Neap:

Trial Examination 2015

VCE Further Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
3	3	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45 Total 60

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white-out liquid/tape.

Materials supplied

Question and answer booklet of 29 pages and a sheet of miscellaneous formulas.

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2015 VCE Further Mathematics Units 3&4 Written Examination 2.

Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Diagrams are not to scale unless specified otherwise.

Core

Question 1 (4 marks)

The number of customers to shop in various types of stores each quarter at the Waterfront Shopping Centre is recorded in the table below. The figures are in thousands.

	1st quarter	**2nd quarter**	**3rd quarter**	**4th quarter**
Supermarket	265	260	275	260
Clothing	128	85	75	140
Fast food	340	260	240	320
Restaurants	130	125	80	120
Souvenirs	160	140	100	110

a. How many people shopped at the souvenir shops in the 3rd quarter? 1 mark

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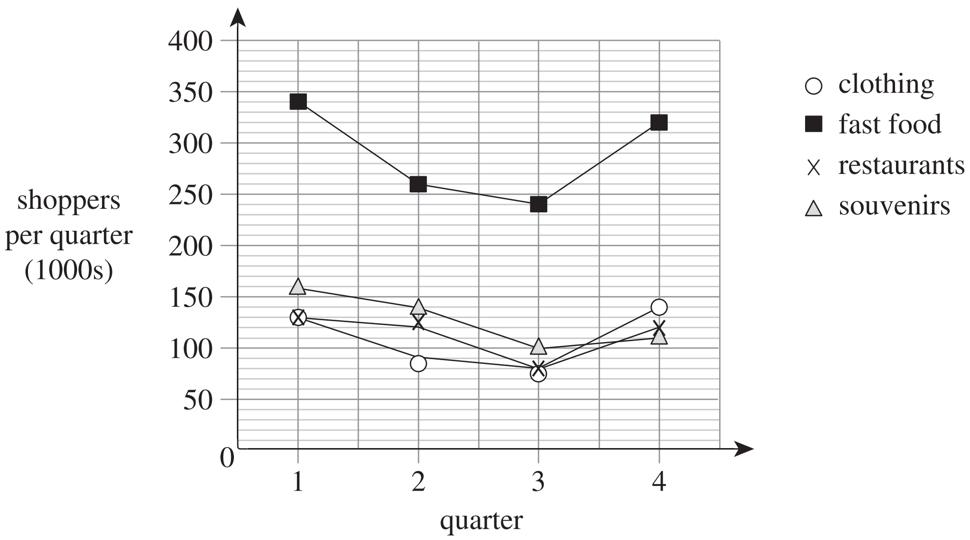
b. Consider the graph below.



i. Comment on the similarities, if any, that you notice in the time series graphs for the clothing, fast food, restaurants and souvenirs shoppers shown above. 1 mark

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ii. Add the data for the supermarket shoppers to the time series graph above. 1 mark

iii. Compare the time series data for the supermarket customers to the other customers. 1 mark

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Question 2 (5 marks)

Sally, one of the store-owners, keeps more detailed figures on the average number of shoppers per day that visit her store over a three-year period. The data is shown below.

	1st quarter	2nd quarter	3rd quarter	4th quarter	Yearly mean
2008	2550	1800	1650	1900	1975
2009	2700	2000	1750	2100	2133
2010	3000	2100	1800	2200	2275

a. Calculate the seasonal index for the 1st quarter to 2 decimal places. 1 mark

b. The seasonal index for the 2nd quarter is 0.92. The sales figures for 2011 are shown below.

