



Trial Examination 2014

VCE Further Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
Core	3	3	15
Section	Number of modules	Number of modules to be answered	Number of marks
Modules	6	3	45

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white-out liquid/tape.

Materials supplied

Question booklet of 27 pages with a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.
Please ensure that you write your **name** and your **teacher's name** in the space provided on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2014 VCE Further Mathematics Units 3&4 Written Examination 2.

Neap Trial Exams are licensed to be photocopied or placed on the school intranet and used only within the confines of the school purchasing them, for the purpose of examining that school's students only. They may not be otherwise reproduced or distributed. The copyright of Neap Trial Exams remains with Neap. No Neap Trial Exam or any part thereof is to be issued or passed on by any person to any party inclusive of other schools, non-practising teachers, coaching colleges, tutors, parents, students, publishing agencies or websites without the express written consent of Neap.

Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

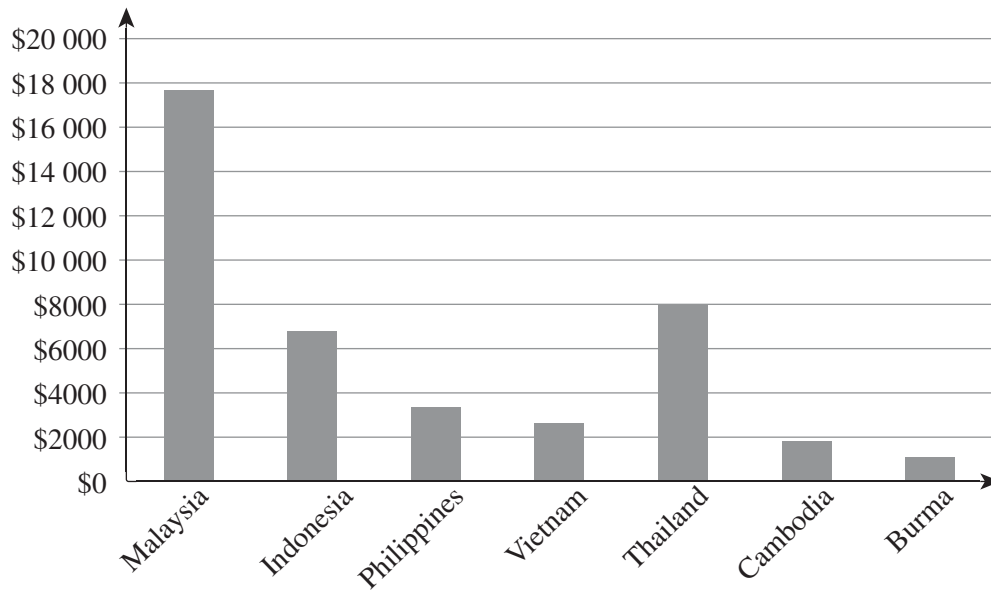
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π , surds or fractions.

Diagrams are not to scale unless specified otherwise.

	Page
Core	3
 Module	
Module 1: Number patterns	7
Module 2: Geometry and trigonometry	10
Module 3: Graphs and relations	15
Module 4: Business-related mathematics	19
Module 5: Networks and decision mathematics	22
Module 6: Matrices	26

Core**Question 1 (2 marks)**

The graph below shows the 2013 GDP per capita of various South-East Asian countries in US dollars.



- a. By what margin does the GDP per capita of Malaysia exceed that of Thailand? 1 mark

- b. What percentage increase will be required in the Philippines GDP per capita before it matches the current Indonesian GDP per capita? 1 mark

Question 2 (4 marks)

The following table outlines economists' predictions for per capita GDP of Indonesia in 2015. 40 economists were surveyed and their predictions were grouped as shown

GDP prediction	Number of economists
4.0	1
4.5	9
5.0	11
5.5	7
6.0	6
6.5	4
7.0	2

- a. Determine the median of these forecasts. 1 mark

- b. Write down a 5-figure summary of the data and hence find the interquartile range (IQR). 2 marks

- c. By performing appropriate calculations, identify if any outliers exist. 1 mark

