VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2014 Further Mathematics GA 3: Written examination 2

GENERAL COMMENTS

The selection of modules by students in 2014 is shown in the table below.

MODULE	% 2014
1 – Number patterns	25
2 – Geometry and trigonometry	66
3 – Graphs and relations	44
4 – Business-related mathematics	32
5 – Networks and decision mathematics	49
6 – Matrices	82

The 2014 Further Maths examination 2 presented opportunities for students of all abilities to start well with the Core and each module. Questions then became more challenging as students progressed through each module.

Full marks were awarded for each complete and correct answer, whether or not this was accompanied by working.

For a two-mark question, if the answer was incorrect, working out may have earned a method mark, but such working needed to clearly show a significant, logical and correct step towards the question solution.

A question that required the use of an answer from a prior question scored full marks for the correct answer. An incorrect answer due to using an incorrect answer from a previous question may have earned consequential marks. If working out had shown the correct application for the consequential question of a reasonable, but incorrect, prior answer, full marks may still have been awarded.

To be eligible for consequential mark consideration:

- the use of the incorrect prior answer must not trivialise the consequential question
- there must be a calculation that shows the correct use of the incorrect prior answer
- the resulting consequential answer must match the written calculation and be rounded as required
- the consequential answer must be reasonable within the context.

Students were expected to follow the rounding instructions specified in the questions. However, some questions did not include any instructions on how to round the answer. This occurred, for example, when the correct answer to the question calculated to be a whole number or the context of the question required a whole number. For instance, a number of elephants cannot be a decimal number, except if the question refers to an average. Similarly, if the correct answer to a question is a terminating decimal such as 23.475, then this question may not have any instructions about rounding and the complete number, with three decimal places in this example, must be written.

The price of an item or account should be written correct to the nearest cent if there are no rounding instructions included in the question. In general, when the answer for a previous question is expected to be rounded, then students should use that rounded final answer for the consequential question. Intermediate values should not be rounded within a question. Intermediate values should be retained and the final answer should only be rounded as required.

Any work or answer that had been crossed out was not assessed, unless the crossing out was negated by a written instruction such as ‘Please ignore the crossing out’ or ‘Please mark this after all’.

Concepts that caused difficulty for students included:

- interpreting a large number shown on the calculator, such as 2.944E7
- converting a decimal fraction to a percentage with one decimal place
- plotting a regression line on a grid from a given equation
- plotting 5330 on minor grid intervals of 1000
- interpreting the slope of a regression line in terms of the two variables
- substitution into a regression equation that requires the use of logs

- calculator simplification of a fraction in the form $\frac{3082-4370}{1560}$
- answering the question as asked; for example, responding with a number of items when a percentage was required
- poor organisation of working out shown for a question that required several separate calculations.

SPECIFIC INFORMATION

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding errors resulting in a total less than 100 per cent.

Core

Question 1a.

Marks	0	1	Average
%	7	93	1

19%

Question 1b.

Marks	0	1	Average
%	44	56	0.6

29 440 000 people

23% × 128 000 000 = 29 440 000

The answer is expected to be written in full and not, for example using technology syntax such as 2.944E7, a technology representation of scientific form. Many students gave the percentage as the answer rather than the required number. Technology syntax is not to be used in providing answers; standard mathematical notation is to be used.

Question 1c.

Marks	0	1	Average
%	59	41	0.4

All three countries have approximately the same percentage: 67%, 64% and 64%

Most students were able to explain that, because the percentages were all close to each other in the 15–64 age group, there was no association between the percentage of people in this age group and the country in which they lived. However, some students contradicted the given statement and claimed that ‘there **was** an association because…’

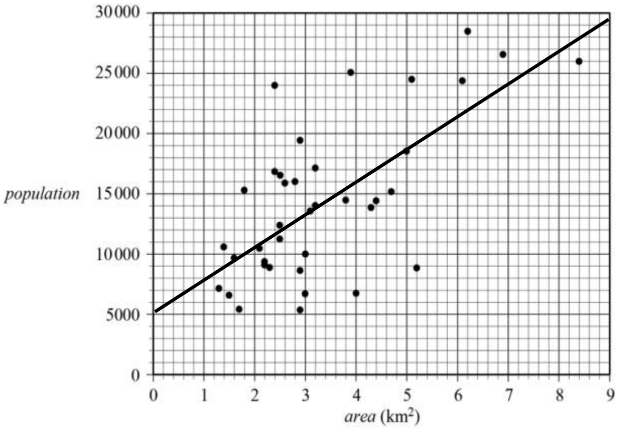
Question 2a.