	1st quarter	2nd quarter	3rd quarter	4th quarter	Yearly mean
2011	3200	2600	2100	2700	2650

Given the seasonal index is 0.92, calculate the deseasonalised figure for the 2nd quarter of 2011, rounding to the nearest whole number. 1 mark

c. Complete the table below by calculating a three-point moving median for the 2008 and 2009 figures. Centre if necessary and round your final answer to the nearest whole number. 2 marks

2008 Q_1	2550	
2008 Q_2	1800	
2008 Q_3	1650	
2008 Q_4	1900	
2009 Q_1	2700	
2009 Q_2	2000	
2009 Q_3	1750	
2009 Q_4	2100	

d. Describe the trend, if any, that the moving median shows. 1 mark

Question 3 (6 marks)

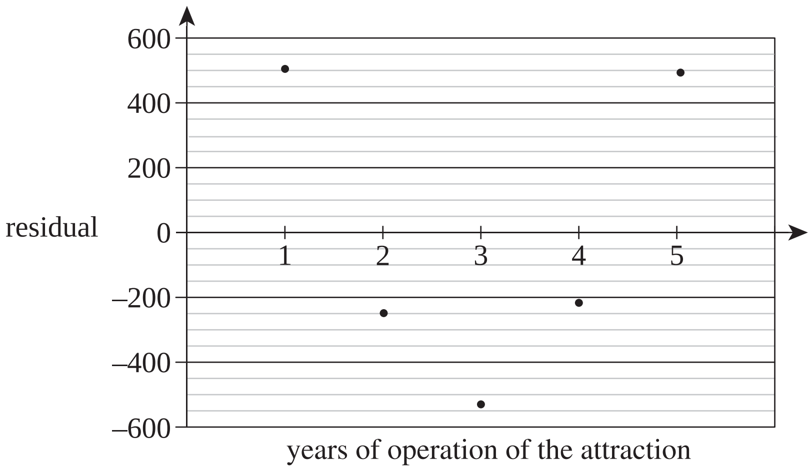
After several delays, an exciting new attraction opens at Waterfront at the end of 2009. Sally experiences substantial growth in shopper numbers to her store over the first five years of the attraction's operation.

Year	2010	2011	2012	2013	2014
Average visitors per day	2275	3000	4200	6000	8200
Years of operation of the attraction	1	2	3	4	5

a. Find the regression equation and complete the formula below. 2 marks

average shoppers per day = ____________ + ____________ × (years of operation of the attraction)

b. The residual plot for this data is shown below.



What conclusion, if any, can be made from the residual plot? 1 mark

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c. Apply an x^2 transformation to the original data.

Does this transformation improve the fit of the regression equation? Justify your answer and write down the new equation. 2 marks

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d. Use a three-point moving mean to predict the number of shoppers in Sally's store for 2015. 1 mark

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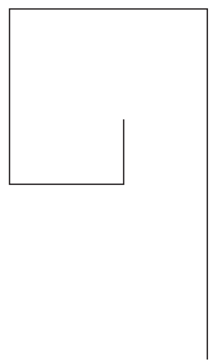
END OF SECTION A

Module 1: Number patterns

Question 1 (4 marks)

Claire makes a pattern by drawing a line of length 9 cm, and then turning 90 degrees left and drawing a further line of length 6 cm. A second 90-degree left turn is then made and a new line of length 4 cm follows. This process is repeated, with each turn being 90 degrees to the left and each ensuing line being $\frac{2}{3}$ of the length of the previous line. The first five such lines are shown in the diagram below.



a. What is the exact length of the fifth line? 1 mark

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b. What is the total length of the lines after the seventh line is drawn, to two decimal places? 2 marks

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c. If it were possible to extend this pattern infinitely, is there a limit as to the maximum total length of the pattern? If so, determine this precise limit. 1 mark

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Question 2 (7 marks)

Jason suggests an alternative pattern. The first line will be also of length 9 cm, but each succeeding line will be 1 cm shorter in length. Thus the second line has length 8 cm and the third has length 7 cm.

a. Find the length of the fifth line of this pattern. 2 marks

b. Find the total length of the lines after the seventh line is drawn of this pattern. 1 mark

c. What is the maximum total length of this pattern? 1 mark

Jason wants to change his pattern so that the total length is 36 cm. He will achieve this by making his first line shorter, but will still decrease each successive segment by 1 cm.

d. Determine the length of the first line so that Jason achieves his objective. 3 marks

Question 3 (4 marks)

Arthur has a third option for the pattern. In order to find successive lengths, he decides to use a difference equation of the form $t_{n+1} = \frac{1}{2}(t_{n-1} - t_n) + 2$, keeping the 90-degree left turn.

a. Arthur decides to draw the first and second lines as lengths 3 cm and 1 cm respectively.

