2016 Teacher-exclusive Exam For 2016-18 VCE Study design



# Units 3 and 4 Further Maths: Exam 1

# **Practice Exam Question and Answer Booklet**

Duration: 15 minutes reading time, 1 hour and 30 minutes writing time

Structure of book:

Section	Number of questions	Number of questions to be answered	Number of Modules	Number of Modules to be answered	Number of marks
А	24	24			24
В			4	2	16
				Total	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, rulers, a bound reference and a calculator.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied:

• This question and answer booklet of 30 pages, including a formula sheet.

Instructions:

- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

## Instructions

Answer all questions by circling your choice.

Choose the response that is correct or that best answers the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

# Section A – Core

# **Data analysis**

#### Question 1

The English test scores of the students in classes 12C and 12D are recorded and compared.

The most suitable way to display this data would be a

- A. Back-to-back stemplot.
- B. Frequencies table.
- C. Scatter plot.
- D. Segmented bar chart.
- E. Histogram.

#### Question 2

A factory produces highly standardised cups with their mean radius being 4.6 cm and the standard deviation 0.12 cm. The distribution is approximately normal.

Cups with radius smaller than 4.36 cm or larger than 4.96 cm will be discarded.

There are 2000 cups made on a particular day.

The estimated number of discarded cups is

- А. З
- B. 6
- C. 16
- D. 53
- E. 100

Consider the following boxplot



Compared to the median, the mean of this distribution is

- A. Greater, because the distribution is negatively skewed.
- B. Greater, because the median is affected by skewedness.
- C. Greater, because the mean is affected by the upper outliers.
- D. Equal, because the distribution is approximately normal.
- E. There is not enough information.

#### Use the following information to answer Questions 4-7.

The following table shows the data collected from a random sample of six financial aid recipients drawn from the population of all aid recipients at the University of Melbourne. The variables in the table are:

- Average the average achieved by each student.
- Sex the sex of the driver (female, male)
- Year the academic year the student is in (1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>)
- *Type of aid* the type of aid received (academic, equity, both)
- *Citizenship* the student's citizenship (Australian, non-Australian)

Average (%)	<i>Sex</i> (F = female, M=male)	Academic year (1 = 1 <sup>st</sup> , 2 = 2 <sup>nd</sup> , 3 = 3 <sup>rd</sup> )	<i>Type of aid</i> (1 = academic, 2 = equity, 3 = both)	<i>Citizenship</i> (1 = Australian, 0 = non- Australian)
89.25	F	2	1	0
78.50	М	1	2	1
80.45	М	3	1	0
88.25	М	1	3	1
64.75	F	1	2	1
75.85	F	2	2	1

#### Question 4

The mean,  $\bar{x}$ , the standard deviation,  $s_x$ , and the interquartile range, IQR, of the variable 'average', for these students are closest to

Α.	$\bar{x} = 79.5$	$s_x = 9.0$	IQR = 12.4
В.	$\bar{x} = 80.45$	$s_x = 9.0$	IQR = 12.4
C.	$\bar{x} = 79.5$	$s_x = 9.0$	IQR = 64.75
D.	$\bar{x} = 79.5$	$s_x = 8.9$	IQR = 12.4
E.	$\bar{x} = 79.5$	$s_x = 8.2$	IQR = 12.4

The number of discrete numerical variables in this data set is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

### Question 6

The number of ordinal variables in this data set is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

#### Question 7



The histogram above displays the distribution of the population for 233 countries plotted on a log scale.

The percentage of countries with population less than 100,000 people is closest to

- A. 11%
- B. 97%
- C. 3%
- D. 18
- E. 14%

#### Use the following information to answer Questions 8 and 9.

The following scatterplot shows the relationship between the level of HCG (mg/L) and estrogen (ng/L) in women taking contraceptive pills



#### Question 8

To linearise the above data, which of the following transformations might be suitable?

- A. Log y.
- B. Log x.
- C. 1/x.
- D. y2.
- E. 1/x<sup>2</sup>.

#### Question 9

Eventually, 1/y transformation is decided to be used. This gives a coefficient of determination of  $r^2 = 0.4466$ .

Correct to 3 decimal places, the Pearson's coefficient of correlation is

- A. 0.668.
- B. -0.668.
- C. 0.199.
- D. 0.264.
- E. -0.256.