Marks	0	1	Average
%	14	86	0.9

Population

Question 2b.

Marks	0	1	Average
%	64	36	**0.4**



This question was not answered well by many students. Many seemed to draw the line by eye, with no reference to the least squares regression equation given in the question. Of those who did use the equation, some drew the line through two points that were needlessly close to each other with a resulting inaccurate line.

Students are encouraged to use the whole grid when plotting a straight line, plotting a point at *area* = 0 and *area* = 9.

The grid interval on the *population* axis seemed to cause problems for a number of students, when they attempted to plot the line with an intercept of 5330.

Question 2c.

Marks	0	1	2	Average
%	55	8	37	**0.8**

On average, population increases by 2680 people for each additional square kilometre of area.

Most students were able to find the gradient of 2680 from the equation, although many were unable to explain what it meant. A common unacceptable answer was ‘The population increases by 2680 people for every increase in area’. Instead, the slope represented the increase in population for a specific increase in area of **one unit** or 1 km^2.

Other students drew their own line and then used it to calculate a slope that was usually incorrect. Some students inappropriately referred to the data being skewed. The term ‘skewed’ only applies to univariate data plots, whereas this question referred to a bivariate data plot.

Question 2di–2dii.

Marks	0	1	2	Average
%	48	21	31	**0.9**

2di.
–9360 people
6690 – 16 050 = –9360

The negative sign was required.

2dii.
44.6%

Many students simply wrote an answer of 45% without showing any working. This was not correct to one decimal place as required by the question and the mark was not awarded. Some students wrote 44.6% = 45%, thus giving their final answer as 45%. Other common, incorrect answers were 40% and 66.8%.

Question 3a.

Marks	0	1	Average
%	57	43	0.5

$population = 7.7 + 7.7 \times \log_{10}(area)$

A log transformation, using base 10, was required, as indicated in the question. Students need to distinguish between use of the base 10 logarithm and the natural logarithm when using technology to carry out computations.

Question 3b.

Marks	0	1	Average
%	65	35	0.4

23 000 people
$7.7 + 7.7 \times \log_{10}(90) = 22.7476... \approx 23$ (thousand)

Of those students who performed the log transformation to find the correct values in Question 3a., many then failed to use the log function in this final calculation and calculated $7.7 + 7.7 \times 90 = 700.7$

Question 4a.

Marks	0	1	Average
%	70	30	0.3

Weak, negative, linear

The most common error by students who got the correct strength and form was to assume the direction was positive. The scatterplot is trending downwards as area increases. Another common, incorrect answer offered a discussion about the coefficient of determination and the percentage of the variation in population. Others referred to skewness, which is not applicable to a bivariate data plot.

Question 4bi.–4biii.

Marks	0	1	2	3	Average
%	27	37	19	17	**1.3**

4bi.
−0.8

$$\frac{3082-4370}{1560}=-0.825...$$

4bii.
1 suburb
2.5% × 38 = 0.95

Some incorrect answers were greater than the number of suburbs in the city.

4biii.
2 suburbs

Very few students gave the correct answer, which relied upon calculating the area that was 'two standard deviations or more above the mean', found from 3.4 + 2 × 1.6. This then meant counting the suburbs (points) on the graph that had an area greater than or equal to 6.6 km^2. Some incorrect answers were greater than the number of suburbs in the city.

Module 1 – Number patterns

Question 1a.–1d.

Marks	0	1	2	3	4	Average
%	11	12	14	22	42	**2.7**

1a.
20 000 km^2

It was evident that a number of students did not understand that L_n was the area at the end of year n and gave 19 800 km^2.

1b.
1%

A common, incorrect answer was 99%.

1c.
0.99 × 20 000 = 19 800

1d.
198 km^2
19 800 – 19 602 = 198

2a.–2d.