Show that the resulting shape is a single rectangle. 2 marks

b. Arthur decides to change the first two terms so that the third and fourth terms are 3 and 2 respectively.

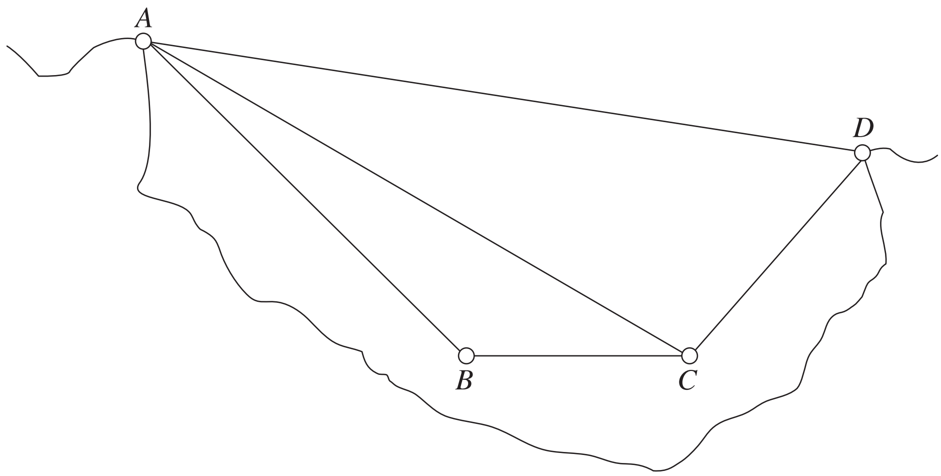
Find the first term, *a*, and the second term, *b*. 2 marks

END OF MODULE 1

Module 2: Geometry and trigonometry

Question 1 (5 marks)

The Slender Valley triathlon begins with a swim in the bay. This is shown in the diagram below.



Swimmers begin at Archer Point (*A*) and swim to Danby (*D*) via Brian Rocks (*B*) and Cradle Inlet (*C*). *B* is 3 km from *A* on a bearing of 135°T. *C* is 1 km due east of *B*. *A* is 5 km from *D* on a bearing of 290°T.

a. How far east is *B* from *A*? Answer to the nearest metre. 1 mark

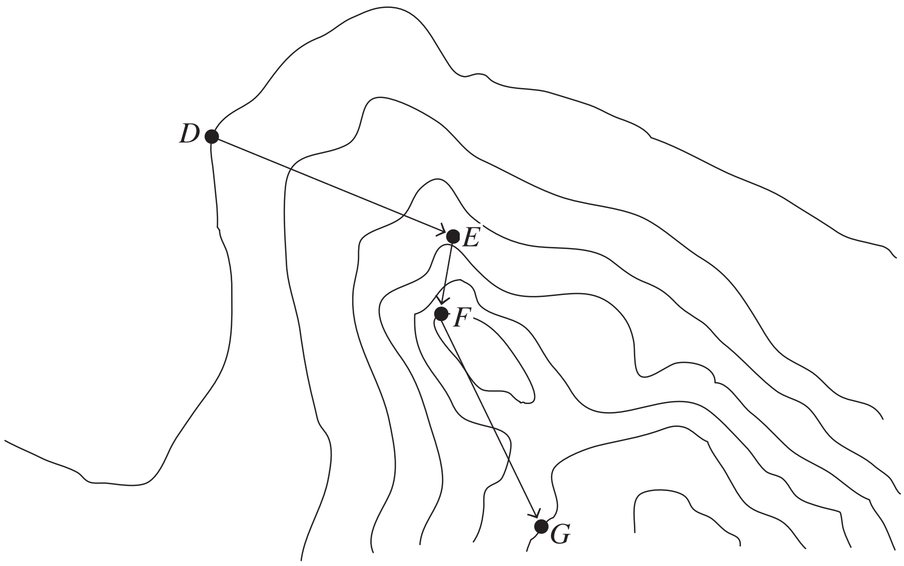
b. What is the bearing and distance of *C* from *A*? Answer to the nearest metre and tenth of a degree. 2 marks