The average daily ice cream sales and the average daily temperature in the past year are related according to the following equation:

*Ice cream sales =* 30.5 x *temperature* + 645.99

Given that that correlation coefficient is 0.89, the standard deviation in ice cream sales is \$120.2 and the average temperature for the year is 20.4°C, the **standard deviation for temperature** and the **mean ice cream sales** respectively are closest to

- A. 4.5 °C and \$725.8
- B. 4.5 °C and \$1268.2
- C. 3.5 °C and \$1268.2
- D. 3.5 °C and \$725.8
- E. 5.5 °C and \$725.8

#### Question 11

The following boxplot represents the test scores of a Year 12 class in English and Maths.



Which of the following statement is true about the information given above?

- A. There is a positive correlation between the Maths and English scores.
- B. There are more people enrolled in Maths than in English.
- C. The range of scores in Maths is larger than that in English.
- D. The test scores in Maths are generally more variable than that in English.
- E. The highest score in Maths is equal to the lowest score in English.

#### Use the following information to answer questions 12 – 16

Given below are the months in a year and their respective seasonal indices

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
SI	1.75	1.5	1.25	1.2	1.1		0.82	0.8	0.78	0.75	0.6	0.5

#### Question 12

The seasonal index for June is

- A. 0.95
- B. 1.05
- C. 0.80
- D. 0.96
- E. 1

In addition to seasonality, which other fluctuations can be observed within a year of the data above?

- A. Increasing trend.
- B. Decreasing trend.
- C. Cycles.
- D. Outlier.
- E. There is not enough information.

#### Question 14

The seasonal index for April is 1.2.

To adjust for seasonality, the actual data value for April should be

- A. Increased by 20%.
- B. Decreased by 20%.
- C. Increased by 80%.
- D. Decreased by 2 %.
- E. Decreased by 17%.

#### Question 15

The deseasonalised value of July is \$46,000

The actual data value of July is closest to

- A. \$37,720.
- B. \$56,100.
- C. \$83,720.
- D. \$8,280.
- E. \$37,700.

Consider the following time series plot



#### Using three-median smoothing, the smoothed time series plot will look most like





# **Recursion and financial modelling**

#### Question 17

Given the following terms in a sequence:

 $P_2 = 4000, P_3 = 5900, P_4 = 8940$ 

The recurrence relation in the form of  $P_{n+1} = c \times P_n + d$ , and the first term of the sequence,  $P_1$ , respectively are

- A.  $P_{n+1} = 1.6 \times P_n 500, P_1 = 2812.5$
- B.  $P_{n+1} = 1.4 \times P_n 500, P_1 = 4000$
- C.  $P_{n+1} = 1.6 \times P_n 500, P_1 = 2070.3125$
- D.  $P_{n+1} = 1.6 \times P_n + 500, P_1 = 2812.5$
- E.  $P_{n+1} = 1.6 \times P_n 500, P_1 = 4000$

#### Question 18

Which of the following recurrence relations will generate a sequence whose values decay geometrically?

- A.  $P_1 = 3000, P_{n+1} = -1.6 \times P_n$
- B.  $P_2 = 3000, P_{n+1} = 0.6 \times P_n$
- C.  $P_0 = 3000, P_n = 0.6 \times P_{n-1} + 100$
- D.  $P_2 = 3000, P_{n+1} = 1.6 \times P_n$
- E.  $P_0 = 3000, P_{n+1} = P_n + 100$
- F.  $P_2 = 3000, P_{n+1} = -1.6 \times P_n$

#### Question 19

Lena invested \$20000 in a perpetuity that returns \$220 per quarter. Interest is compounded **every three months.** 

The annual interest rate of Lena's investment is closest to

- A. 1.1%
- B. 13.2%
- C. 4.4%
- D. 2.2%
- E. 6.6%

#### Use the following information to answer Questions 20 and 21.

Jacob took out a reducing balance loan of \$12000 to buy a new car.

Interest was charged at a rate of 12% per annum, compounding monthly.

His loan is to be fully repaid in four years, with equal monthly payments.

#### Question 20

The amount of monthly payments made by Jacob is closest to

- A. \$316.00
- B. \$1066.19
- C. \$3075.37
- D. \$315.31
- E. \$250.61

After three years, Jacob will have reduced the balance of his loan by approximately

- A. 30%
- B. 70%
- C. 75%
- D. 50%
- E. 25%

#### Question 22

Ben has \$5000 that he plans to invest for three years.

One bank offers to pay him interest at the rate of 6.25% per annum compounding weekly.

Assuming that there are 52 weeks in a year, the effective annual interest rate for this investment would be closest to

- A. 6.45%
- B. 6.43%
- C. 6.42%
- D. 6.40%
- E. 6.25%

#### **Question 23**

Pallav borrowed \$60000 at an interest rate of 4.5% per annum.

Interest is calculated monthly on the reducing balance of the loan.

