2015
VCE
Further Mathematics
Trial Examination 2
Suggested Answers

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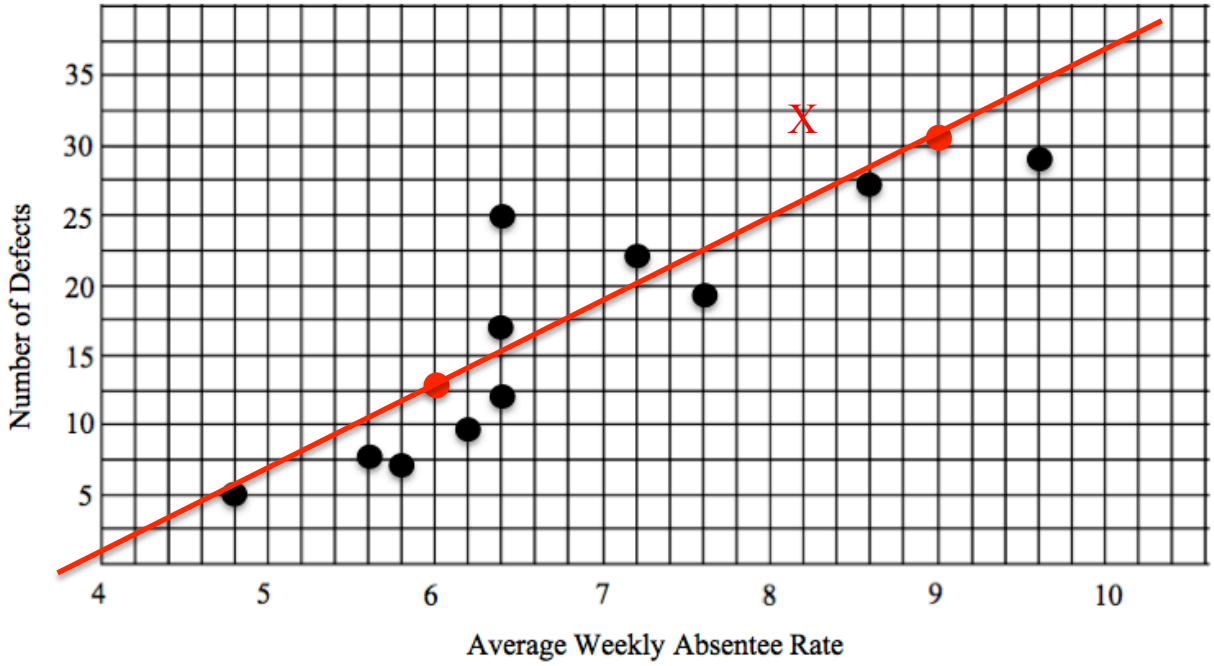
Core: Data analysis

Question 1

a. X shows this point



(1 mark)

b.
Use calculator to get
Number of defects = 6 × Average weekly absentee rate − 23

(1 mark)

c.
To draw the line, find 2 points.
e.g. When $x = 9$, $y = 54 - 23 = 31$. Point is (9,31)
When $x = 6$, $y = 36 - 23 = 13$. Point is (6,13)
Mark these 2 points on the graph and draw the line through them.

(1 mark)

d.
The gradient predicts that the number of weekly defects increases by 6 for each additional average weekly absentee rate.

(1 mark)

e.
Residual value = Actual value – Predicted value.
Actual value = 32
Predicted value = 6 × 8.2 – 23 = 26.2
Residual = 32 – 26.2 = 5.8

(1 mark)

Core: Data analysis

Question 2

a.

$$\text{Unacceptable} = \frac{7}{20} \times 100 = 35\%$$

(1 mark)

b.

