



# Units 3 and 4 Further Maths: Exam 1

## Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email [practiceexams@ee.org.au](mailto:practiceexams@ee.org.au).

## Section A – Core

### Data analysis

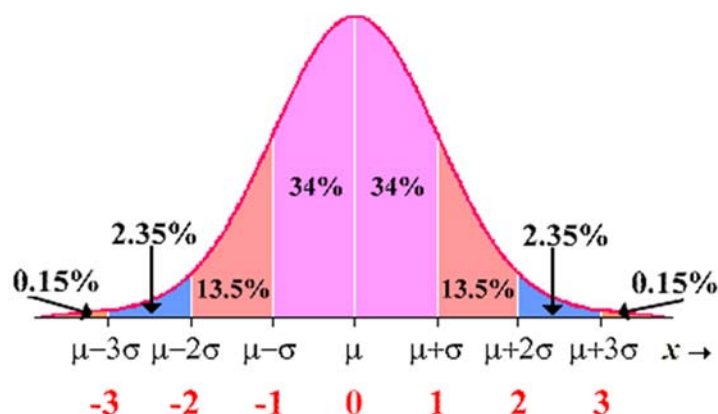
#### Question 1

The correct answer is A.

Back-to-back stem plot as it can be used to represent data from a categorical variable (classes) and numerical variable (scores).

#### Question 2

The correct answer is D.



4.36cm is 2 standard deviations below the mean. 4.96cm is 3 standard deviations above the mean.

The percentage of cups outside this range is  $2.5\% + 0.15\% = 2.65\%$ . In 2000 cups, 2.65% equates to 53 cups.

#### Question 3

The correct answer is C.

Only the mean is affected by skewedness and outliers. In this case, there are outliers in the upper extreme and hence, mean is drawn upwards. Hence it is greater than the median.

#### Question 4

The correct answer is A.

The results can be achieved from statistics function in the CAS.

$\bar{x}$	=79.508333
$\sum x$	=477.05
$\sum x^2$	=38333.863
$\sigma_x$	=8.209873
$s_x$	=8.9934652
$n$	=6
$\min X$	=64.75
$Q_1$	=75.85
$Q_3$	=88.25

The IQR can be worked out to as  $88.25 - 75.85 = 12.4$

**Question 5**

The correct answer is A.

None. The academic year is ordinal, type of aid and citizenship are both categorical.

**Question 6**

The correct answer is B.

Academic year.

**Question 7**

The correct answer is E.

The histogram is on the log scale. Hence,  $\log(100,000) = 5$ .

There are 33 countries below this value.

The percentage is 14.16% ~ 14%.

**Question 8**

The correct answer is A.

The graph can be linearised by either compressing the vertical axis or expanding the horizontal axis. A ( $\log y$ ) is the only one to satisfy this.

**Question 9**

The correct answer is B.

The reciprocal transformation makes larger number smaller e.g.,  $4 > 3$  but  $\frac{1}{4} < \frac{1}{3}$ . Hence, the relationship is reversed from positive in the original distribution to negative.

$$r = -\sqrt{r^2} = -\sqrt{0.4466} = -0.668$$

**Question 10**

The correct answer is C.

$$y = mx + c$$

$$m = r \times \frac{s_y}{s_x}$$

$$\therefore 30.5 = 0.89 \times \frac{120.2}{s_x}$$

$$\therefore s_x = \frac{120.2}{\frac{30.5}{0.89}} = 3.5$$

$$\bar{y} = m\bar{x} + c$$

$$\text{ice cream sale} = 30.5 \times 20.4 + 645.99 = \$1268.2$$

**Question 11**

The correct answer is D.

A is wrong because the data are not paired up and hence, correlation cannot be determined.

B is wrong because the boxplot represents percentages, not actual numbers.

C is incorrect due to outliers present in the English scores

D is correct because despite the outliers in the English scores, Maths scores has longer whiskers and interquartile range. The focus of the statement here is '**generally**'.

E is incorrect.

### Question 12

The correct answer is A.

The sum of all seasonal indices must equal to the number of seasons per year, i.e. 12.

### Question 13

The correct answer is B.