Marks	0	1	2	3	4	Average
%	10	16	23	27	24	**2.4**

2a.
$\frac{0.68}{0.8}=0.85$ *or* $\frac{0.578}{0.68}=0.85$

A suitable calculation that resulted in 0.85 was required.

2b.
0.35 km^2

$0.8 \times 0.85^5 = 0.3549...$

Some students substituted n = 5 in the rule $t_n = ar^{n-1}$, but, in this rule, n represents the term number in a geometric sequence. This question required the value of t_{2019}, which is five years after 2014, yet it is the sixth term in the sequence, where the first term is t_{2014}. Students who used the rule $t_n = ar^{n-1}$ should have substituted n = 6.

2c.
2.97 km^2

$$S_5 = \frac{0.8\left(0.85^5 - 1\right)}{0.85 - 1} = 2.966...$$

2d.
2022

Year	**Area**	
2020	3.623…	
2021	3.880…	
2022	4.098…	this is the first year where area of desert > 4

An answer of nine years was not accepted. The question referred to the year, rather than the number of years.

Question 3a.–3b.

Marks	0	1	2	3	Average
%	21	26	14	40	1.7

3a.
0.85 × 14 000 + 500 = 12 400

Some calculations were written incorrectly; for example, $0.85 \times 14\,000 = 11\,900 + 500 = 12\,400$, where the expressions either side of the first equals sign were not equal.

3b.
In 2016, it is expected that 11 040 km^2 will be available for 5618 elephants.

$\frac{11\,040}{6618} \approx 1.97$

Only about 1.97 km^2 per elephant will be available and this is less than the 2 km^2 needed.
Therefore, overpopulation is expected in 2016.

Year n	H_n	E_n	**Area needed** $2 \times E_n$
2014	14 000	5000	10 000
2015	12 400	5300	10 600
2016	11 040	5618	11 236

In 2016:
Area available, H_{2016} = 11 040
Area needed, $2 \times E_{2016}$ = 11 236
$H_{2016} < 2 \times E_{2016}$, therefore overpopulated since the available area is less than the area needed.

Many students found that 5618 elephants are predicted for 2016 but went no further. Of those who were able to progress further, some were unable to draw a conclusion from their calculations about overpopulation.

Question 3c.–3e.

Marks	0	1	2	3	4	Average
%	41	17	28	3	11	1.3

3c.
300 elephants

Constant population occurs when $P_{n+1} = P_n$
Solve:
$P_n = 1.06P_n - k$
$\therefore\ k = 1.06P_n - P_n$
$\therefore\ k = 0.06P_n$

Then, when $P_n = 5000$ constantly,
$k = 0.06 \times 5000 = 300$

3d.
400 elephants

Remove 100 more than needed for a constant population in Question 3c.
300 + 100 = 400
or
Solve $P_n - 100 = 1.06P_n - k$

When $P_n = 5000$,
$4900 = 1.06 \times 5000 - k$
$\therefore k = 400$

3e.
48 elephants

In 2016, 11 040km^2 will support 5520 elephants.
$P_{2016} = 1.06\ P_{2015} - k$
$\therefore 5520 = 1.06\ P_{2015} - k$ … equation 1
and
$P_{2015} = 1.06\ P_{2014} - k$ … equation 2

Then, substituting equation 2 into equation 1:
$5520 = 1.06\ (1.06 \times P_{2014} - k) - k$
$\therefore 5520 = 1.06^2 \times P_{2014} - 2.06k$
$\therefore 5520 = 1.06^2 \times 5000 - 2.06k$
Solve to find $k = 47.57\ldots$

Very few students were able to complete this question. A common incorrect answer was 98.

Module 2 – Geometry and trigonometry

Question 1a.–1b.

Marks	0	1	2	Average
%	12	26	62	1.5

1a.
8 m^2

Some students split the trapezium into a triangle and rectangle to find the correct total area.
The rule for the area of a trapezium is:

$$A = \frac{1}{2}(a+b)\times h$$
$$= \frac{1}{2}(5+3)\times 2$$
$$= 8$$

The rule for a trapezium is a version of *Area* = *length* × *width*, where *length* is the average of the two uneven sides.