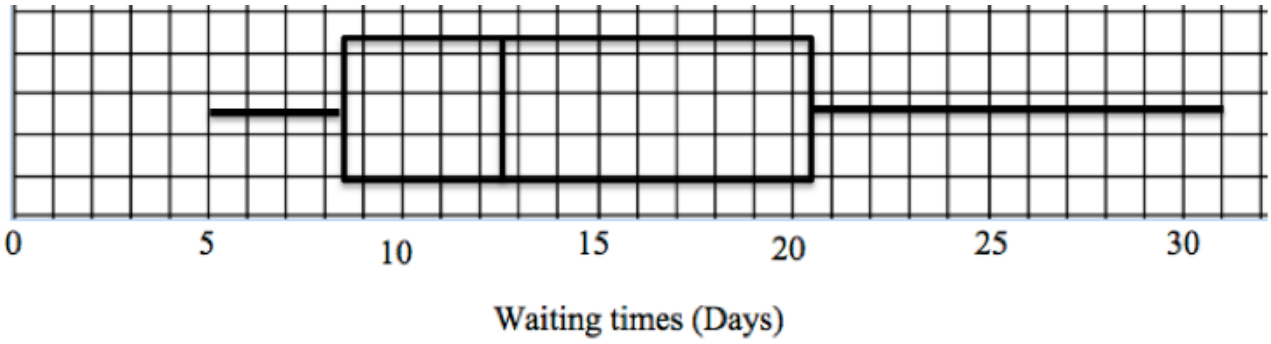
Use calculator to get
Mean = 16.3

(1 mark)

Standard deviation = 7.7

(1 mark)

c.



Minimum = 5
LQ = 8.5
Median = 12.5
UQ = 20.5
Maximum = 31

1 mark for correct values
1 mark for correct positioning.

(2 marks)

Core: Data analysis

Question 3

a.
Because the normal distribution graph is symmetrical about the mean, 50% will have waiting times of less than 15 days.

(1 mark)

b.
$z = \frac{19-15}{2} = 2$
19 is 2 standard deviations above the mean of the standard normal curve.
95% lie within ± 2 standard deviations of the mean. So 5% lie outside. Half of these, i.e. 2.5% lie above 19.
2.5% have to wait more than 19 days.

(1 mark)

c.
$z_1 = \frac{11-15}{2} = -2$
$z_2 = \frac{21-15}{2} = 3$
Less than -2 is 2.5%
Greater than 3 is 0.15%
Outside -2 and 3 is 2.5 + 0.15 = 2.65
Between -2 and 3 is 100 – 2.65 = 97.35%

(1 mark)

d.
2.5% has $Z_1 = 2$ on standard normal curve and 12 on the normal curve.
16% has $Z_2 = -1$ on standard normal curve and 3 on the normal curve.

$Z_1 = 2 = \frac{12-\bar{x}}{s} \Rightarrow 2s = 12 - \bar{x}$

$Z_2 = -1 = \frac{3-\bar{x}}{s} \Rightarrow -s = 3 - \bar{x}$

Use calculator to solve these 2 simultaneous equations
This gives
Mean = 6, Standard deviation = 3

(2 marks)

Module 1: Number patterns and applications

Question 1

a.
From the graph it can be seen that when the number of years is 1, the projected number of car sales is 1500.

(1 mark)

b.
From year 1 to year 3, the number of car sales increases by 1000. This is an arithmetic sequence because the graph is a straight line. Hence, the increase in sales each year is a constant of 500.

(1 mark)

c.
$a = 1500$

$d = 500$

$n = 10$

$t_n = a + (n-1) \times d$

$t_{10} = 1500 + (10-1) \times 500 = 6000$

(1 mark)

d.
$S_n = \frac{n}{2}[2a + (n-1) \times d]$

$S_{10} = \frac{10}{2}[2 \times 1500 + (10-1) \times 500] = 37500$

(1 mark)

e.
$1500 + (n - 1) \times 500 > 15600$
$15 + 5n - 5 > 156$
$5n > 146$
$n > 29.2$

In the 30th year of production.

(1 mark)

Module 1: Number patterns and applications

Question 2

a.
$2100 \div 1500 = 1.4$
$2940 \div 2100 = 1.4$
Since these both give the answer 1.4, there is a common ratio, and so the sequence is geometric.

(1 mark)

b.
$t_n = ar^{n-1}$

$t_6 = 1500 \times (1.4)^5 = 8067$

(1 mark)

c.
Number of cars expected to be sold = 37500
Number of cars produced = S_{10}

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{1500(1.4^{10} - 1)}{1.4 - 1} = 104720$$

Number of cars not sold = 104720 – 37500
= 67220

(1 mark)

d.
95% of 8067.36 = 7664

(1 mark)

e.
$4000 + (n-1) \times 500 > 8067 \times 0.95^{n-1}$

Use solve on calculator to solve equation

$4000 + (n-1) \times 500 = 8067 \times 0.95^{n-1}$

This gives $n = 5.7$

n is an integer greater than 5.7

$n = 6$

The sixth year of production is the first year, so the 11th year of production is when $n = 6$.
11th year of production is the answer.

(1 mark)

Module 1: Number patterns and applications

Question 3

a.