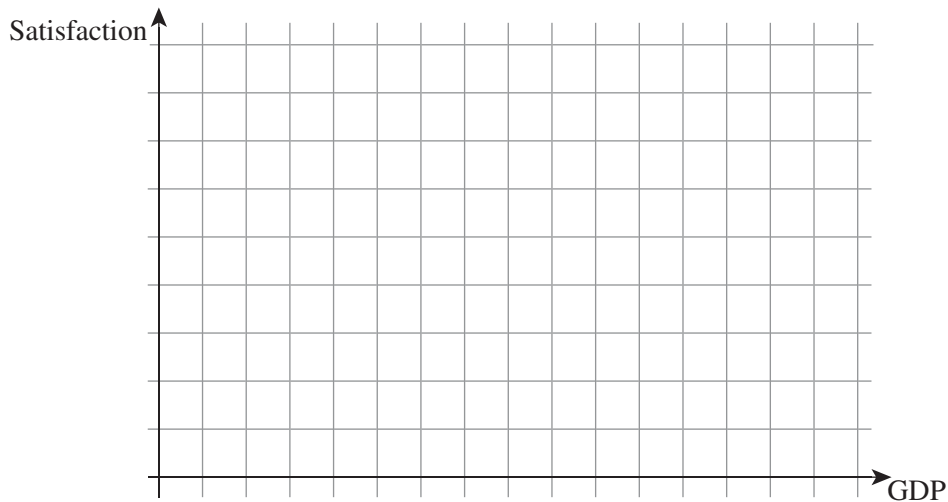
Question 3 (9 marks)

The GDP of a randomly-chosen country from South-East Asia is compared with the results of surveys on people’s feelings of satisfaction in that country. The results are shown below.

Year	GDP (\$m)	Satisfaction
2005	246	410
2006	261	513
2007	263	582
2008	272	626
2009	306	615
2010	296	654
2011	299	673
2012	312	665
2013	302	681
2014	334	685

- a. Find the correlation between GDP and satisfaction rating. Give your answer to 2 decimal places. 1 mark

- b. Sketch a scatterplot of satisfaction against GDP. Draw in a regression line on this scatterplot. 2 marks



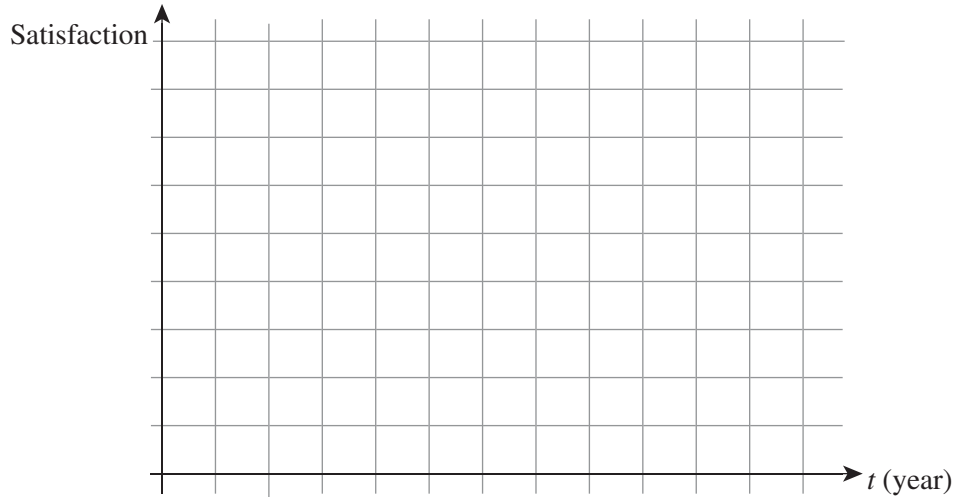
It is suggested that a y^2 transformation be applied so that the data is linearised.

