Engage Education Foundation

Units 3 and 4 Further Maths: Exam 2

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 1 hour 30 minutes writing time

Structure of book:

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Number of marks
Core	4	4			15
Modules			6	3	45
				Total	60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied:

- This question and answer booklet of 32 pages, with a sheet of miscellaneous formulas.

Instructions:

- Detach the formula sheet from this book during reading time.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Write all your answers in the spaces provided in this booklet.

Instructions

This examination consists of a core and six modules. Students should answer all questions in the core then select three modules and answer all questions within the modules selected.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.
Diagrams are not to scale unless specified otherwise.

Core: Data analysis

Question 1

The stem and leaf plot below shows the distribution of 14 peoples' hourly wages in a particular country.

Key 4|5 = $45/hr

1	1 5
2	4 6 9
3	2 2 8 9
4	5 7 7 8
5	1

a. Calculate the interquartile range (IQR) of the above distribution. Show all working.

1 mark

b. What are the values of the upper and lower fences? Show all working.

2 marks

Total: 3 marks

Question 2

The table below shows the percentage of people in a particular country, by wage bracket, for the years 1960 and 2000.

Wage bracket ($/hr)	Year	
	1960	2000
10 - 20	53.6%	28.7%
21- 30	22.8%	38.0%
31 – 40	8.5%	21.1%
41- 50	4.2%	8.3%
>51	0.9%	3.9%

a. If there were 3 000 000 people in this country and all of them were working, how many people were earning more than $30/hr in 1960?

1 mark

b. Does the information in the above table support the opinion that people's wages are associated with the year in which they are employed? Use percentages to support your answer.

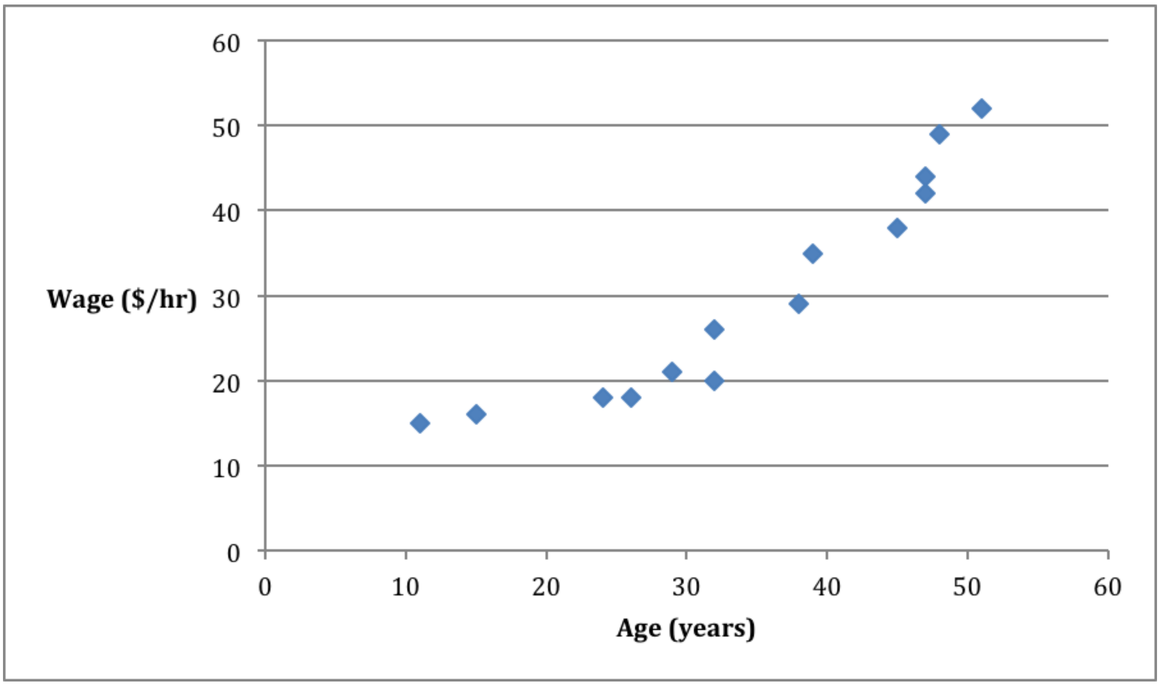
2 marks

Total: 3 marks

Question 3

The peoples' wages are now combined with their ages. This is shown in the table and scatterplot below.