$p = 1.1$ and $q = 2000$, $k = 45000$

$I_2 = 1.1 \times 45000 + 2000 = \$51500$

(2 marks)

b.

Salary forms a geometric sequence and bonus form an arithmetic sequence.

Geometric sequence has $a = 48000, r = 1.08$

Arithmetic sequence has $a = 2000, d = 180$

$48000 \times 1.08^{n-1} + 2000 + (n-1) \times 180 > 200000$

On calculator use solve to solve the equation

$48000 \times 1.08^{n-1} + 2000 + (n-1) \times 180 = 200000$

This gives $n = 19.2$

n is an integer greater than 19.2

$n = 20$

Her income will first exceed \$200000 in her 20th year of working for the company.

(2 marks)

c.

$I_2 = 1.05 \times 30000 + k$

$I_3 = 1.05(1.05 \times 30000 + k) + k = 41751$

Use solve on calculator to get

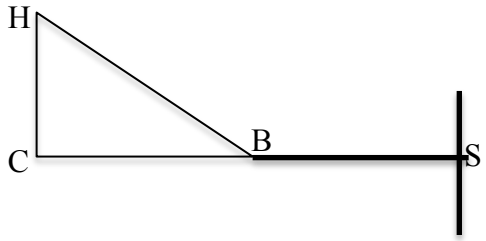
$k = 4232$

(1 mark)

Module 2: Geometry and trigonometry

Question 1

a.



$HB^2 = HC^2 + CB^2 = 80^2 + 30^2$

$HB = \sqrt{80^2 + 30^2} = 85.4 \text{ m}$

(1 mark)

b.

$\tan \angle HBC = \frac{80}{30}$

$\angle HBC = \tan^{-1}\left(\frac{80}{30}\right) == 69.44$

$\angle HBS = 180 - 69.44 = 111^0$

(1 mark)

c.

Triangle *HBS* is isosceles, so base angles are equal. All angles in this triangle add to 180^0.

$110.56 + 2 \times \angle HSB = 180$

$\angle HSB = 35^0$

(1 mark)

d.

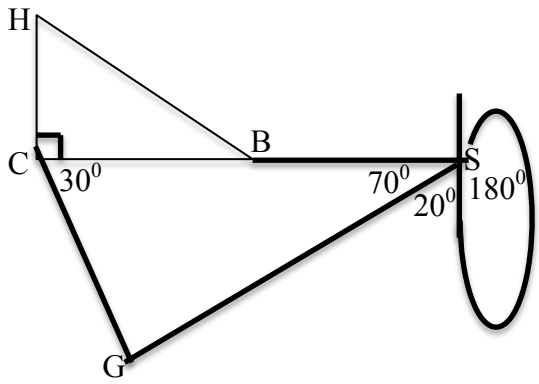
Measuring the angle in a clockwise direction from north at *S* we get $270 + 35 = 305^0$T

(1 mark)

Module 2: Geometry and trigonometry

Question 2

a.



$\angle CSG = 70^0$

(1 mark)

b.

$\angle CGS = 180 - (70 + 30) = 80^0$

(1 mark)

c.

$CS = 30 + 85.4 = 115.4$

$$\frac{115.4}{\sin 80^0} = \frac{GS}{\sin 30^0}$$

$GS = 59 \text{ m}$

(1 mark)

d.

$$\text{Area} = \frac{1}{2} \times 80 \times 30 + \frac{1}{2} \times 115.4 \times 58.6 \times \sin 70^0$$

$\text{Area} = 4377 \text{ m}^2$

(1 mark)

Module 2: Geometry and trigonometry

Question 3

a.

OA is the radius of the largest bowl.
OA = 14 cm.

(1 mark)

b.

$$V = \frac{1}{2} \times \frac{4}{3} \times \pi \times 14^3$$
$$V = 5747 \text{ cm}^3$$

(1 mark)

c.

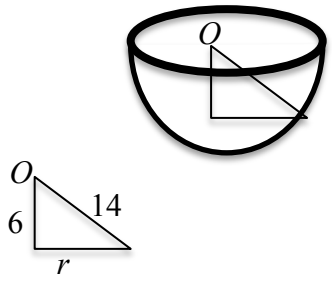
Ratio of lengths = $\sqrt[3]{64} : \sqrt[3]{1} = 4:1$

Diameter of small bowl = $\frac{1}{4} \times 28 = 7$

Radius = 3.5 cm.