The loan will be fully repaid with monthly payments of \$1500.

Which one of the following statements is **not** true?

- A. His first payment reduces the loan by less than \$1500.
- B. His second payment reduces the loan by more than the first payment.
- C. Repaying \$1800 monthly would enable Pallav to fully repay the loan in less than three years.
- D. His final payment will be less than \$1400.
- E. He will fully repay the loan in less than four years.

#### Question 24

Juno invested \$10000 in an annuity at an interest rate of 6% per annum. Interest is compounded monthly and Juno withdraws \$400 per month, after the interest has been calculated.

The **total** amount of interest Juno will have earned by the end of the 3<sup>rd</sup> month (i.e. after three monthly withdrawals) is closest to

- A. \$150.75
- B. \$8950.75
- C. \$100.00
- D. \$144.74
- E. \$189.46

# **Section B - Modules**

# Module 1 – Matrices

#### Question 1

Fiona, Jack and Cindy are trading books. Fiona receives 1 from Jack and 3 from Cindy and gives 2 to Jack and 4 to Cindy. Cindy gives Jack 4 books. Jack gives Cindy 5 books.

If a 3 x 3 matrix is used to described this trading, it could most likely look like



- C. Ill only.
- D. I or III.
- E. II or III.

#### Question 2

The total cost of two carrots and four eggplants is \$5.25.

The total cost of six carrots and three eggplants is \$7.80.

Let *x* be the cost of a carrot and *y* be the cost of an eggplant.

The inverse matrix that has to be multiplied with the matrix equation  $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} h \\ k \end{bmatrix}$ solve for x and y is

to

A.	$\begin{bmatrix} -\frac{1}{6} & \frac{2}{9} \\ \frac{1}{3} & -\frac{1}{9} \end{bmatrix}$	D. $\begin{bmatrix} -\frac{1}{6} & \frac{1}{3} \\ \frac{2}{9} & -\frac{1}{9} \end{bmatrix}$
B.	$\begin{bmatrix} -2 & 6 \\ 4 & -3 \end{bmatrix}$	E. $\left[\frac{103}{120}\right]$
C.	$\begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix}$	$\begin{bmatrix} 120\\ 53\\ 60 \end{bmatrix}$

#### Use the following information to answer Questions 3 and 4.

A dessert shop sells four types of cakes, Cupcake (C), Black forest cake (F), Macaron (M) and Tea cake (T). The regular customers buy cake each per day and choose the type of cake they buy according to the following transition matrix.

Choose today  

$$C \quad F \quad M \quad T$$

$$T = \begin{bmatrix} 0.75 & 0.6 & 0.02 & 0.45 \\ 0 & 0.1 & 0.28 & 0.25 \\ 0.15 & 0.05 & 0.55 & 0.25 \\ 0.1 & 0.25 & 0.15 & 0.05 \end{bmatrix} T$$
Choose tomorrow

#### Question 3

What is the probability that amongst the customers who buy a cupcake today, one will buy a macaron tomorrow?

- A. 0.02.
- B. 0.55.
- C. 0.1.
- D. 0.15.
- E. 0.05.

#### Question 4

After running for several years, the shop decides to change one of its cake with another creative one. It has decided to replace the one that is the least liked by its regulars.

By examining the long-term number of buyers of each type of cake, which one should the shop replace?

- A. Cupcake.
- B. Black forest cake.
- C. Macaron.
- D. Tea cake.
- E. None of the above, they are all well-liked.

#### Question 5

Consider the following set of simultaneous equations

 $\begin{cases} hx + 2y = 5\\ -3x + ky = 6 \end{cases}$ 

Where h and k are constants.

These simultaneous equations will have a solution with which of the following sets of h and k?

A. 
$$h = -\frac{3}{2}, k = 4$$
.  
B.  $h = -3, k = 2$ .  
C.  $h = -6, k = 1$ .  
D.  $h = 1, k = -6$ .  
E.  $h = -\frac{1}{2}, k = -\frac{3}{4}$ 

There are four towers in four different cities A, B, C and D. Their communication can be summarised by the matrix below

А	В	C	) [	)
0	1	0	1]	А
1	0	1	1	В
0	1	0	0	С
1	1	0	0	D

How many of the two-step communication links between the towers will be redundant?

- A. 2
- B. 4
- C. 6
- D. 8
- E. 10

# Question 7

Which of the following pairs of towers require at least three steps to communicate with each other?

- A. B and C; A and D.
- B. C and D; A and B.
- C. A and C; C and D.
- D. C and B; A and B.
- E. A and D; A and C.