The seasonal index describes how the value of each season (month in this case) compares to the annual average (with 1 being equal). The decreasing trend of the SI represents the decreasing trend of the seasonal data itself within a year.

### Question 14

The correct answer is E.

Assume the data for April were 100 (standing for 100%).

To get a deseasonalised data value  $\rightarrow 100/1.2 = 83$ .

Compared to the original 100%, the value has been decreased by 17%.

Hence, 17%

### Question 15

The correct answer is A.

Deseasonalised = actual/seasonal index.

Hence, actual = deseasonalised x seasonal index = \$37,720.

### Question 16

The correct answer is D.

This can be done graphically.

## Recursion and financial modelling

### Question 17

The correct answer is A.

From what is given in the question, a set of simultaneous equations can be worked out as

$$5900 = c \times 4000 + d$$

$$8940 = c \times 5900 + d$$

Solve by CAS

$$\begin{cases} 5900=c*4000+d \\ 8940=c*5900+d \end{cases}_{c, d}$$

$$\{c=1.6, d=-500\}$$

So B and D are incorrect.

To find P1, substitute P1 and P2 into the equation

$$4000 = 1.6 \times P_1 - 500$$

$$\text{Hence, } P_1 = 2812.5$$

### Question 18

The correct answer is B.

A geometrically decaying sequence has a recurrence relation in a form  $P_{n+1} = R \times P_n$  with

$$0 < R < 1$$

B is the only one that fits.

### Question 19

The correct answer is C.

Perpetuity means the return is exactly the same as the interest.

$$\text{Hence the quarterly interest rate is } 220/20000 \times 100 = 1.1$$

$$\text{As there are 4 periods per year, annual interest rate} = 1.1 \times 4$$

The annual interest rate is 4.4%.

### Question 20

The correct answer is A.

The financial solver can be used

### Compound Interest

N	48
I%	12
PV	12000
PMT	-316.0060252
FV	0
P/Y	12
C/Y	12

### Question 21

The correct answer is B.

Note: Finance Solver can be used for this question

The amount of loan balance of Jacob after  $n$  payments can be represented by the following recursion relation

$$P_{n+1} = \left(1 + \frac{12}{12 \times 100}\right) \times P_n - 316, P_0 = 12000$$

After 3 years means after 36 payments,

The screenshot shows a financial calculator interface with the following elements:

- Buttons for navigation and calculation:  $\nabla$ ,  $\leftarrow$ ,  $\rightarrow$ ,  $\text{all...}$ ,  $\text{bn...}$ ,  $\left[ \left[ \right] \right]$ ,  $\left[ \left[ \right] \right]$ ,  $\left[ \left[ \right] \right]$ ,  $\Sigma \text{an}$ .
- Mode selection: Recursive (selected), Explicit.
- Formula:  $a_{n+1} = \left(1 + \frac{12}{12 \cdot 100}\right) \cdot a_n - 316$
- Initial value:  $a_0 = 12000$
- Other parameters:  $b_{n+1} = 0$ ,  $b_0 = 0$ ;  $c_{n+1} = 0$ ,  $c_0 = 0$ .
- Table of values:
 

n	$a_n$
33	4381.7
34	4109.5
35	3834.6
36	3556.9
37	3276.5
38	2993.3
39	2707.2
40	2418.3
41	2126.5

The balance left is \$3556.9. Hence, the amount reduced is \$8443.1, which equates to around 70%

### Question 22

The correct answer is A.

This is straight off from the formula of effective annual interest rate.

### Question 23

The correct answer is C.

A is true because interest is calculated before the payment is made, hence, the true reduction to the balance will always be the payment take away the interest.

B is true because the balance is reduced by the second payment compared to the first one and hence, the interest is reduced. Thus, the amount of reduction is larger.

C is false. Consider the following results from financial solver.

#### Compound Interest

N	35.67509541
I%	4.5
PV	60000
PMT	-1800
FV	0
P/Y	12
C/Y	12

This means that after 35 payments (just 1 month under 3 years), there is still one last monthly payment left before the loan is written off. Hence, it takes exactly 3 years.

D is true.

First, we need to find out how long it takes to write off the loan.