(1 mark)

d.



r is the radius of the water.
Since the radius of the dish is 14 and the depth of the water is 8, then the vertical part of the triangle is 14 – 8 = 6.
Using Pythagoras,
$r^2 + 6^2 = 14^2$
$r = 12.65$ cm

(1 mark)

Module 2: Geometry and trigonometry

Question 4

a.

Let the height of the small pyramid that is cut off the top be h.

The small pyramid cut off and the large original pyramid are similar figures, so their sides are in the same ratio.

Height of large pyramid = $h + 40$

Base of small pyramid = 30.

Base of large pyramid = 50

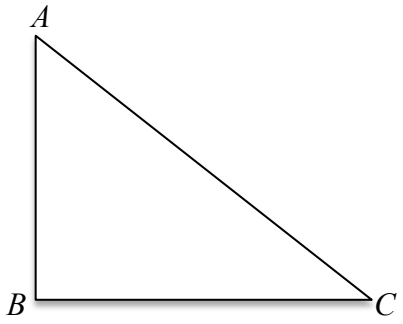
$$\frac{h}{h+40} = \frac{30}{50}$$

$$h = 60$$

Height of original pyramid = 60 + 40 = 100 cm.

(1 mark)

b.



AB is the height of the pyramid = 100

BC is half the distance across the side of the base = 25

AC is the height of a face of the pyramid.

$$AC^2 = 100^2 + 25^2$$

$$AC = 103.08$$

Area of triangular face = ½ × 50 × 103.08

Area of triangular face = 2577 cm^2

(1 mark)

c.

Ratio of lengths = 3 : 5

Ratio of areas = 9 : 25

Area of face of small pyramid that was cut off = $\frac{9}{25} \times 2576.94 = 927.7$

Area of side of glass case = $2576.94 - 927.7 = 1649.24$

Area of 4sides of glass case = $4 \times 1649.24 = 6596.96$

Area of 4sides of glass case and top and bottom = $1649.24 + 50^2 + 30^2 = 9996.96$ cm^2

= 1 m^2

(1 mark)

Module 3: Graphs and relations

Question 1

a.

$10x + 20y \le 160$

$\Rightarrow x + 2y \le 16$

(1 mark)

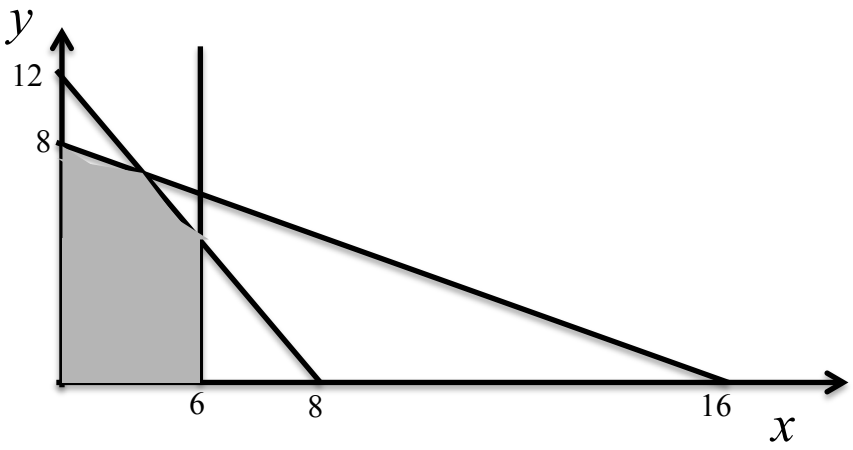
b.

$6x \le 36$

$\Rightarrow x \le 6$

(1 mark)

c. and **d.**



Missing inequality is $x \le 6$

(1 mark)

e.

$P = 200x + 120y$

(1 mark)

f.

Corner points satisfying the constraints.
(0,0), (0,8), (6,0) and the point of intersection of the lines, $3x + 2y = 24$ and $x = 6$. This gives the point $x = 6$, $y = 3$
Also, the point of intersection of the lines
$3x + 2y = 24$ and $x + 2y = 16$
Using a calculator to solve these simultaneous equations gives $x = 4$ and $y = 6$.
Profits are \$0, \$960, \$1200, \$1560 and \$1520
Maximum profit is \$1560

(2 marks)

Module 3: Graphs and relations

Question 1

g. Make 6 snow jackets and 3 rain jackets. (1 mark)	**h.** The corner points would now be (0,8), (0,3), (4,6) and the point of intersection of the lines, $3x + 2y = 24$ and $y = 3$. This gives the point $x = 3$, $y = 6$ Profit = $260x + 100y$ By inspection, (4,6) will give a larger profit than the other 3 points. P = \$1640 So, the profit will increase by 1640 – 1560 = \$80 (1 mark)