1b.
12.8 m

Question 2a.–2d.

Marks	0	1	2	3	4	Average
%	6	9	10	22	53	3.1

2a.
180 – (45 + 60) = 75

A suitable calculation that resulted in 75 was required.

2b.

$$\frac{AX}{\sin 45°} = \frac{3.16}{75°} \text{ or } \frac{AX}{45°} = \frac{\sqrt{8} \approx 2.83}{60°}$$

2c.
2.31 m

$XC = 1.84$

$$AX = \sqrt{(3-1.84)^2 + 2^2} = 2.312...$$

2d.
3.2 m^2

$$\frac{1}{2}\times 3.16\times 2 = 3.16 \approx 3.2$$

Question 2e.

Marks	0	1	2	Average
%	52	21	28	0.8

17 m^2

Roof: $\frac{1}{2}(3+1.84) \times 2 = 4.84$

Walls: $1.8(1.84+2+3)=12.312$

Total = $4.84+12.312=17.152\approx 17$

Many students misread the question and covered the nesting and eating spaces. Some others who covered only the eating space as required, then incorrectly included the wall *AX*.

A method mark was available in this two-mark question if the final answer was incorrect. Many calculations were set out poorly and a method mark could not be allocated. Students are encouraged to clearly label each step in an extended calculation and to draw supporting diagrams where applicable.

Question 3a.–3c.

Marks	0	1	2	3	4	5	Average
%	17	15	23	31	6	8	2.2

3a.
157 cm^2

SA = $\frac{1}{2}\times 4\times\pi\times 5^2=157.07...$

Some students calculated the surface area of a full sphere, while others used an incorrect radius or formula.

3b.
1440 cm^3

Cylinder = $\pi\times 5^2\times 15=1178.097...$

Hemisphere = $\frac{1}{2}\times\frac{4}{3}\times\pi\times 5^3=261.799...$

A method mark was available in this two-mark question if the final answer was incorrect. Many calculations were poorly set out and a method mark could not be allocated. Some students used the wrong formula for the volume of a sphere, while others did not halve this for a hemisphere.

3c.
761 cm^2

Volume factor $\frac{\text{feed container}}{\text{water container}}=\frac{4}{3}$

Linear factor $\frac{\text{feed container}}{\text{water container}}=$ cube root of volume factor = $\sqrt[3]{\frac{4}{3}}\approx 1.1006...$

Area factor $\frac{\text{feed container}}{\text{water container}}=$ (Linear factor)^2 = $\left(\sqrt[3]{\frac{4}{3}}\right)^2\approx 1.2114...$

Therefore, the surface area of feed container

$=\left(\sqrt[3]{\frac{4}{3}}\right)^2\times$ surface area of water container

$=\left(\sqrt[3]{\frac{4}{3}}\right)^2\times 628=760.76...$

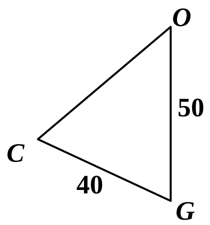
This question was not answered well. Many students only converted the volume factor of $\frac{3}{4}$ or $\frac{4}{3}$ into a linear factor, while others calculated the area factor as $\sqrt{\text{volume factor}}$, which was inappropriate. Many students did not attempt conversion of the volume factor.

Some students rounded too early within the question, such as rounding the linear factor to 1.1 before squaring it again to produce the incorrect surface area of 759.88 cm^2.

Question 4

Marks	0	1	2	Average
%	66	12	22	**0.6**

228°



$\angle OGC = 65°$

By the cosine rule, $OC = 49.0869\ldots$
By the sine rule, $\angle COG = 47.6\ldots°$
Then, the bearing of the chicken from its owner = 180 + 47.6 = 227.6…°

Many students who began a calculation for this question found the correct length of *OC*, but then few were able to correctly find the required bearing. A common, incorrect answer was 065°.

Module 3 – Graphs and relations

Question 1a.–1c.

Marks	0	1	2	3	Average
%	2	8	24	66	**2.6**

1a.
0.04 kg

1b.
25 kg

$100 \times 0.05 + 400 \times 0.05 = 25$

1c.
$0.06x + 0.04y \geq 180$

A number of students wrote $0.6x + 0.4y \geq 180$, but this was incorrect.