### Question 8

A friendly tennis tournament is organised amongst the members of the Walkers family – Jack (J), Adam (A), Bridget (B), Emily (E) and Sierra (S). Each player plays against another once. The results are summarised in the matrix below:

- Jack beats Adam and Emily but loses to others.
- Bridget also beats Adam and loses to Emily and Sierra.
- Sierra also beats Emily but loses to Adam.
- Adam also beats Emily.

By taking into account both the one-step and two-step matrices representing the results, the winner overall of the tournament is?

- A. Jack.
- B. Adam.
- C. Bridget.
- D. Emily.
- E. Sierra.

# Module 2 – Networks and decision mathematics

# Question 1

Which of the following is **not** a planar graph?



The following directed graph represent a set of one-way streets and their intersections are numbered as vertices from 1 to 8.



All intersections can be visited with a walk starting from

- A. Intersection 2.
- B. Intersection 3.
- C. Intersection 4.
- D. Intersection 5.
- E. Intersection 6.

#### **Question 3**

For the graph below to have a Eulerian circuit, an edge must be added to connect between



- A. Vertices 1 and 5.
- B. Vertices 3 and 5.
- C. Vertices 4 and 3.
- D. Vertices 1 and 6.
- E. Vertices 2 and 3.



The minimal spanning tree for the network above does not include the edge that has a weight of

- A. 1.
- B. 2.
- C. 4.
- D. 6.
- E. 8.

#### Use the following information to answer Questions 5 and 6.

Matt is travelling from town A to town B by car. The following network represents all his possible routes. The weight of each edge represents the road's length. The vertices, aside from A and B, represent toll-paying points. The amount of toll paid at each point is \$12.00.



#### Question 5

Matt at first wants to travel on the route with the least toll paid. The minimum toll amount that Matt will pay is

- A. \$72.00.
- B. \$48.00.
- C. \$60.00.
- D. \$36.00.
- E. \$4.00.

#### Question 6

Matt then decides to travel on the shortest route. Due to traffic, it takes him two minutes to travel each kilometre. What is the shortest travel time, in minutes, for Matt?

- A. 82 minutes.
- B. 200 minutes.
- C. 164 minutes.
- D. 99 minutes.
- E. 198 minutes.

Consider the following network representing a pipeline system. The weight of each edge represents its capacity.



The maximum flow through this system is

- A. 20
- B. 22
- C. 24
- D. 25
- E. 29

# Question 8

Four workers, Aaron, Jack, Matt and Harry can paint the wall, the window, the door and the ceiling in different amount of time (in minutes) summarised in the following table.

	Wall	Window	Door	Ceiling
Aaron	40	50	45	45
Jack	30	55	30	50
Matt	45	45	45	50
Harry	35	40	50	45

Given that each worker can only finish one job in a day, what is the minimum time it takes to finish all the jobs during a day?

- A. 150 minutes.
- B. 155 minutes.
- C. 160 minutes.
- D. 165 minutes.
- E. 170 minutes.

# Module 3 – Geometry and measurement

# Question 1

All of the following sets of side lengths make up a right-angled triangle **except** 

- A. 5, 12 and 13.
- B. 13, 84 and 85.
- C. 9, 40, 41.
- D. 33, 56, 65.
- E. 6, 8, 12.

#### Question 2

Jonathan stands at a distance, in metres, from a tall building. The angle of elevation from where he stands to the top of the building is 45°. As he moves 50 metres further away from the building, the angle of elevation is 40°.



The height of the building, in metre, is closest to

- A. 42m.
- B. 460m.
- C. 261m.
- D. 500m.
- E. 353m.

A block of cheese has a shape with its dimensions described in the diagram below.



The shaded area represents a cut made half-way through the length of the cheese block.

The volume of the smaller block after the cut is closest to

- A. 250 cm<sup>3</sup>.
- B. 500 cm<sup>3</sup>.
- C. 750 cm<sup>3</sup>.
- D. 1000 cm<sup>3</sup>.
- E. 1250 cm<sup>3</sup>.

#### Question 4

A group of hikers map out their path. From their starting point, they first travel to the lake that is 12 km away on a bearing of 65°. Then, they travel to cave that is 10 km from the lake, on a bearing of 45°. Afterwards, they hike to the waterfall that is 8 km away from the cave, on a bearing of 70°.

If the hikers want to travel back to the starting point from the waterfall, on what bearing, correct to nearest degree, should they go?

- A. 30°
- B. 60°
- C. 220°
- D. 240°
- E. 260°

A cake has a cylindrical shape with its dimensions depicted in the diagram below.

A cut of 15 cm is made as represented by the shaded area.