c. How far apart are *B* and *D*? 2 marks

Question 2 (3 marks)

The second part of the triathlon is the cycling section.

This starts where the competitors come ashore at Danby (*D*) and continues onward up East Ridge past Egg Rock (*E*), Fantasia Bluff (*F*) and Granite Head (*G*). The map below has a contour interval of 10 m throughout.



The distance from *D* to *E* is 500 m, from *E* to *F* is 100 m and from *F* to *G* is 300 m horizontally along the track.

a. Determine which of the three sections is steepest, and the track's average gradient to two decimal places. 2 marks

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b. Determine the actual distance from *D* to *E* to the nearest 10 cm. Assume a constant gradient. 1 mark

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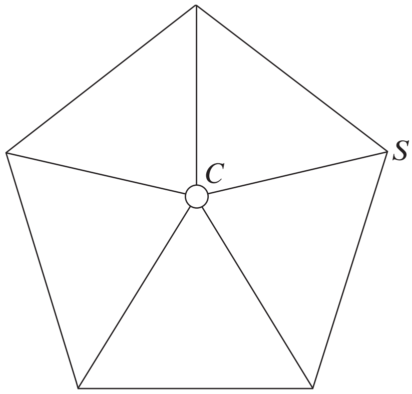
Question 3 (7 marks)

The running stage of the event is actually conducted using a regular pentagonal track. This is shown below.



Runners must start at *S* and finish at the central point, *C*, equidistant from each pentagon vertex. The runners must run around the pentagon before running to *C*.

There are five isosceles triangles within the pentagon.

a. **i.** Show that the largest of the angles within each isosceles triangle is 72°. 1 mark

ii. Find the size of the other angles. 1 mark

The distance around the pentagon is 10 km.

b. What is the distance from *S* to *C*? Answer in km, correct to the nearest metre. 2 marks

The organisers need to clear space for the pentagonal track and are concerned that a large area will need to be cleared.

c. What is the area that must be cleared? Give your answer in km^2, correct to two decimal places. 2 marks

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A scale model of the track is made with each side of the pentagon 20 cm long.

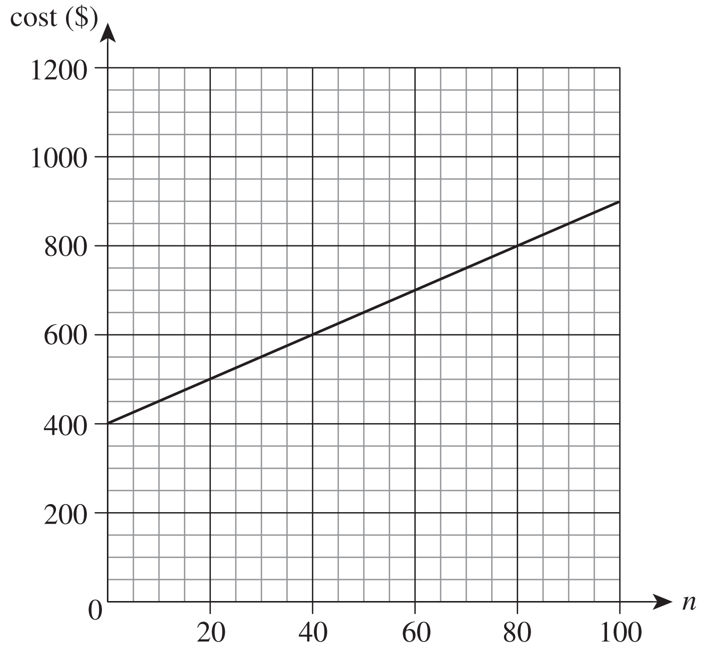
d. What is the angle at C in the scale model? 1 mark