Question 2

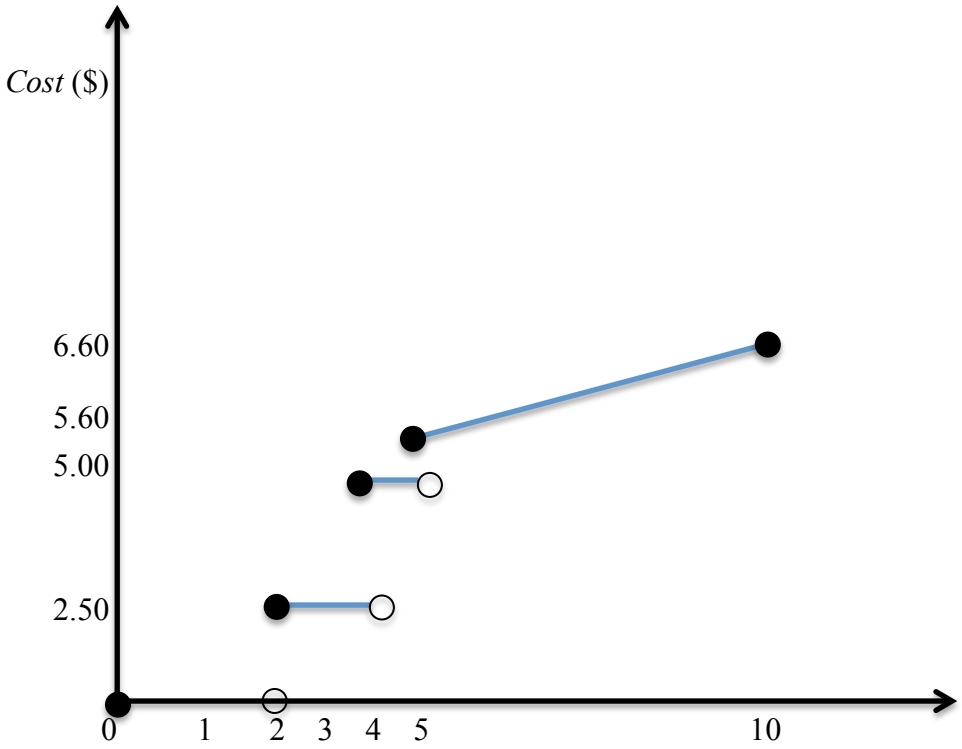
a. 180 + 50 = 230. (1 mark)	**b.** Sale price = \$200 Profit = \$120 Cost price = 200 – 120 = \$\$80 (1 mark)

Question 3

a. Entry cost for week = $(10 + 4 \times 2) \times 7$ = \$126 Cost of lifts = $20 \times 3 + 45 \times 2$ = \$150 Total cost = 126 + 150 = \$276 (1 mark)	**b.** Cost for 8 hours = $5 + 0.1 \times 12$ = \$6.20 (1 mark)
c. Cost of parking for 2 days = 10 Cost of parking for 5 days = 42 – 10 = 32 This would average at more than \$6 for the other 5 days, so use $t \geq 5$. In this case cost for 5 days = $5 \times [5.00 + 0.1(2t - 4)] = 32$ Solve on calculator to get $t = 9$ hours. (1 mark)	

Module 3: Graphs and relations

Question 3 (continued)



(1 mark)

Module 4: Business-related mathematics

Question 1

a.
400,000 – 332,000 = $68,000
She has to use $68,000 of her own money.
This leaves 80,000 – 68,000 = $12,000
(1 mark)

b.
33⅓% × 18,000 = 100 ÷ 300 × 18,000 = $6,000.
(1 mark)

c.
Amount paid = 6000 + 420 × 36 = 21,120.
Cash price = 18,000
Interest = 21,120 – 18,000 = $3,120.
(1 mark)

d.

$$I = \frac{PRT}{100}$$

$$3120 = \frac{(18000 - 6000) \times R \times 3}{100}$$

$$R = 8.7\%$$

(1 mark)

e.

$$\text{Effective interest rate} \simeq \frac{2n}{n+1} \times \text{ flat rate} = \frac{2 \times 36}{36+1} \times 8.6667 = 17\%$$

Question 2

a.
6% of 18,000 = 0.06 × 18,000 = $1080
(1 mark)

b.
Amount to be depreciated by
= 18,000 – 5040 = 12,960
Number of years for this depreciation
= 12960 ÷ 1080 = 12 years.
(1 mark)

c.

$$A = P\left(1 - \frac{r}{100}\right)^n$$

$$5040 = 18000\left(1 - \frac{r}{100}\right)^{10}$$

Use solve on calculator to get r = 12%

(1 mark)

Module 4: Business-related mathematics

Question 3

a.