The volume of the smaller piece after the cut is made is closest to

- A. 11 cm<sup>3</sup>
- B. 20 cm<sup>3</sup>
- C. 230 cm<sup>3</sup>
- D. 300 cm<sup>3</sup>
- E. 8482 cm<sup>3</sup>

Use the following information to answer Questions 6 and 7.

Assume the radius of the Earth is 6400 km.

Points A and B have the coordinates (40°N, 25°E) and (40°N, 30°W) respectively.

#### Question 6

The distance between points A and B is closest to

- A. 6100 km
- B. 4700 km
- C. 5000 km
- D. 3950 km
- E. 2350 km

Question 7

If it is 4.56pm at point A, what is the time at point B?

- A. 4.56am
- B. 1.26pm
- C. 12.56pm
- D. 1.16pm
- E. 12.16pm

A square-based pyramid has the dimension as below



The height of the pyramid, in centimetres, is closest to

- A. 10.4 cm
- B. 12.4 cm
- C. 9.0 cm
- D. 13.7 cm
- E. 17.0 cm

# Module 4 – Graph and relations

# Question 1



The equation of the line shown of the graph is

A. 
$$y = -\frac{2}{3}x + 2$$

B. 
$$y = \frac{2}{3}x - 2$$

$$C. \quad -2x + 3y = 6$$

- D. 2x + 3y = 6
- E. 2x + 3y = -6

# Question 2

A point that lies on the graph of 4x - 5y = 3 is

- A. (1,1)
- B. (-3,-3)
- C. (2,-1)
- D. (1,2)
- E. (3,-3)

# Question 3

A clinic offers psychiatric therapy.

Four hour of therapy costs \$640. Six hour of therapy costs \$720.

Given that the clinic charges an upfront fee plus an hourly flat rate, the flat rate is

- A. \$40
- B. \$45
- C. \$140
- D. \$160
- E. \$480



The equation of the curve above is

A.  $y = \frac{1}{2}x^{2}$ B.  $y = \frac{1}{8}x^{3}$ C.  $y = \frac{1}{16}x^{4}$ D.  $y = \frac{2}{x}$ E.  $y = \frac{4}{x^{2}}$ 

A school has a plan to give each of its students a science kit and/or an arts kit depending on their chosen subject.

One science kit costs \$40.00.

One arts kit costs \$35.00.

The school budget for this program is \$1000.00.

The school is intending to buy at least 15 kits, including at least 5 science kits and 5 arts kits.

Let x be the number of science kits and y be the number of arts kits.

The graph below contains the boundaries of all the constraints.



How many science kits should the school buy if they were to spend the most of the allocated budget?

- A. 5
- B. 6
- C. 7
- D. 10
- E. 20

# Use the following information to answer Questions 6 and 7.

The *incomplete* graph below describes the journey of a train from town A to town B. The train does stop for a period of 10 minutes in the middle of the journey.



The two towns, A and B, are 120 km away from each other.

#### Question 6

Given that after the stop, the train travels at the speed of 100 km/h, how long after it leaves town A (point (0,0)), does the train arrive at town B?

- A. 36 minutes
- B. 60 minutes
- C. 72 minutes
- D. 86 minutes
- E. 120 minutes

The average speed throughout the whole journey is closest to

- A. 84 km/h
- B. 90 km/h
- C. 92 km/h
- D. 95 km/h
- E. 100 km/h

#### Question 8

The shaded region in the graph below represents the feasible region for a linear programming problem.

Which objective function, *Z*, has its maximum value at C?



# **Formula Sheet**

**Core – Data analysis** 

standardised score	$z = \frac{x - \overline{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$ , where $b = r \frac{s_y}{s_x}$ and $a = \overline{y} - b\overline{x}$
residual value	residual value = actual value - predicted value
seasonal index	seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

# **Core – Recursion and financial modelling**

first-order linear recurrence relation	$u_0 = a, \qquad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[ \left( 1 + \frac{r}{100n} \right)^n - 1 \right] \times 100\%$

Module 1 – Matrices

determinant of a $2 \times 2$ matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a $2 \times 2$ matrix	$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ , where $\det A \neq 0$
recurrence relation	$S_0 = \text{initial state}, \qquad S_{n+1} = T S_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	v + f = e + 2
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# Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc\sin(\theta^{\circ})$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$ , where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc \cos(A)$
circumference of a circle	2 <i>πr</i>
length of an arc	$r \times \frac{\pi}{180} \times \theta^{\circ}$
area of a circle	$\pi r^2$
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base $\times$ height
volume of a pyramid	$\frac{1}{3}$ × area of base × height

# Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	y = mx + c

### End of Booklet