Question 1di.–1eii.

Marks	0	1	2	3	4	5	Average
%	37	21	14	10	15	4	**1.6**

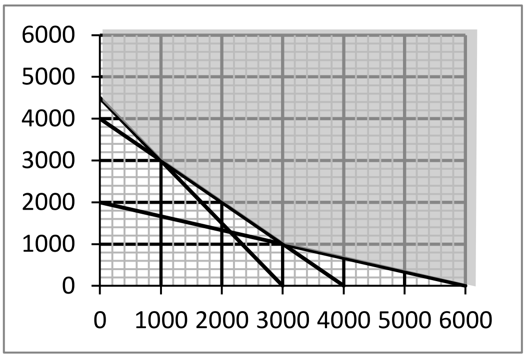
1di.

$0.02x + 0.06y = 120$ or $y = -\frac{1}{3}x + 2000$

The gradient can be found from the line that limits the original Inequality 4, given as $0.02x + 0.06y \geq 120$. The equation for the line is then $0.02x + 0.06y = 120$, which gives $m = -\frac{0.2}{0.6} = -\frac{1}{3}$.

The majority of students did not seem to identify Inequality 4 as being relevant to Line A. Instead, they calculated a gradient from the graph and wrote their answer in the form $y = mx + c$ where $m = \frac{Fall}{Run} = -\frac{2000}{6000} = -\frac{1}{3}$, often with inappropriate rounding.

1dii.

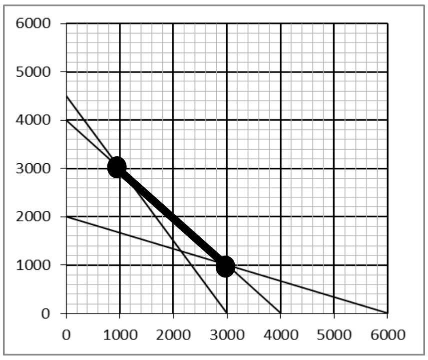


Most students could not identify the correct feasible region. Students are encouraged to test points within their nominated feasible region by substitution into the inequalities.

Instead of shading the feasible region as required, some students shaded outside the feasible region. This was accepted only if the unshaded section was then clearly identified in some way as the correct feasible region. This was usually done with a legend.

1ei.
4000 kg

1eii.



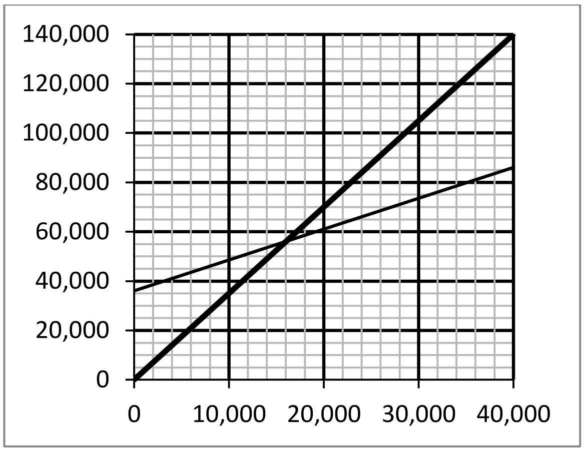
All points along the line interval between, and including, (1000, 3000) and (3000, 1000) are the solution to this linear programming problem and so students needed to draw a line on the graph, as shown. Ideally, the end points should be filled circles to show these points are included.

Of those who were able to identify the correct feasible region, most were able to nominate the two points at the end of the line interval as being solutions. Some went further and identified nine further solutions at intersections of gridlines between the two end points, but these failed to identify the infinite possible solutions depicted by the line interval.

Question 2a. and 2b.

Marks	0	1	2	3	Average
%	32	10	16	43	**1.7**

2a.



A few students appeared to miss this question entirely. Some others were unable to locate the line with sufficient accuracy.

2b.
$9000

$R = 3.5 \times 20\,000$
$C = 1.25 \times 20\,000 + 36\,000$
$P = R – C = 9000$

Question 3a.–3c.