Use TVM solver
$N = 18 \times 12$
$I = 5.2$
$PV = 332000$
$PMT =$
$FV = 0$
$P/Y = 12$
$C/Y = 12$
This gives $PMT = \$2370$

(1 mark)

b.

Use TVM solver
$N = 60$
$I = 5.2$
$PV = 332000$
$PMT = -2370.0769$
$FV =$
$P/Y = 12$
$C/Y = 12$
This gives $FV = \$268334$
Amount owing = 268334 – 150000 = $118,334

(1 mark)

c.

Use TVM solver
$N =$
$I = 5.2$
$PV = 118334$
$PMT = -1916$
$FV = 0$
$P/Y = 12$
$C/Y = 12$
This gives $N = 72$
72 months = 6 years.

(1 mark)

d.

Actual time of repayment will be a little over 72 months.

Use TVM solver
$N = 72$
$I = 5.2$
$PV = 118334$
$PMT = -1916$
$FV =$
$P/Y = 12$
$C/Y = 12$
This gives $FV = -63.93631138$

Use TVM solver
$N = 1$
$I = 5.2$
$PV = 63.93631138$
$PMT =$
$FV = 0$
$P/Y = 12$
$C/Y = 12$
This gives $FV = \$64.22$

(1 mark)

Module 4: Business-related mathematics

Question 4

a.
Use TVM solver
$N = 12$
$I = 6.5$
$PV = 80000$
$PMT =$
$FV = -80000$
$P/Y = 12$
$C/Y = 12$
This gives $PMT = -433.33333$
Monthly repayments = \$433.33

(1 mark)

b.
Amount paid to bank each month
$= 24 \times 433.3333 = 10400$
Amount paid to bank = 10400 + 80000 = 90,400
Profit = 120,000 – 90,400 = \$29,600

(1 mark)

c.
Q = 1530 per month = 1530 × 12 = \$18,360 per year.
P = 200,000 + 29,600 = 229600

$$Q = \frac{P \times r}{100}$$

$$18360 = \frac{229600 \times r}{100}$$

Use solve on calculator to get $r = 8\%$

(1 mark)

Module 5: Networks and decision mathematics

Question 1

a.
Teams defeated by Edithaven besides Carramatta were Aliceville and Beaconstown.

(1 mark)

b.

$$D^1 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

(1 mark)

c.
It gives the number of games won by each of the towns.

(1 mark)

d.

$$D^2 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

(1 mark)

e.
Aliceville is now 2 + 4 = 6, Beaconstown is now 1 + 2 = 3, Carramatta is now 1 + 1 = 2,
Dixonvale is now 5 + 3 = 8, Edithaven is now 3 + 4 = 7

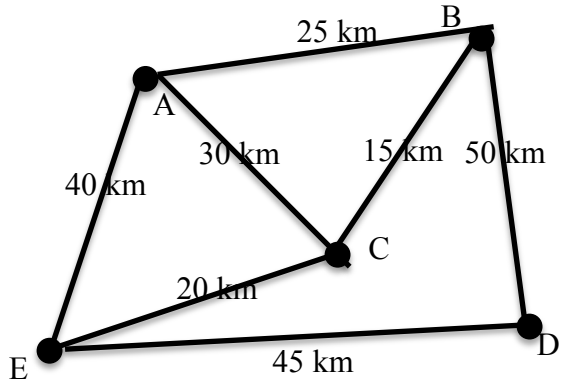
First is Dixonvale
Second is Edithaven
Third is Aliceville
Fourth is Beaconstown
Fifth is Carramatta

(1 mark)

Module 5: Networks and decision mathematics

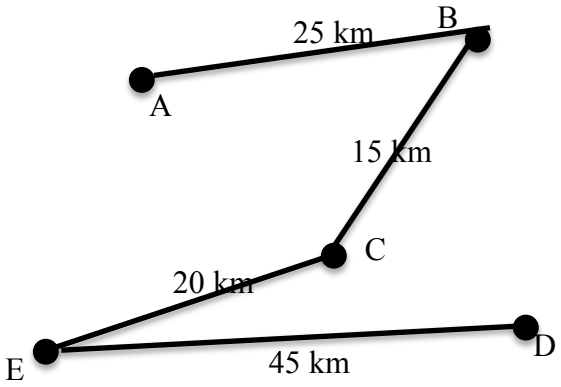
Question 2

a.



