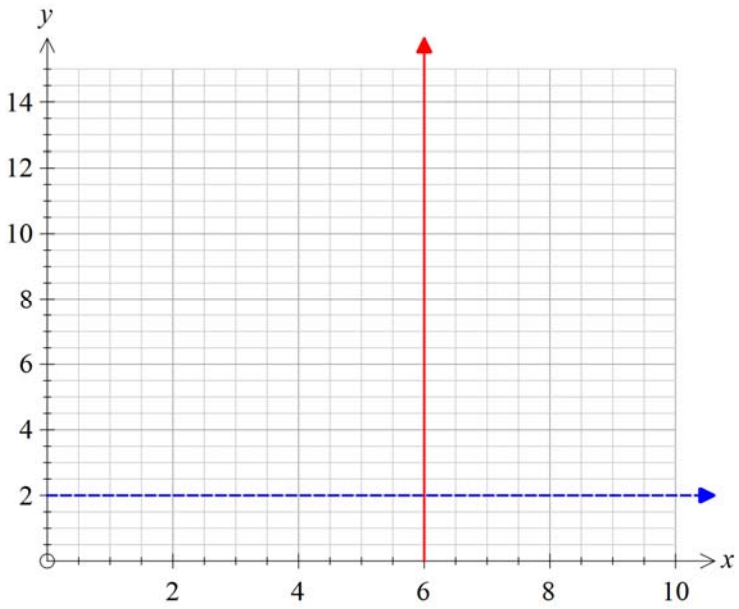
Question 2 (4 marks)

To improve the quality of her dresses, Laura is investigating using a mix of organic cotton and bamboo fabric in her new designs. She visits the local fabric store with a clear idea of what she requires.

Laura would like to purchase no more than 6 metres of organic cotton at $16 a metre and no less than 2 metres of bamboo fabric at $18 a metre. Overall, she would like to spend no more than $200.

The inequalities below represent this information, where x is the amount of organic cotton purchases, in metres, and y is the amount of bamboo fabric purchased, in metres.

Inequality 1	$x \leq 6$
Inequality 2	$y > 2$
Inequality 3	$16x + 18y < 200$



The graph above shows inequality 1 and 2.

a. Complete the graph by sketching inequality 3.

1 mark

(*Answer on the graph above.*)

The fabric store has informed Laura that they have limited stock of bamboo fabric and can only sell her a maximum of 9 metres of this fabric.

b. Represent this as inequality 4 and sketch it onto the graph above.

1 mark

(*Answer on the graph above.*)

c. On the graph above, shade the feasible region for Laura's purchases at the fabric store.

2 marks

(*Answer on the graph above.*)

END OF QUESTION AND ANSWER BOOK