Wage ($/hr)	Age (years)
11	15
15	16
24	18
26	18
29	21
32	20
32	26
38	29
39	35
45	38
47	42
47	44
48	49
51	52



c. Comment on the features of the above scatterplot.

4 marks

d. Suppose a linear regression line is now fitted to the data. What does the resulting residual plot indicate about the line that has been fitted?

1 mark

e. A logy transformation can be applied to linearize the data in the above scatterplot. Determine the new equation of the least squares regression line after the transformation has been performed.

2 marks

f. Calculate the r and r^2 values after the transformation has been applied. What effect has the transformation had on each of these values?

2 marks

Total: 9 marks

Section B

Module 1: Number patterns

Question 1

Farmer Susan breeds sheep on her property in Ballarat. She looks at various ways of maintaining animals but also selling them off to make a living. In her first year she starts off with 50 sheep and sells all of them at the end of the year and buys 4 less sheep at the start of the next year. For example, she has 46 sheep at the start of her second year and 42 sheep at the start of her third year, where each year there is a different batch of sheep.

a. How many sheep will she have in her 4^{th} year?

1 mark

b. Find a rule for S_n that represents the amount of sheep she has at the start of her nth year.

1 mark

c. At the start of which year will she have less than 4 sheep?

1 mark

d. At the start of the 8^{th} year how many sheep in total will she have had looked after?

1 mark

As well as buying and selling sheep she looks at modeling the average weight of her sheep each year. At the start of her first year, she notices that her sheep weigh an average of 48kg. She models the common ratio of the average weight of sheep between each successive year to be represented by $r = 1.1$. For example, in her second year her model predicts that her sheep will weigh an average of 52.8kg and in her third year her sheep will weigh an average of 58.08kg.

e. Is the sequence describing the average weight of sheep every year an example of an arithmetic or geometric sequence?

1 mark

f. What will be the average weight of her sheep in her 10^{th} year?

1 mark

g. Why is this model of average weight of sheep each year unrealistic?

1 mark

Total: 7 marks

Question 2

One summer, Susan's farm experiences a rabbit infestation. Due to the rapid breeding of rabbits the population of rabbits can be described using a difference equation:

$$R_{n+1}=2 \times R_n \text{ , } R_4 = 48$$

Where R_n denotes the number of rabbits in the nth year and R_4 denotes the number of rabbits in the fourth year.

a. If R_1 = b, determine the value of b.

1 mark

b. The difference equation above generates a geometric sequence. Find the values for *p* and *q* such that the number of rabbits in the nth year can be found using the rule: $R_n = q \times p^{n-1}$

2 mark

At a population level of 48 rabbits, the rabbits damage the vegetation for the sheep to eat so they pose a threat to Susan's farm. To counter the growing rabbit population she plays loud music during the day scare off the rabbits. However, this only slows the rabbit population and she still needs to physically remove some rabbits. If the rabbit population is now modeled as:

$$R_n = 1.5 \times R_n - c \text{ , } R_1 = 48$$

Where 'c' is the number of rabbits she plans to physically remove each year.

c. Find the value for c such that the rabbit population stabilizes each year. E.g. stays at 48.

1 marks

Total: 4 marks

Question 3

The following is a Fibonacci-related sequence

a, 5 , b , 12 …

a. Find the values for ‘a’ and ‘b’.

2 mark

b. With your values of ‘a’ and ‘b’ in part (a) write a difference equation describing the Fibonacci-related sequence above. Let t_n be the nth term of the series

1 marks

c. Find the sum of the first 7 terms of this sequence.

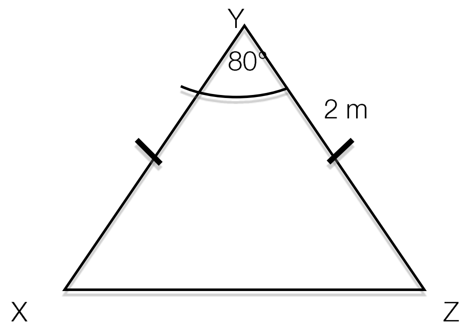
1 marks

Total: 4 marks

Module 2: Geometry and trigonometry

Question 1

Triangle XYZ is an isosceles triangle and is shown below:



a. What is the value of angle YXZ?