END OF MODULE 2

Module 3: Graphs and relations

Question 1 (7 marks)

Abaci Industries specialise in making office machinery. The graph below shows the cost of making standard cash boxes.



a. **i.** Calculate the gradient of the line shown above. 1 mark

ii. Write an equation for the cost, C, of making n cash boxes. 1 mark

b. The revenue comes from selling the cash boxes. Each is sold for \$8.

i. Write down an equation for the revenue. 1 mark

ii. The price is changed and the new revenue equation is $R = 12n$.
Draw this line on the graph above. 1 mark

iii. How many boxes would need to be made and sold in order to make a profit? 1 mark

c. It is decided that the break-even point should occur when exactly 80 boxes are made and sold. The selling price will need to be altered to achieve this.

What should the price be to achieve this objective? 2 marks

Question 2 (8 marks)

The company makes both standard and deluxe cash boxes. Each has a different construction time, and the cost of the materials varies also. The company knows how many of each box it can sell and, as such, will not make more than this number. The details are as follows:

Box type	Hours to make	Cost of materials	Selling price	Maximum number
standard	2	\$4	\$9	150
deluxe	3	\$8	\$14	100

The company will make x standard and y deluxe boxes. They have 360 hours each week available to make either box. They have a weekly budget of \$840 for materials.

Some of the resulting constraints are as follows:

$$2x + 3y \leq 360$$
$$x \leq 150$$
$$y \leq 100$$

a. **i.** Explain the first of these constraints. 1 mark

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ii. Identify one more constraint. 1 mark

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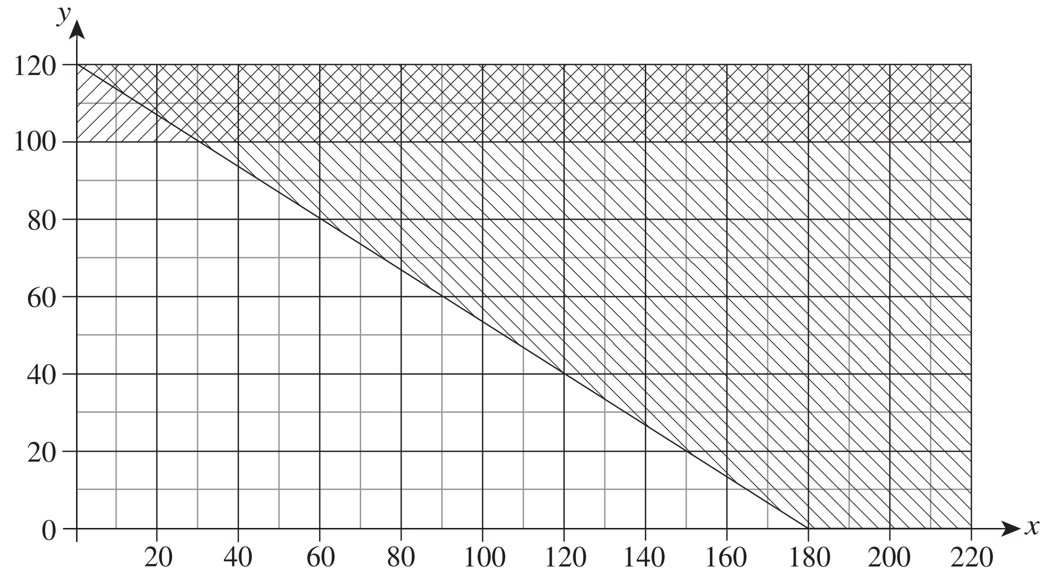
Some of the constraints have been graphed below.