#### Compound Interest

N	46.8086378
I%	4.5
PV	60000
PMT	-1400
FV	0
P/Y	12
C/Y	12

$N = 46.8$  and hence, it takes 47 payments.

Now, we consider what is the balance after the 46<sup>th</sup> payment.

N	46
I%	4.5
PV	60000
PMT	-1400
FV	-1128.267205
P/Y	12
C/Y	12

Hence, D is true.

E is true since 47 payments is one month short of 4 years.

#### Question 24

The correct answer is D.

To work this out, an amortisation table needs to be done. However, it is also worth noting that the withdrawal is made after the interest has been calculated, so it goes like this interest -> withdrawal -> interest -> withdrawal -> interest -> withdrawal. So we are looking for the sum of 3 interest earnings.

Payment number	payment made	interest earned	reduction in principal	balance of annuity
0	0	0	0	10000
1	400	50	350	9650
2	400	48.25	351.75	9298.25
3	400	46.49125	353.50875	8944.74125

The sum of the first 3 interest earnings equals to \$144.74

## Section B – Modules

### Module 1 – Matrices

Question 1

The correct answer is C.

**Answer should be D as both 1 and 3 are correct.**

Question 2

The correct answer is A.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix}$$

Hence, its inverse matrix is

$$\begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} -\frac{1}{6} & \frac{2}{9} \\ \frac{1}{3} & -\frac{1}{9} \end{bmatrix}$$

Question 3

The correct answer is D.

Question 4

The correct answer is B.

$$\begin{bmatrix} 0.75 & 0.6 & 0.02 & 0.45 \\ 0 & 0.1 & 0.28 & 0.25 \\ 0.15 & 0.05 & 0.55 & 0.25 \\ 0.1 & 0.25 & 0.15 & 0.05 \end{bmatrix} \xrightarrow{50*} \begin{bmatrix} 1000 \\ 1000 \\ 1000 \\ 1000 \end{bmatrix}$$

$$\begin{bmatrix} 2049.254127 \\ 450.3486627 \\ 1007.149793 \\ 493.2474181 \end{bmatrix}$$

One thing that might confuse students in this question will be that the initial state matrix is missing. However, in long-term, the initial state values are insignificant and hence, any will do.

As from here, black forest cake is the least favourite.

Question 5

The correct answer is E.

The simultaneous equation can be rewritten in matrix form as

$$\begin{bmatrix} h & 2 \\ -3 & k \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$



In order for a solution to exist, the inverse matrix of the first matrix has to exist.

Hence, its determinant has to be different from zero.

For its determinant to be zero,  $h \times k = -6$ .

E is the only option in which  $h \times k \neq -6$ .

### Question 6

The correct answer is D.

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}^2$$

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

The redundant links are 2, 3, 1, 2 in the diagonal. Hence, a total of 8.

### Question 7

The correct answer is C.

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}^3$$

$$\begin{bmatrix} 2 & 4 & 1 & 3 \\ 4 & 2 & 3 & 4 \\ 1 & 3 & 0 & 1 \\ 3 & 4 & 1 & 2 \end{bmatrix}$$

We look for the values of 1 that are not in the diagonal.

### Question 8

The correct answer is E.

Using all the clues, student should be able to construct the following matrix

$$D = \begin{array}{c} J \quad A \quad B \quad E \quad S \\ \begin{array}{l} J \\ A \\ B \\ E \\ S \end{array} \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

From there, student only needs to determine  $D + D^2$  which is

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \wedge 2$$

$$\begin{bmatrix} 0 & 1 & 1 & 2 & 1 \\ 1 & 0 & 2 & 2 & 1 \\ 1 & 2 & 0 & 2 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix}$$

Jack has 5.

Adam has 6.

Bridget has 6.

Emily has 3.

Sierra has 8.

Hence Sierra wins overall

## Module 2 – Networks and decision mathematics

### Question 1

The correct answer is A.

All other networks may be re-drawn so no paths cross. Network A cannot be redrawn to fit planar limitations.

### Question 2

The correct answer is C.

The path that is taken to visit all 8 intersections is 4-1-2-3-8-7-5-6.