---

1 mark

b. Using Heron's formula, find the area of triangle XYZ to 2 decimal places.

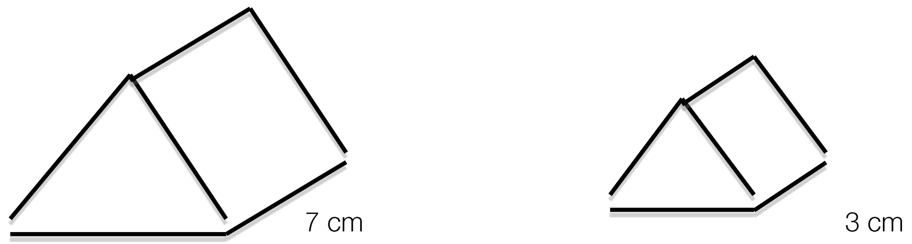
---

3 marks

Total: 4 marks

Question 2

Two similar blocks are shown below:



The smaller block has a total surface area of 8 cm^2 and a volume of 11 cm^3.

a. What is the total surface area of the larger block?

2 marks

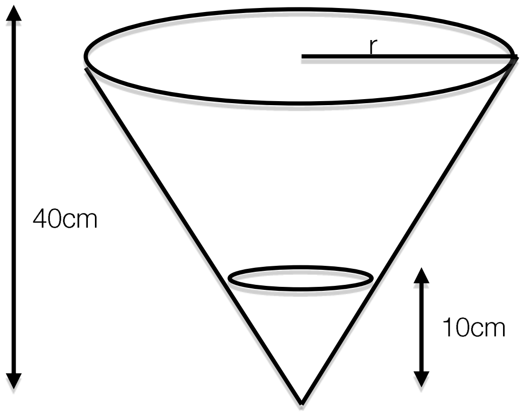
b. What is the volume of the larger block?

2 marks

Total: 4 marks

Question 3

A 40 cm tall miniature cone water tank currently holds 280 cm^3 of water that reaches 10cm on the cone.



a. What is the radius of this cone at the point where the water reaches, to 2 decimal places?

3 marks

b. What is the actual radius of the cone?

2 mark

c. What is the maximum volume of liquid the cone water tank can hold?

2 marks

Total: 7 marks

Module 3: Graphs and relations

Question 4

Shadow Hero is a company, which manufactures footballs and volleyballs. In any week, the production is subject to a number of constraints. By letting x represent the number of footballs and y represent the number of volleyballs manufactured in a week, the constraints can be listed as inequations. The inequation that can be formed from constraint A has been given below:

Constraint A: The total number of balls manufactured will not exceed 100:

$$x + y \leq 100$$

Constraint B: The total number of balls manufactured will not be less than 60.

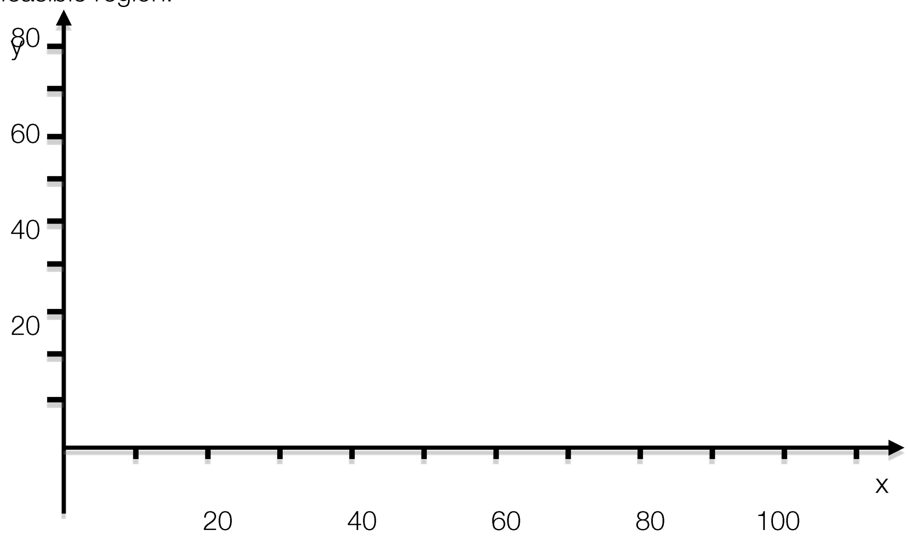
Constraint C: The number of footballs manufactured will be no more than 80.