- c. Conduct this transformation and obtain the new equation and write it below 2 marks

$$\text{satisfaction}^2 = \boxed{} \times \text{GDP} + \boxed{}$$

- d. By performing appropriate calculations, determine whether this equation can be expected to provide a better prediction of satisfaction than that of the equation in part b. 1 mark

- e. Sketch a time series graph of satisfaction on the axes below. 1 mark



- f. Choose to employ either deseasonalisation or 3-point moving mean of the satisfaction data in the right-most column on the table below (whichever is more appropriate). If any particular data point exists, leave its space in the table empty. 1 mark

Year	GDP (\$)	Satisfaction	Smoothed/Deseasonalised
2005	246	413	
2006	261	513	
2007	263	582	
2008	272	626	
2009	306	615	
2010	296	654	
2011	299	673	
2012	312	665	
2013	302	681	
2014	334	685	

- g. Justify your choice of either deseasonalisation or 3-point moving mean in the table above. 1 mark

END OF CORE

MODULES**Module 1: Number patterns****Question 1 (7 marks)**

The population of fish in Tulo Lake is such that it constantly approaches its capacity. In 2014, the number of fish (in thousands) is 10. The difference equation governing annual populations is

$$t_{n+1} = 60 - 0.4(60 - t_n) \quad t_1 = 10$$

- a. How many will there be in the years 2015 and 2016? 2 marks

- b. Determine the maximum population of fish within the lake. 1 mark

The breeding rate is altered so that the difference equation reads

$$t_{n+1} = 60 - k(60 - t_n) \quad t_1 = 10$$

- c. Find k if the population in 2015 is 30 thousand. 2 marks

- d. After how many years does the population reach 57? 2 marks

Question 2 (8 marks)

Tulo Lake fisheries are a new company but in 2013 and 2014 they received orders for 3.5 and 4.1 tonnes of salmon respectively. The company expects that the orders will increase at this same number of tonnes every year.

- a. Assuming that their own predictions are correct, what will be the orders for 2015 and 2016? 2 marks

- b. In what year will the salmon orders first reach 9 tonnes or more? 1 mark

- c. Find the total orders for the years 2013 to 2019. 1 mark

The company believes that it will not be able to service orders of over 9 tonnes. They will limit their orders to exactly 9 tonnes.

- d. What will be the total mass of fish that they can deliver in the years 2013 to 2030? 2 marks

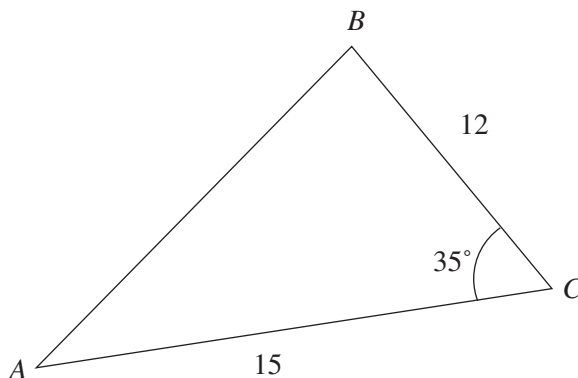
An alternative projection of future fish orders is proposed by a new employee. He believes that, after 2014, the orders will increase annually by 10%.

- e. Determine whether the orders projected by this employee exceed those projected originally by the company for the first 7 years. Determine the difference. 2 marks

END OF MODULE 1

Module 2: Geometry and trigonometry

Question 1 (2 marks)

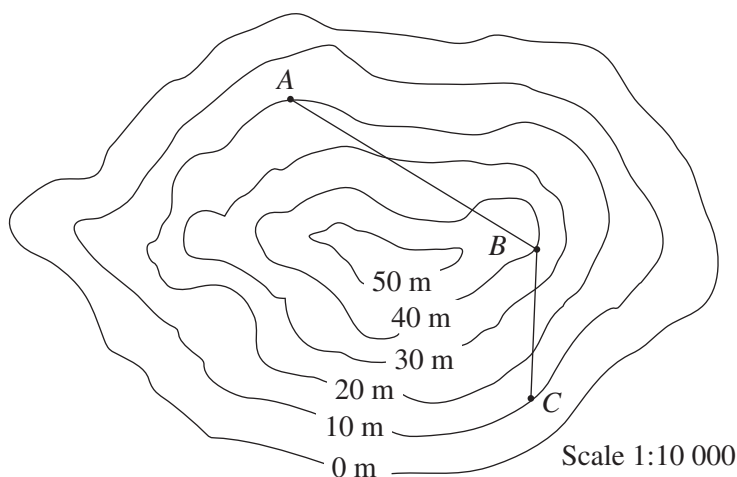


For the triangle ABC above, find:

- a. the area. Give answer correct to 2 decimal places. 1 mark

- b. the side length AB . Give answer correct to 2 decimal places. 1 mark

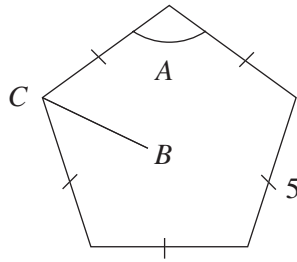
Question 2 (2 marks)



Points A , B and C are shown on a contour map.

- a. The horizontal distance AB measures 4 cm.
 What is the horizontal distance between A and B in meters? 1 mark

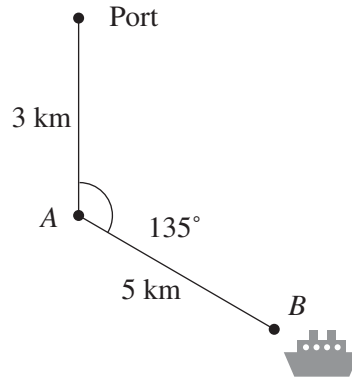
- b. Find the angle of elevation for the line BC if the direct distance from B to C is 200 m. 1 mark

Question 3 (3 marks)

- a. In the regular pentagon of side length 5 cm, find the angle A correct to 2 decimal places. 1 mark

- b. If B is situated in the centre of the pentagon and the distance CB is 8 cm, find the area of the pentagon. 2 marks

Question 4 (3 marks)

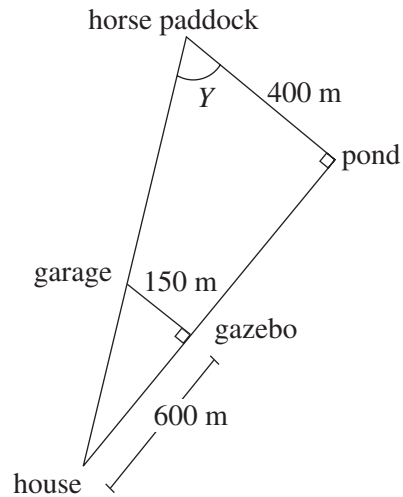


A ship leaves Port and travels south for 3 km to point *A*. It then changes direction and travels on a bearing of 135 degrees for 5 km to a point *B* where it stops.

- a. When the ship is at point *B*, how far is the ship from the port? 1 mark

- b. What bearing must the ship travel on to return to port from point *B*? 1 mark

- c. The ship leaves point *B* and makes it back to port.
 What is the area of the triangle formed by the ship's journey? 1 mark

Question 5 (3 marks)

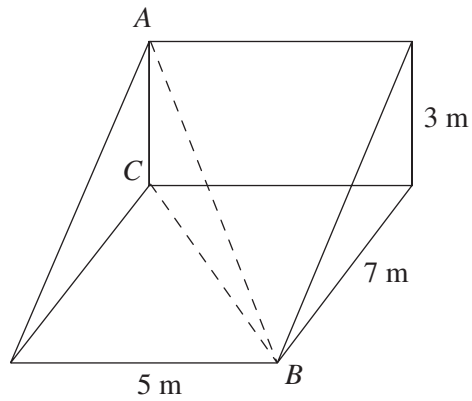
A land surveyor submits the following measurements for a set of roads on a large triangular block of land. The surveyor has unfortunately left out some critical measurements required for planning but claims there is enough information there to work out the required information.