b. Complete the graph, including all of the constraints listed. 2 marks

c. If 70 standard boxes are made in one week, what is the maximum number of deluxe boxes that can be made? 1 mark

d. Determine the number of each type of box that should be made so revenue is maximised. 2 marks

e. Determine the maximum weekly revenue. 1 mark

END OF MODULE 3

Module 4: Business-related mathematics

Question 1 (3 marks)

Kirsten purchased a set of car tyres for $780. A 25% deposit was required.

a. Calculate the size of the deposit. 1 mark

b. Calculate the amount owing immediately after the deposit is paid. 1 mark

The amount owing after the deposit will be repaid in five equal repayments.

c. Calculate the amount that will be owing after the third of these five repayments. 1 mark

Question 2 (6 marks)

Kirsten purchased a car for \$32 000. She considered depreciating the car at a flat rate of 7.5% per annum of the purchase price.

a. **i.** Calculate the annual depreciation amount. 1 mark

ii. Calculate the value of the car after two years. 1 mark

iii. After how many years will the car be worth \$20 000? 2 marks

Kirsten considered reducing balance depreciation at a rate of 6.5% per annum.

b. **i.** Calculate the value of the car after two years. 1 mark

ii. By what amount did the car depreciate during the third year? Write your answer correct to the nearest cent. 1 mark

Question 3 (2 marks)

Kirsten agreed to an interest-only loan of \$32 000 for her car. Interest is charged monthly and Kirsten's repayments are \$117.33 per month.

a. Calculate the annual interest rate on this loan. Write your answer correct to one decimal place. 1 mark

b. Calculate how much is still owing on this loan immediately after the twelfth repayment. 1 mark

Question 4 (4 marks)

Kirsten takes out a reducing balance loan of $690 000 to purchase a new home. The loan will be fully repaid with monthly repayments over 25 years. The interest rate is 5.3% per annum.

a. Calculate the monthly repayment. Write your answer correct to the nearest cent. 1 mark

b. Calculate the amount owing immediately following the 50th repayment. Write your answer correct to the nearest dollar. 1 mark

c. Calculate the total interest paid immediately following the 100th repayment. Write your answer correct to the nearest dollar. 2 marks

END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1 (4 marks)

a. Draw a complete planar graph with 4 vertices. 1 mark

b. How many edges does the graph from part **a.** have? 1 mark

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c. Draw a subgraph of the graph from part **a.** that is connected and has one vertex of degree 3 and no vertices of degree 2. 1 mark

d. Consider a network that has 4 vertices. The degrees of the vertices are d_1, d_2, d_3 and d_4.

If E is the number of edges in the network, then it can be shown that $E = \frac{d_1 + d_2 + d_3 + d_4}{C}$,

where C is a constant for all networks with 4 vertices.