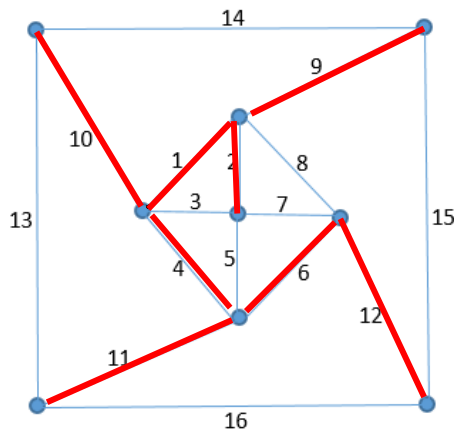
### Question 3

The correct answer is D.

A Eulerian circuit is one in which all the vertices are connected and each has an even degree. It requires a path to start and finish at the same vertex, and use each edge exactly once. To do this, a connection must be added between 1 and 6.

**Question 4**

The correct answer is E.



**Question 5**

The correct answer is B.

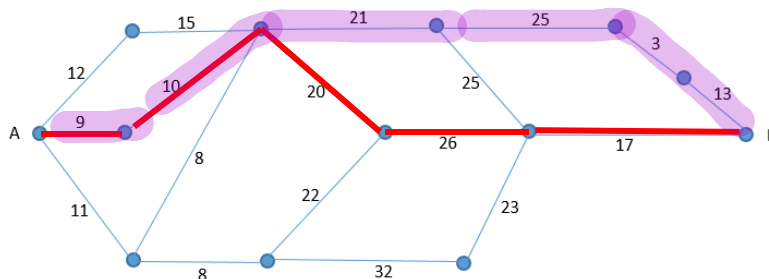
The minimum number of toll points is 4.  $4 \times 12 = \$48$

**Question 6**

The correct answer is ~~C~~.

**No answer, should be  $81 \times 2 = 162$  mins**

The shortest route can be worked out using Dijkstra algorithm.



The total distance is 82 km. It takes 2 minutes to travel each. Hence, 164 minutes for the whole journey.

**Question 7**

The correct answer is A.

Examination has to be done to determine cut with minimal capacity.

**Question 8**

The correct answer is B.

The answer can be worked out using the Hungarian algorithm.

Harry – wall, Matt – window, Jack – door, Aaron – ceiling

$$35+45+30+45 = 155$$

### Module 3 – Geometry and measurement

#### Question 1

The correct answer is E.

The side lengths that make up a right-angled triangle are ones that fit the Pythagorean theorem. The only option that does not fit is E.

#### Question 2

The correct answer is C.

Let  $d$  be the original distance of the man from the tower and  $h$  is the height of the building.

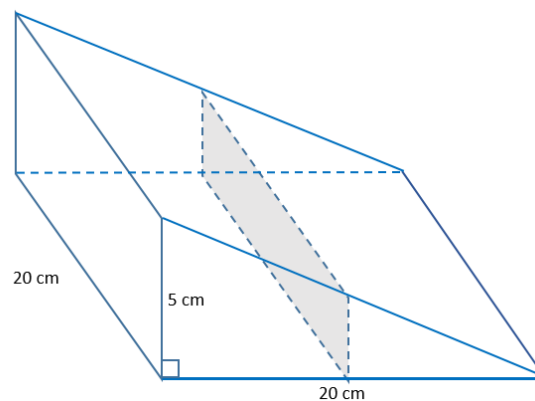
The following set of simultaneous equations can be worked out

$$\begin{cases} d \cdot \tan(45) = h \\ (d+50) \cdot \tan(40) = h \end{cases} \Big|_{d, h}$$

$$\{d=260.7513076, h=260.7513076\}$$

#### Question 3

The correct answer is A.