- a. Find the distance between the house and the garage. 1 mark

- b. Find the distance between the gazebo and the pond. 1 mark

- c. Find the angle Y . 1 mark

Question 6 (2 marks)



For the diagram above:

- a. find the length AB . 1 mark

- b. find the angle ABC . 1 mark

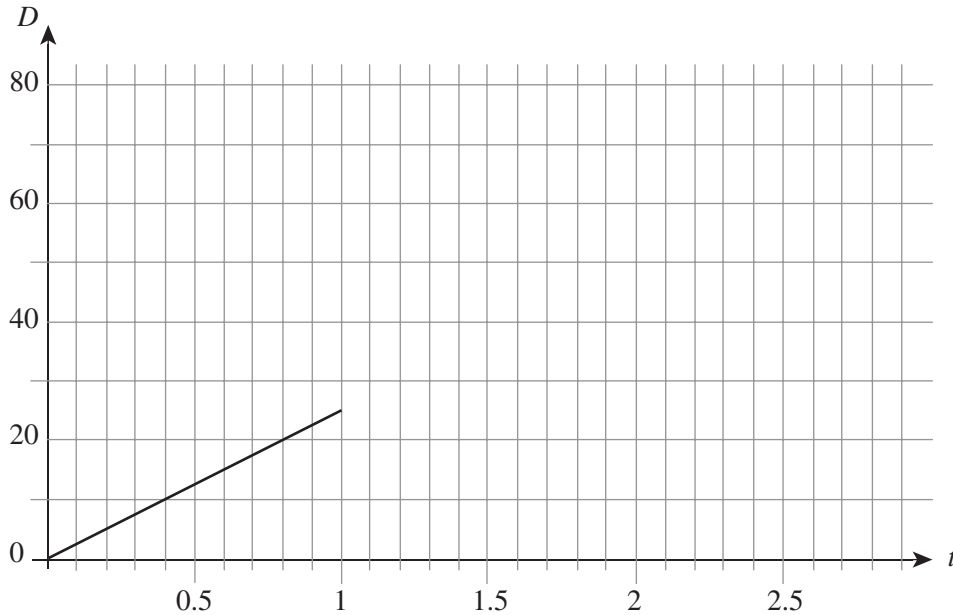
END OF MODULE 2

Module 3: Graphs and relations**Question 1 (7 marks)**

Worrel Wedding Commemorations organise tours for married couples to various resorts on the occasion of their anniversary. The most popular tour is to Tagaytay Hotel which is 80 km from the central city. The journey takes place in a tour bus and the bus leaves at 6 pm on Friday night.

The first hour involves travelling at a speed of 25 km/h. After that, the bus travels for 30 mins at 60 km/h. The last part of the journey is at 75 km/h and this speed is maintained until the destination is reached.

Part of the journey is shown on the axes below.



- a. Complete the graph. 2 marks
- b. At what time will the bus arrive at Tagaytay Hotel? 1 mark

- c. Write the average speed of the bus over the entire journey in km/h. Answer correct to one decimal place. 1 mark

A hybrid equation is developed by tour operator Helen. She sends this in an email to her assistant but some characters are unreadable due to software incompatibility. Her assistant receives the following equation with gaps as shown by letters a , b and d .

$$D(t) = \begin{cases} 25t & 0 \leq t \leq 1.0 \\ 60t + a & 1.0 \leq t \leq b \\ 75t - 57.5 & b \leq t \leq d \end{cases}$$

d. State the values of a , b and d . Exact values are required.

3 marks

Question 2 (8 marks)

The activities on the week-long tour are organised by Grace. She knows that each couple will have a total of 1000 minutes to spend on wilderness activities. They can spend the time climbing volcanoes and in cave exploration. The staff employed are able to conduct a maximum of 5 volcano tours, each of 2.5 hours duration. They are also able to conduct a minimum of 5 cave tours, each of 100 minutes duration. Due to known demand, Grace knows that the number of cave tours should not exceed twice those of volcano tours and that volcano tours should not exceed the number of cave tours.

Let x be the number of cave tours and y be the number of volcano tours.

The following are two of the inequality constraints.