(1 mark)

b.



(1 mark)

c.

25 + 15 + 20 + 45 = 105 km.

(1 mark)

d.

There are more than 2 odd vertices, so it will not be possible for Mimi to travel on each road only once.

(1 mark)

Module 5: Networks and decision mathematics

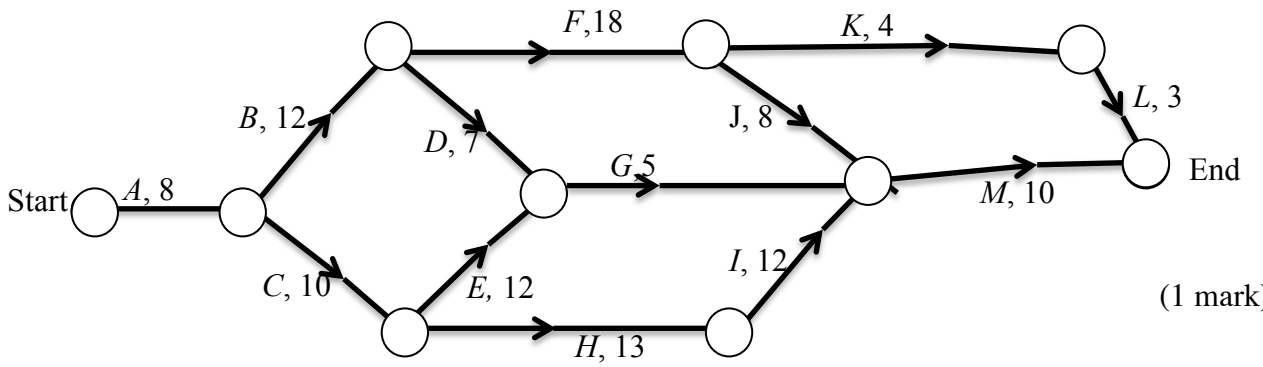
Question 3

a.

D and E.

(1 mark)

b.



(1 mark)

c.

Critical path is A – B – F – J – M.
Shortest time to complete project
= 8 + 12 + 18 + 8 + 10 = 56 days.

(1 mark)

d.

Float time = Latest start time – Earliest start time
Latest start time = 56 – 10 – 5 = 41
Earliest start time = 8 + 10 + 12 = 30
Float time = 41 – 30 = 11 days.

(1 mark)

e.

Extra workers reduce the number of days.
Reducing the number of days on an activity that does not lie on the critical path will not reduce the total time. Hence, F, which lies on the critical path is the one to be reduced.

(1 mark)

f.

To get to the start of activity M.
A – C – H – I = 8 + 10 + 13 + 12 = 43
A – C – E – G = 8 + 10 + 12 + 5 = 35
A – B – D – G = 8 + 12 + 7 + 5 = 32
For A – B – F – J to still be part of the critical path it cannot be less than 43.
Hence, F = 15 which means it is reduced by 3 days.
Cost of 3 days reduction for F
= 800 × 3
= $2,400

(1 mark)

Module 6: Matrices

Question 1

a.

$$X = \begin{array}{c} \\ May \\ Nat \\ Owen \end{array} \begin{array}{c} \begin{array}{cccc} A & B & O & M \end{array} \\ \begin{bmatrix} 6 & 5 & 3 & 1 \\ 2 & 3 & 3 & 4 \\ 5 & 4 & 1 & 2 \end{bmatrix} \end{array}$$

(1 mark)

b.

$$Y = \begin{array}{c} \\ A \\ B \\ O \\ M \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0.8 & 0.5 \\ 0.6 & 0.7 \\ 0.5 & 0.3 \\ 3.1 & 4.5 \end{bmatrix} \end{array}$$

(1 mark)

c.

$$XY = \begin{bmatrix} 12.4 & 11.9 \\ 17.3 & 22 \\ 13.1 & 14.6 \end{bmatrix}$$

(1 mark)

d.

The cost of the fruit bought by Natalie in shop A.

(1 mark)

e.