Find the value of C. 1 mark

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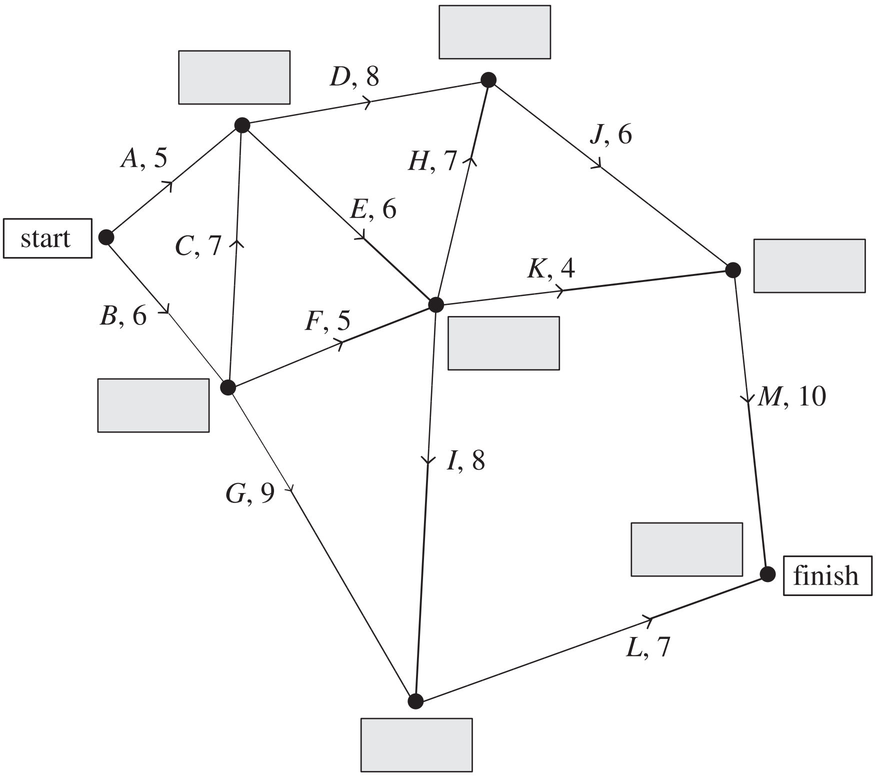
Question 2 (6 marks)

A construction project involves tasks *A*–*M*.

a. The number of days required for each task and their order of precedence is presented in the network diagram below.

Perform a forward and backward scan, recording the results in the shaded rectangular boxes located at each vertex of the diagram. Record the earliest and latest starting time at each vertex, using a comma to separate the numbers in each box. 2 marks



b. List the tasks that are on the critical path. 1 mark

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c. What is the minimum time required to complete the project? 1 mark

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d. Which activity has the greatest float time? 1 mark

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e. Extra resources can be allocated to task *M*. If this is done, the duration of task *M* will be reduced.

What is the maximum reduction in completion time of the entire project that can be achieved by reducing the duration of task *M*? 1 mark

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Question 3 (3 marks)

A government has three projects (*A*, *B* and *C*) that it has invited companies to tender (or bid) for.

The companies tender using code numbers rather than their company names. There are three companies eligible, and each is restricted to undertaking just one of the three projects. They may, however, tender for any or all of the projects.

The government receives bids as set out in the following table.

	Project *A*	**Project *B***	**Project *C***
Company 1	$1 million	$3 million	$4 million
Company 2	$4 million	$2 million	$1 million
Company 3	$1 million	$4 million	$3 million

The government is obliged to allocate the projects to the companies in a way that minimises the total cost to themselves.

a. Draw a bipartite graph to present the information in the above table. The amounts of each tender should be written on the corresponding edge of the graph. 1 mark

b. Use the Hungarian algorithm to find how to allocate the projects so that the total cost is minimised. Draw a matrix to show the effect of each step in the algorithm, then write the the numbers 1–3 in the table below to indicate which company should be allocated each project. 1 mark

	Project *A*	**Project *B***	**Project *C***
Company			

c. Calculate the minimum total cost to the government for all three projects. 1 mark

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Question 4 (2 marks)

A project requires five tasks to be completed: *A*, *B*, *C*, *D* and *E*. The immediate prerequisite tasks required for each of these are listed in the table below.