Marks	0	1	2	3	4	Average
%	41	34	10	8	6	**1**

3a.
$34.80

$10.8 + 4(8–2) = 34.8$

The expected answer needed to be written correct to the nearest cent and so students needed to include the zero at the second decimal place. Some students wrote their answer as $34.8, which may be any amount between $34.75 and $34.84 inclusive when rounded to the nearest ten cents. Therefore, $34.8 is not necessarily equal to $34.80.

3b.
$10.8 + 4(10 – 2) = 42.8$

3c.
15.2 kg

Solve:
revenue ≥ cost

$42.8 + 2(n – 10) \geq 3.5n$
$\therefore\ n \leq 15.2$

Some students rounded 15.2 up to 16, but this is too much since 15.2 is the maximum as calculated.

Module 4 – Business-related mathematics

Many students did not read questions carefully and therefore missed out on some marks for questions in this module. This generally occurred where the required period of time was misread, such as an interest rate per month being read as a rate per year.

Students generally gave monetary answers correct to the nearest cent, except where instructed otherwise.

Question 1a.–1c.

Marks	0	1	2	3	Average
%	3	11	40	46	2.3

1a.
20%

The most common incorrect answer was 80%.

1b.
$330

$150 + 12 \times 15 = 330$

1c.
$15

Many students misread the 5% rate of interest as an annual rate, rather than a monthly rate and obtained the incorrect answer of $1.25.

Question 1di. and 1dii.

Marks	0	1	2	Average
%	9	59	32	1.3

1di.
$17 000

1dii.
3.5%

$$\frac{125.12}{42700} \times 12 \times 100 = 3.499...$$

Question 2a.–2cii.

Marks	0	1	2	3	4	Average
%	18	16	23	31	13	2.1

2a.
3.75%

A common, unacceptable answer was 0.0375%.

2b.
$20 000

Many students did not understand what a perpetuity is. A perpetuity balance remains constant since only the interest earned is withdrawn in each compounding period.

2ci.
$772.50

$1.03 \times 750 = 772.5$

The answer must have been written correct to the nearest cent and so the zero at the second decimal place was required in this question.

2cii.
$558

$(\text{value in 2014}) \times (1+0.03)^{10} = 750$

$\text{therefore, the value in 2014} = \dfrac{750}{1.03^{10}} = 558.0704...$

A common incorrect answer was $1008, calculated from $750 \times (1+0.03)^{10}$

Another common incorrect answer was $553 calculated from $750 \times (1-3\%)^{10}$

Question 3a.–3cii.

Marks	0	1	2	3	4	Average
%	23	31	21	16	10	1.6

3a.
$14 450

3b.
6.9%

A common error was to treat this as a simple interest question, despite the question stating that there were 'four years of compounding interest'.

3ci.
0.006 885

Answers equivalent to 0.006 were accepted in the first box, commonly $\dfrac{7/100}{12}$ or $\dfrac{7/12}{100}$. However, many students wrote only $\dfrac{7}{100}$ or $\dfrac{7}{12}$.

A number of students added a power to complete a formula to find the account balance at the end of 12 months, rather than the first month as required.

3cii.
$75 443

N = 12
I % = 7.2
PV = 60 000
PMT = –885
FV = 75 443.014…
P/Y = 12
C/Y = 12

Question 4

Marks	0	1	2	Average
%	88	5	8	0.2

78%

The interest component of next month's repayment

$= \frac{4.5\%}{12} \times 143\,585.33$

= \$538.4449…

The reduction in the principal next month
= repayment – interest component
= 2500 – 538.44
= \$1961.56

The percentage of next month's repayment that will reduce the loan balance

$= \frac{1961.56}{2500} = 0.7846... \approx 78.46\%$

Many students made poor attempts at this question, gave a single number as the answer or did not attempt it at all. A common incorrect answer was 22%, which was the interest component of the next repayment. A method mark was available for an incorrect answer only if the working out of a significant step towards a solution could be followed, but this was rare.

Module 5 – Networks and decision mathematics

Questions 1a.–1b.

Marks	0	1	2	Average
%	0	2	97	2

1a.
2

1b.
Miniature trains

Questions 2a.–2b.