Constraint D: The number of volleyballs manufactured will be no more than 50.

a. Write inequations for constraints B, C and D listed above.

3 marks

b. On the axes provided below, draw a graph showing all the constraints and clearly indicate the feasible region.



3 marks

c. Describe briefly what the feasible region represents within the context of this problem?

1 marks

d. The profit that Shadow Hero can expect in any given week is $30 for each football manufactured and $20 for each volleyball manufactured. Write an expression for the profit, P, in terms of x and y.

1 mark

e. Using your graph, determine the number of balls of each type that need to be manufactured in order for Shadow Hero to maximise its profit.

2 marks

f. Determine the maximum profit that can be made in any week.

2 marks

Total: 11 marks

Question 5

Manufacturing problems have beset Shadow Hero and profits have slipped. The profit that Shadow Hero can now expect is any given week is $15 for each football manufactured and $15 for each volleyball manufactured.

a. Write a new expression for profit.

1 mark

b. Explain why there are now many possible values of x and y that maximise profit.

1 marks

c. Write three specific solutions for the number of balls of each type that need to be manufactured in order for profit to be maximised.

2 marks

Total: 4 marks

Module 4: Business-related mathematics

Question 1

A student takes out $4000 loan to purchase a bike. The bank charges a simple interest rate of 6.5% per annum.

a. How much interest is the student paying after 3 years? (rounding to 1 decimal place)

1 mark

b. Find the interest paid after 5 years as a percentage of the original loan amount

1 mark

The student feels that they will have trouble paying back the loan over five years at that interest rate so the bank instead offers a compound interest rate of 6% per annum.

c. Over a 5-year period, explain which offer is the cheaper option for the student.

1 mark

Total: 3 marks

Question 2

The bike that the student purchased is originally valued at $4500 and the value of the bike depreciates 60c for every kilometre ridden on the bike.

a. If the student rides the bike 5 kilometres every day for a year, how much is the bike worth after a year?

____

____

1 mark

The student finds a second-hand bike online that is valued at $5000 but however depreciates using a reducing balancing depreciation at 16% per annum.

b. After 3 years, how much does this second-hand bike cost? (rounding to 1 decimal place)

____

____

1 mark

c. Consider the original bike priced at $4500 that depreciated at 60c for every kilometre and the second-hand bike priced at $5000 that depreciates uses reducing balance depreciation at 16% per annum. If the student wished to sell the bike in 4 years, which bike should the student purchase now?

____

____

3 mark

Total: 5 marks

Question 3

Laura takes a $12 000 loan from the bank with a flat interest rate of 4% per annum. She hopes to pay the bank back in monthly instalments over three years.

a. Calculate how much Laura will pay in her monthly instalments. Round to the nearest cent.

2 marks

b. Find the effective rate of interest per annum charged over these three years. Write you answer as a percentage, correct to one decimal place.

1 mark

c. Explain why the effect interest rate per annum is higher than the flat interest rate per annum.

1 mark

Total: 4 marks

Question 4

The price of a jacket on Monday is $600. However, on the Tuesday there is a discount of 20% on all clothes items including the Jacket. On Wednesday, the jacket prices rises up by 35% due to demand.

a. Comparing the final price on Wednesday to the original price on Monday has the jacket price decreased or increased overall and by how much?

2 marks

b. If jacket was already two years old and had depreciated each year using the reducing depreciation method at a rate of 2.4% per annum. What was the original value of the jacket?