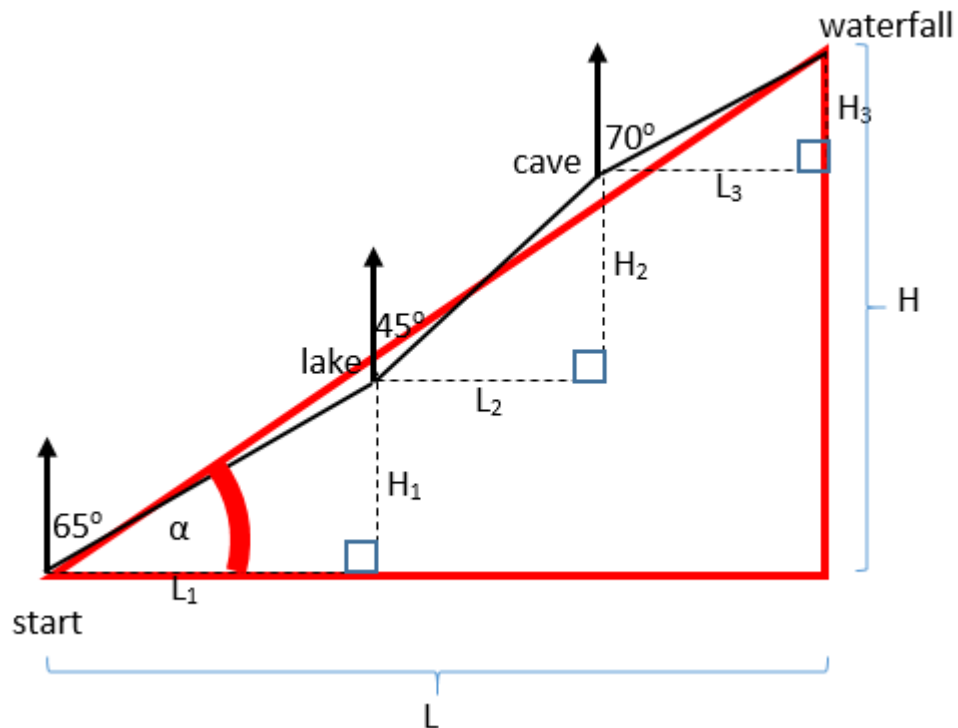
The length of the new block is 10cm, the height is 2.5cm and the width stays the same.

Hence, its volume is  $\frac{1}{2} \times 2.5 \times 10 \times 20 = 250$

#### Question 4

The correct answer is D.

The diagram below represents the problem and how to approach it



$$L = L_1 + L_2 + L_3$$

$$L = 12 \times \sin(65) + 10 \times \sin(45) + 8 \times \sin(70) = 25.46$$

$$H = H_1 + H_2 + H_3$$

$$H = 12 \times \cos(65) + 10 \times \cos(45) + 8 \times \cos(70) = 14.88$$

$$\alpha = \tan^{-1}\left(\frac{H}{L}\right) = \tan^{-1}\left(\frac{14.88}{25.46}\right) = 30.3^\circ$$

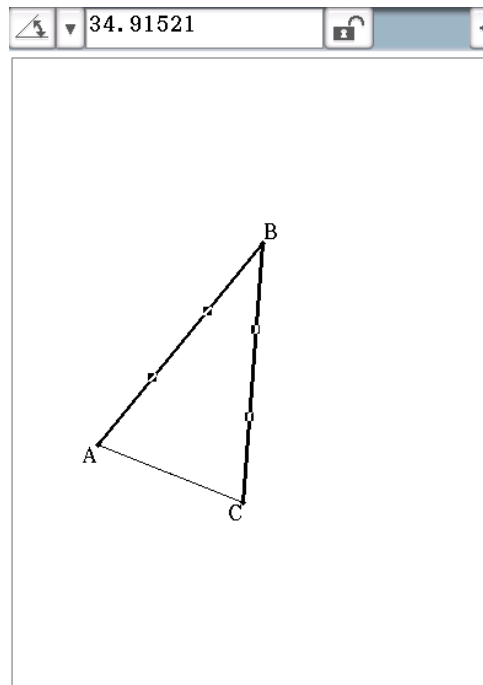
The bearing of the start from the waterfall is  $270 - 30.3 = 239.70 \sim 240^\circ$ .

### Question 5

The correct answer is C.

The geometry function of the CAS can be used to find most figures of the question.

The angle at the centre is  $34.91^\circ$



The area of the isosceles is  $178.86 \text{ cm}^2$ .

The area of the sector is  $\pi \times 25^2 \times \frac{34.91}{360} = 190.40 \text{ cm}^2$ .