Task	Prerequisite tasks to be completed prior to this task
A	
B	
C	*A*, *B*
D	*A*, *B*
E	*C*, *D*

a. Draw a directed graph to describe this project. Label the edges of your digraph with a letter for each task and indicate the start and finish of the project. 1 mark

b. List all the paths that could be critical paths for this project. 1 mark

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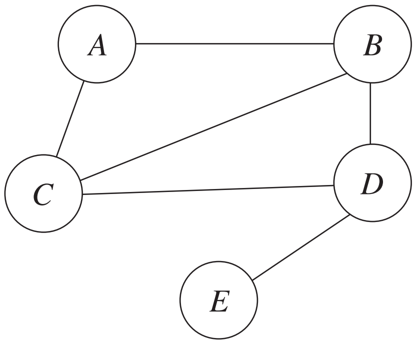
END OF MODULE 5

Module 6: Matrices

Question 1 (8 marks)

The diagram below shows the electricity cables connecting five towns. However, one wire is missing from the diagram.



The matrix below represents this system. A value of 1 indicates that a cable exists between the towns indicated in the row and column headings.

$$\begin{array}{c} \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \end{array}$$

a. Between which two towns is there an electricity cable missing? 1 mark

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b. Find the sum of all the elements in row 2. 1 mark

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c. In terms of electrical cables between towns, what does the sum of row 2 represent? 1 mark

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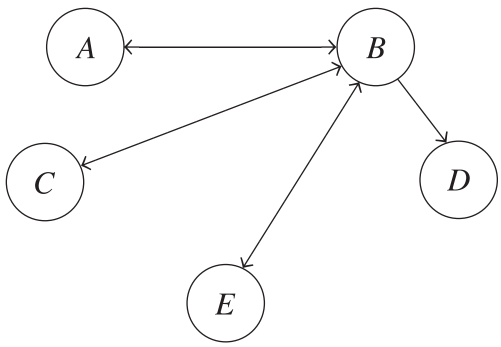
An engineer used the matrix shown on the previous page and noted that it seems inadequate, as it ignores direction of current. In fact, some of the cables are single-directional; in these cases, no current can flow in the opposite direction. He produces the new matrix below.

$$F = \begin{array}{c} \text{current from town} \\ \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} \text{ current to town}$$

The engineer draws a diagram illustrating this arrangement but it is incomplete, as only the cables connected to town B are shown. A double-ended arrow indicates cabling in both directions.



d. Complete the diagram above. 1 mark

e. Calculate the sums of all rows. What do these sums represent? 1 mark

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The engineer notes that some towns are more susceptible to power problems as they are reliant on a single connection. He thus calculates F^2 as a means of determining second-order supply.

f. Calculate F^2. 1 mark

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$$F^2 = \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

The engineer wants to multiply by a matrix G so that F^2G gives a 5×1 matrix with the sums of all the rows of F^2. He will use this matrix to evaluate supply connection vulnerability.

g. Determine matrix G. 1 mark

h. Which town has the most connections and is thus least vulnerable to a supply connection failure? 1 mark

Question 2 (7 marks)

The populations of the five towns vary over time due to population movements. Some increase while others decrease. The transition matrix T shows how the populations change annually.

$$T = \begin{array}{c} \text{from town} \\ \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \left[\begin{array}{ccccc} 0.93 & 0.01 & 0 & 0 & 0 \\ 0.01 & 0.92 & 0 & 0.01 & 0.05 \\ 0.02 & 0 & 0.97 & 0.01 & 0.01 \\ 0.02 & 0 & 0 & 0.83 & 0.02 \\ 0.02 & 0.07 & 0.03 & 0.15 & 0.92 \end{array}\right] \end{array} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} \text{ to town}$$

a. What proportion of residents of A travel to live in C every year? 1 mark

The populations of the five towns in 2014 are given in the table below.

Town	*A*	*B*	*C*	*D*	*E*
Population	3200	2000	1800	2500	500

b. Find the population of E in 2015. 1 mark

c. In what year does the population of E first exceed that of D? 2 marks

d. Determine the long-term populations of each town. 2 marks

The council overseeing the five towns wishes the total population to remain constant at their 2014 values, not at the long-term stable values found in part **d.**

e. How many people will need to be either encouraged to settle into or to leave each town every year for this to occur? 1 mark

END OF QUESTION AND ANSWER BOOKLET