Marks	0	1	2	Average
%	10	46	44	1.4

2a.

Task	Andrew	Brianna	Charlie	Devi
publicity	3	2	0	0
finances	0	1	2	2
equipment	0	4	3	2
catering	1	2	3	0

2b.
The minimum number of lines to cover all zeros is less than four.

Students' explanations needed to refer to the stage in the process of the Hungarian algorithm. This required reference to the required minimum number of lines though zeroes.

Some students simply stated that there was 'no clear allocation to Brianna'. Such reference to an 'allocation' does not explain how allocations might be attempted at this first stage of the algorithm. Further, if there is 'no clear allocation to Brianna', it also follows that there are no clear allocations to anybody at this stage of the algorithm.

Another common but unacceptable answer was 'there are not enough zeroes'. This answer does not indicate how many zeroes might be enough or how the zeroes would be used. Even if the table had up to 12 zeroes in three lines or columns, the Hungarian algorithm indicates that at least one further step is needed.

Questions 2ci. and 2cii.

Marks	0	1	2	Average
%	22	14	64	**1.4**

2ci.
Equipment

A common incorrect answer was publicity.

2cii.
36 hours

A common incorrect answer was 21, the total of all the numbers on Table 3.

Questions 3ai.–3c.

Marks	0	1	2	3	4	Average
%	6	8	12	28	45	**3**

3ai.
Bower, Eden

3aii.
910 km

3b.
270 km
Bower – Clement – Derrin – Eden

3c.
Between Bower and Derrin

A common incorrect answer was between Bower and Clement.

Questions 4a.–4e.

Marks	0	1	2	3	4	5	Average
%	21	21	18	18	15	7	2.1

4a.
7 hours

Activities *A* and *D* are predecessors. A common incorrect answer was 11 hours, which is the earliest finishing time of *F*.

4b.
18 hours
Latest starting time of *L*
= length of critical path – duration of *L*
= 21 – 3 = 18

4c.
2 hours
LST – EST = 13 – 11

4d.
4 hours

Activity *X* is an immediate predecessor of activity *G*
EST of *G* = 11 = LST of *X*

EST of *X*
= LST of *X* – duration of *X*
= 11 – 7 = 4

4e.
$270

The longer paths are:
A-C-G-K = 21 hrs (this is the critical path)
A-D-E-H-K = 20 hrs
A-D-F-J-M = 19 hrs
A-D-E-I-M = 18 hrs
B-E-H-K = 18 hrs
B-F-J-M = 17 hrs

Reducing any path that includes *A* below 18 hours is pointless since *B-E-H-K* becomes a critical path at 18 hrs.
The critical path *A-C-G-K* can be reduced to 18 hours if *A* is reduced by three hours.
Max. reduction = 3 hrs × $90 = $270

Module 6 – Matrices

Questions 1a.–1d.

Marks	0	1	2	3	4	Average
%	1	3	12	30	54	3.4

1a.
4 × 2

Some students reversed the two numbers, writing 2 × 4, which was not accepted.

1b.
1850 adults

1c.
Total number of adult females living in this small city

1d.
Number of columns in V = number of rows in P

Questions 1e. and 1f.

Marks	0	1	2	Average
%	20	25	55	1.4

1e.
1360 × 0.45 + 1460 × 0.55 = 1415

Some students copied the full given matrix multiplication and replaced the *w* with 1415. This was not acceptable.

1f.
6021 votes
1415 + 1812 + 988 + 1806 = 6021
Ms Aboud's votes = the sum of the elements in the product $V \times P$.

Questions 2ai.–2b.

Marks	0	1	2	3	Average
%	6	7	59	28	2.1

2ai.
20%

2aii.
25%

2b.
1164
5% of those who chose Ms Aboud in January + 40% of those who chose Mr Choi in January
= 5% × 6000 + 40% × 2160 = 1164

Many students were unable to answer this question correctly, often giving an answer without showing any working. The most common incorrect answer was 396, the increase in Mr Broad's total votes from January to February. This number includes an allowance for the loss of votes from Mr Broad to one of the other candidates in the month and is, therefore, fewer than the number of voters whose votes had changed to Mr Broad.