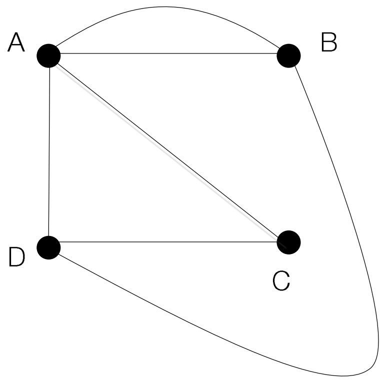
1 mark

Total: 3 marks

Module 5: Networks and decision mathematics

Question 1

Four friends live near each other in a neighbourhood. The network belows shows the various ways each friend can get to one another's houses. The edges shows the possible roads and each vertex is someone's house labelled A, B, C and D.



a. Complete the following adjacency matrix describing the allowed links.

	A	B	C	D
A				
B				
C				
D				

2 marks

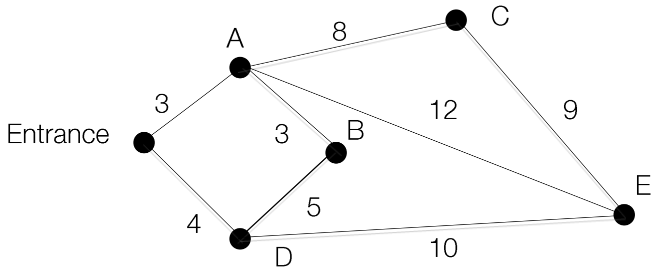
b. How many edges are there in the network above?

1 mark

Total: 3 marks

Question 2

A museum is set up with various art pieces labelled A, B, C, D and E. There is one entrance that also acts as an exit. The vertices in the network diagram below show connected walkways between each art piece. The numbers on the edges represent the time in minutes of how long to reach each art piece.



a. How many vertices have an even degree?

1 mark

b. What is the shortest time from vertex D to vertex C? Write down this path.

2 mark

c. What condition does a graph need to have for an Euler Path to exist?

1 mark

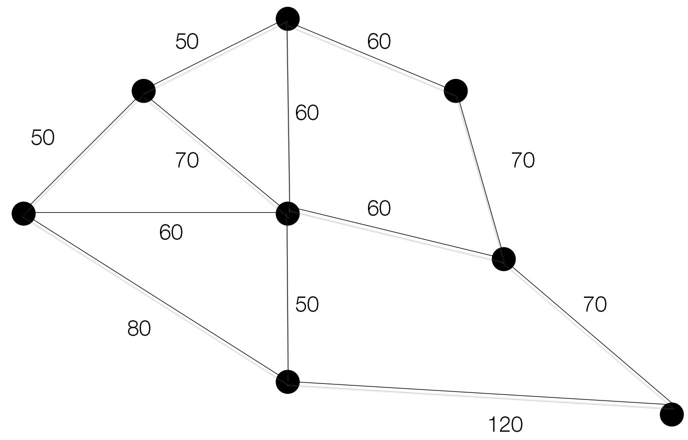
d. Does an Euler Path exist starting from the entrance?

1 mark

Total: 5 marks

Question 3

The network below represents a section of the Square Kilometre Array (SKA), a massive system of connected telescopes. Each vertex is a telescope connected to one another by wires. Each edge represents the shortest distance between telescopes with the number on the edge representing the distance in metres.



a. If all the telescopes were to be connected using the minimum amount of wire, highlight on the diagram where these wires would go.

1 mark

b. Calculate the minimum total length of wire needed to connect all the telescopes.

---

1 mark

c. If the wire cost $24 per metre, how much would the wiring of this section of the SKA cost?

---

1 mark

Total: 3 marks

Question 4

At a bakery a company wishes to allocate four tasks to four of its employees so they can maximize the time taken to complete each task. The table below summarises the amount of time each person (A, B, C and D) takes to do each task (P, Q, R and S) on average.

	P	Q	R	S
A	10	12	14	8
B	12	14	12	8
C	8	10	15	10
D	11	8	8	14

Find the optimal allocation if each task can only be allocated to one person and state the maximum time.

4 marks

Module 6: Matrices

Question 1

A is matrix: $\begin{vmatrix} 3 & 6 & 1 \\ 2 & 3 & 2 \\ 5 & 4 & 3 \end{vmatrix}$

a. Multiply $\begin{vmatrix} 1 & 1 & 1 \end{vmatrix}$ by matrix A.