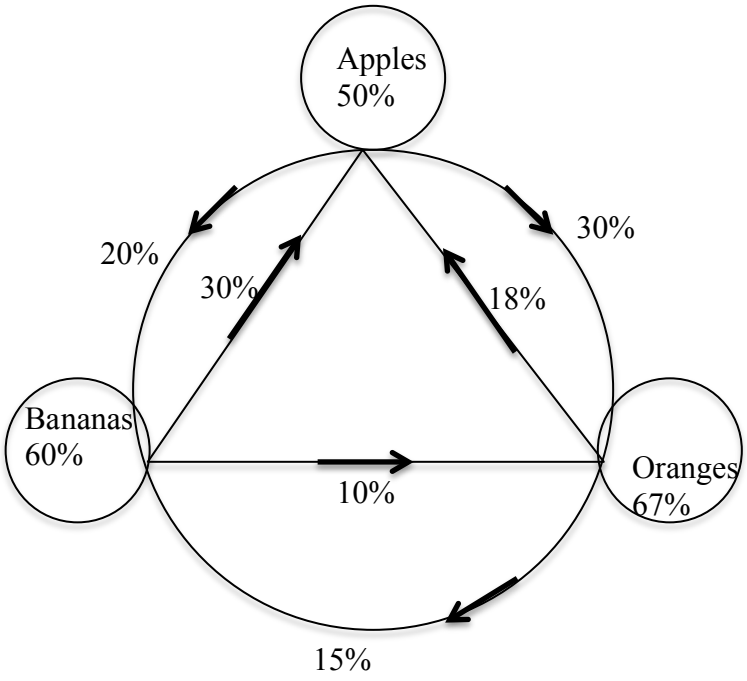
13.1 + 14.6 = $27.70

(1 mark)

Module 6: Matrices

Question 2

a.



(1 mark)

b.

$$\begin{array}{cc} & \begin{matrix} \textit{Apple} & \textit{Banana} & \textit{Orange} \end{matrix} \\ \begin{matrix} \\ \textit{To} \\ \\ \end{matrix} & \begin{matrix} \textit{Apple} \\ \textit{Banana} \\ \textit{Orange} \end{matrix} \begin{bmatrix} 0.5 & 0.3 & 0.18 \\ 0.2 & 0.6 & 0.15 \\ 0.3 & 0.1 & 0.67 \end{bmatrix} \end{array}$$

From

(1 mark)

c.

$$S_0 = \begin{matrix} \textit{apples} \\ \textit{bananas} \\ \textit{oranges} \end{matrix} \begin{bmatrix} 300 \\ 100 \\ 500 \end{bmatrix}$$

(1 mark)

d.

$$\begin{bmatrix} 0.5 & 0.3 & 0.18 \\ 0.2 & 0.6 & 0.15 \\ 0.3 & 0.1 & 0.67 \end{bmatrix} \times \begin{bmatrix} 300 \\ 100 \\ 500 \end{bmatrix} = \begin{bmatrix} 270 \\ 195 \\ 435 \end{bmatrix}$$

195 people are expected to buy only bananas next week.

(1mark)

Module 6: Matrices

Question 2 (continued)

e.

$$\begin{bmatrix} 0.5 & 0.3 & 0.18 \\ 0.2 & 0.6 & 0.15 \\ 0.3 & 0.1 & 0.67 \end{bmatrix}^3 \times \begin{bmatrix} 300 \\ 100 \\ 500 \end{bmatrix} = \begin{bmatrix} 277 \\ 255 \\ 368 \end{bmatrix}$$

Expect 368 people to buy oranges only in 3 weeks time.

(1mark)

f.

$$\begin{bmatrix} 0.5 & 0.3 & 0.18 \\ 0.2 & 0.6 & 0.15 \\ 0.3 & 0.1 & 0.67 \end{bmatrix}^{100} \times \begin{bmatrix} 300 \\ 100 \\ 500 \end{bmatrix} = \begin{bmatrix} 286 \\ 272 \\ 342 \end{bmatrix}$$

Expect 286 people to buy apples only in the long term.

(1mark)

Question 3

a.

The decoding matrix will be the inverse of A.

$$D = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

(1mark)

b.

AB = C

(1mark)

c.

DAB = DC

(1mark)

d.

$$DAB = DC$$
$$IB = DC$$
$$B = DC$$
$$B = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 20 & 10 & 67 \\ 35 & 19 & 114 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 20 \\ 5 & 7 & 7 \end{bmatrix}$$

(1mark)

End of Suggested Solutions 2015 VCE Further Mathematics Trial Examination 2

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