Hence, the area of the segment is  $190.4 - 178.86 = 11.54 \text{ cm}^2$ .

The volume is  $11.54 \times 20 = 230.8 \sim 230 \text{ cm}^3$

### Question 6

The correct answer is B.

The angle at the centre is  $55^\circ$ .

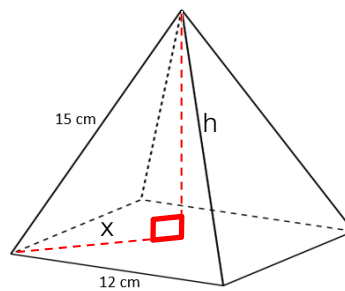
The radius at latitude  $6400 \times \cos(40) = 4902.68$

Hence, the distance between 2 points is  $2 \times \frac{4902.68}{360} \times 55 = 4706 \text{ km}$

### Question 7

The correct answer is D.

There is a longitudinal difference of  $55^\circ$  and hence, 220 minutes given  $1^\circ$  is the equivalent of 4 minutes, with point A being ahead of point B.

**Question 8**

The correct answer is B.

$$x = \frac{1}{2} \sqrt{12^2 \times 2} = 6\sqrt{2}$$

$$h = \sqrt{15^2 - (6\sqrt{2})^2} = 12.37 \text{ cm}$$

**Module 4 – Graphs and relations****Question 1**

The correct answer is E.

$$m = \frac{0 - (-2)}{-3} = -\frac{2}{3}$$

$$y = -\frac{2}{3}x - 2$$

$$2x + 3y = -6$$

**Question 2**

The correct answer is B.

Examination by substitution has to be done.

$$4x - 5y = 3$$

$$y = \frac{4x - 3}{5}$$

$$y = \frac{4(-3) - 3}{5} = -3$$

**Question 3**

The correct answer is A.

Let  $C$  = cost,  $t$  = number of hours of therapy,  $b$  = hourly rate, and  $a$  = upfront fee

$$C = a + b \times t$$

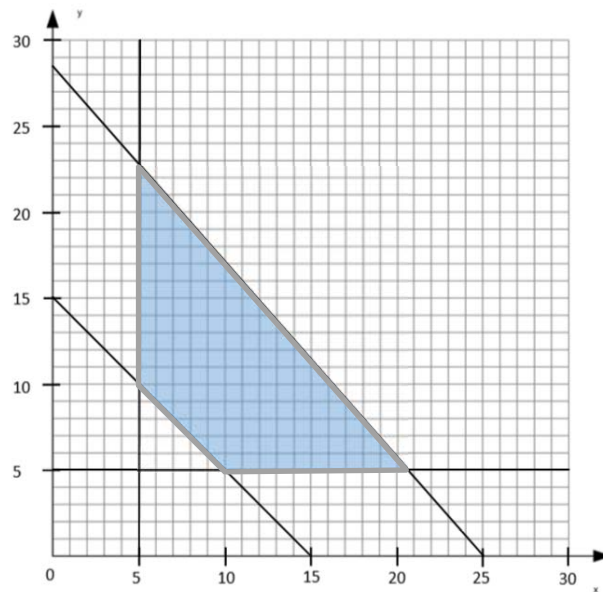
$$640 = a + b \times 4 \text{ and } 720 = a + b \times 6$$

Solving by CAS,  $b = 40$ .

**Question 4**

The correct answer is C.

Substitution and further examination of points aside from the one given has to be done, or each equation may be graphed on a CAS.

**Question 5**

The correct answer is C.

Sketching the regions using CAS or otherwise. Examination of integer points close to the vertices.  
Objective function is  $C = 40x + 35y$ . Maximum occurs at (7,20)

**Question 6**

The correct answer is D.

The train has travelled through 60 km. That leaves another 60 km to travel.

The time it takes for this is  $\frac{60}{100} \times 60 = 36$  minutes.

This together with 50 minutes previously gives 86 minutes.

**Question 7**

The correct answer is A.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\frac{120}{\frac{86}{60}} = 83.7 \sim 84 \text{ km/h}$$

**Question 8**

The correct answer is B.

Examination of points needs to be done.