1 mark

b. Explain the effect on the elements of A caused by this product of matrices.

1 marks

Total: 2 marks

Question 2

Kiki Car Rentals allows customers to hire out cars for 1, 2, 3, 4 or 5 days. The following table gives the number of rentals for each type of car for 1, 2, 3, 4 or 5 days over a month:

Number of Rentals:

	Small	Medium	Large	4WD
1 Day	50	75	30	25
2 Days	81	63	134	42
3 Days	83	143	64	32
4 Days	87	104	119	59
5 Days	45	32	90	86

a. Write the values from this table as a 5 x 4 ‘number of rentals matrix, N.

1 mark

b. Construct a matrix, S, such that the product SN will give a 1 x 5 matrix whose elements represent the total number of rentals in the month for each of small, medium, large or 4WD vehicles

1 mark

c. Calculate and record the matrix SN.

1 marks

Total: 3 marks

Question 3

Giggity Grocers has only one other competitor, Fritzy Food Market in their town. At present Giggity Grocers has 55% of the fresh produce market and Fritzy Food Market has 45%.

Giggity Grocers started an advertising campaign in order to increase its market share. After several weeks of the campaign, data was recorded that showed 90% of customers who shopped from Giggity Grocers this time, shopped from Giggity Grocers last time and 40% of those who shopped from Fritzy Food market last time shopped from Giggity Grocers this time.

a. Use the information above to complete the following table showing the change from one week to the next:

		Before Advertising	
		Giggity Grocers	Fritzy Food Market
After Advertising	Giggity Grocers	0.9	
	Fritzy Food Market		

2 marks

b. From the table above, construct a transition matrix, T.

1 mark

c. If Giggity Grocers started with 55% of the market, construct an initial-state column matrix S_0.

1 mark

d. Assuming that the choice of purchasing fresh produce depends entirely on the choice made for the previous purchase of fresh produce:

i. Find the percentage, correct to 1 decimal place, of the market captured by Giggity Grocers after one transition period.

2 marks

ii. Find the percentage, correct to 1 decimal place, of the market captured by Giggity Grocers after 5 transition periods.

2 marks

e. After how many transition periods is a steady stare reached for percentages that are correct to 1 decimal place?

1 mark

f. What is the steady state percentage for Giggity Grocers?

1 mark

Total: 10 marks

Formula Sheet

Core: Data analysis

Standardised score:	$z = \frac{x - \bar{x}}{s_x}$
Least squares line:	$y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
Residual value:	$\text{residual value} = \text{actual value} - \text{predicted value}$
Seasonal index:	$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

Arithmetic series:	$a + (a + d) + \cdots + (a + (n-1)d) = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$
Geometric series:	$a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$
Infinite geometric series:	$a + ar + ar^2 + ar^3 + \cdots = \frac{a}{1 - r}, \lvert r \rvert < 1$

Module 2: Geometry and trigonometry

Area of a triangle:	$\frac{1}{2}bc \sin A$
Heron's formula:	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a + b + c)$
Circumference of a circle:	$2\pi r$
Area of a circle:	πr^2
Volume of a sphere:	$\frac{4}{3}\pi r^3$
Surface area of a sphere:	$4\pi r^2$
Volume of a cone:	$\frac{1}{3}\pi r^2 h$
Volume of a cylinder:	$\pi r^2 h$
Volume of a prism:	$\text{area of base} \times \text{height}$
Volume of a pyramid:	$\frac{1}{3}\text{area of base} \times \text{height}$
Pythagoras' theorem:	$c^2 = a^2 + b^2$
Sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

Gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Equation: $y = mx + c$

Module 4: Business-related mathematics

Simple interest: $I = \dfrac{PrT}{100}$

Compound interest: $A = PR^n$, where $R = 1 + \frac{r}{100}$

Hire purchase: $\text{effective rate of interest} \approx \dfrac{2n}{n+1} \times \text{flat rate}$

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

Determinant of a 2 x 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Inverse of a 2 x 2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$

End of Booklet

Looking for solutions? Visit www.engageeducation.org.au/practice-exams

To enrol in one of our Further Maths lectures head to: http://engageeducation.org.au/lectures/