Questions 2ci.–2d.

Marks	0	1	2	3	Average
%	22	22	26	30	1.7

2ci.

$$S_3 = \begin{bmatrix} 4900 \\ 4634 \\ 2466 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 0.75 & 0.10 & 0.20 \\ 0.05 & 0.80 & 0.40 \\ 0.20 & 0.10 & 0.40 \end{bmatrix}^2 \times \begin{bmatrix} 6000 \\ 3840 \\ 2160 \end{bmatrix} = \begin{bmatrix} 4900.2 \\ 4633.8 \\ 2466 \end{bmatrix}$$

2cii.
The number of preferences for each candidate (predicted) for March

An incorrect answer given by some students was ‘The number of voters expected to change their votes in March’.

2d.
5303

$$S_{\text{June}} = S_6$$
$$= T^{(6-1)} \times S_1$$
$$= \begin{bmatrix} 0.75 & 0.10 & 0.20 \\ 0.05 & 0.80 & 0.40 \\ 0.20 & 0.10 & 0.40 \end{bmatrix}^5 \times \begin{bmatrix} 6000 \\ 3840 \\ 2160 \end{bmatrix} = \begin{bmatrix} 4334.05... \\ 5302.94... \\ 2362.99... \end{bmatrix}$$

Many students used $S_{\text{June}} = S_6 = T^6 \times S_1$ to find the answer of 5404, which was incorrect.

The state matrix for January was given as S_1. Then, $S_2 = T \times S_1$, $S_3 = T^2 \times S_1$, $S_4 = T^3 \times S_1$, … , $S_n = T^{n-1} \times S_1$.
The power by which the transition matrix must be raised is one less than the number of the required state matrix.

Some students wrote the complete matrix as their answer. This did not demonstrate an understanding of the correct required element in the matrix.

Questions 3a. and 3b.

Marks	**0**	**1**	**2**	**3**	**Average**
%	80	6	10	4	**0.4**

3a.
50%

This question concerned the percentage the voters who would have changed their preferred candidate from Mr Broad to Mr Choi. From May to June, this represented 10% of the voters.

If Mr Choi withdrew from the election, the percentage of voters who stayed with Mr Broad as their preferred candidate would rise from 80% to 85%, an increase of 5%, as shown in T_1. This 5% is half (or 50%) of the voters who would have changed from Mr Broad to Mr Choi, had Mr Choi not withdrawn.

Many students seemed unable to identify the sub-group who were expected to change their votes from Mr Broad to Mr Choi. The most common incorrect answer was 5%, which represents the percentage of Mr Broad’s total votes that reverted back to Mr Broad.

3b.
6451

May preferences are in $S_{\text{May}} = S_5 = T^4 S_1$, where T is the original transition matrix.

June preferences are in $S_{\text{June}} = T_1 \times S_{\text{May}}$, where T_1 is the modified transition matrix after Mr Choi had withdrawn.

$S_{\text{June}} = T_1 \times S_{\text{May}}$

$= T_1 \times \left(T^4 S_1\right)$

$$= T_1 \times \left(\begin{bmatrix} 0.75 & 0.10 & 0.20 \\ 0.05 & 0.80 & 0.40 \\ 0.20 & 0.10 & 0.40 \end{bmatrix}^4 \times \begin{bmatrix} 6000 \\ 3840 \\ 2160 \end{bmatrix} \right)$$

$$= T_1 \times \begin{bmatrix} 4454 \\ 5154 \\ 2392 \end{bmatrix}$$

$$= \begin{bmatrix} 0.75 & 0.15 & 0.60 \\ 0.25 & 0.85 & 0.40 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 4454 \\ 5154 \\ 2392 \end{bmatrix}$$

$$= \begin{bmatrix} 5549 \\ 6451 \\ 0 \end{bmatrix}$$

This question was poorly answered, with very few instances seen of working out that might earn a method mark where the final answer was incorrect. The most common error was $S_{\text{June}} = (T_1)^5 \times S_1$ or $S_{\text{June}} = (T_1)^6 \times S_1$ rather than $S_{\text{June}} = T_1 \times S_{\text{May}}$.