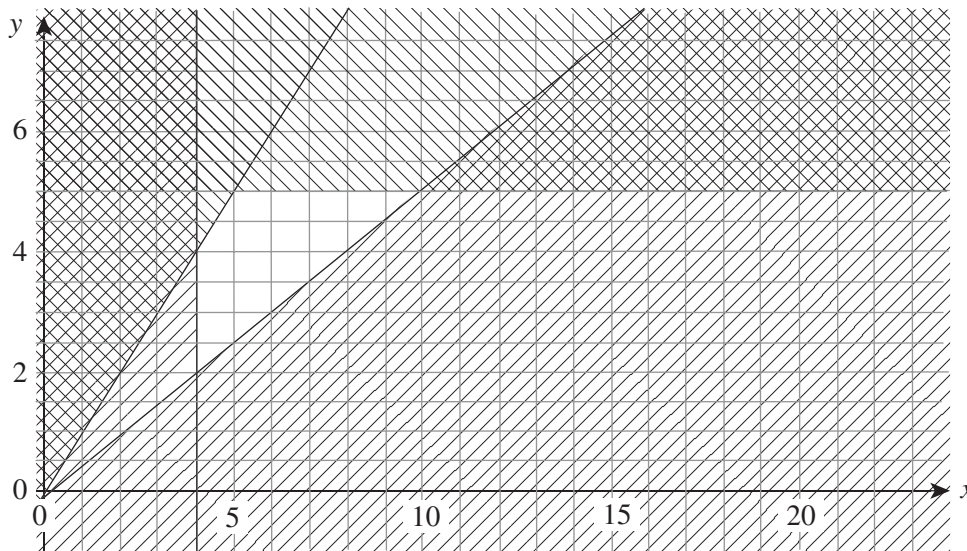
$$50x + 150y \leq 1000$$

$$y \geq \frac{x}{2}$$

a. Write down two other constraints.

2 marks

The graph below shows all of the constraints except one.



b. Draw in the missing inequality graph on the same axes.

1 mark

c. The points $(4, 2)$ and $(8, 4)$ are both corner points for the unshaded region.

Determine the other corner points of the unshaded region. Write down coordinates of each. 2 marks

The operational profits are \$50 for a cave tour and \$200 for a volcano tour.

- d.** Determine the greatest profit that can be made and state the mean number of tours that should be run to achieve this. 3 marks

END OF MODULE 3

Module 4: Business-related mathematics**Question 1 (2 marks)**

Holly is employed in the finance department of a hospital.

The bank statement below indicates the transactions in one of the hospital's bank accounts for March.

Date	Transaction	Withdrawal	Deposit	Balance
1 March	Opening balance			\$747.00
7 March	Payment – <i>BPAY</i>			\$339.00
18 March	Deposit – <i>donation</i>		\$350	\$684.00
31 March	Closing balance			\$684.00

- a. Write down the amount that was withdrawn on 7 March. 1 mark

Interest is calculated on the minimum monthly balance at the rate of 2% per annum.

- b. Calculate the interest for March. Write your answer correct to the nearest cent. 1 mark

Question 2 (7 marks)

Holly purchased an X-ray machine for the hospital for \$340 000.

The X-ray machine will be depreciated after Holly considers flat rate depreciation and reducing balance depreciation.

Flat Rate Option

- a. The X-ray machine is depreciated at a flat rate of 20% of the purchase price each year.
- i. By what amount will the X-ray machine depreciate each year? 1 mark

 - ii. Calculate the depreciated value of the machine after two years. 1 mark

 - iii. How many complete years are needed until the value of the machine first falls to below \$100 000? 1 mark

Reducing Balance Option

- b. The value of the X-ray machine, V , after t years is given by the formula $V = 340\,000 \times 0.78^t$
- i. Write down the annual depreciation rate as a percentage. 1 mark

 - ii. Calculate the value of the X-ray machine after four years. Write your answer correct to the nearest dollar. 1 mark

 - iii. At the end of which year will the value of the X-ray machine first be less using flat rate depreciation than it would be using reducing balance depreciation? 2 marks

Question 3 (4 marks)

Holly was asked to invest \$65 000 on behalf of the hospital. She was offered a simple interest investment, for three years, which would earn \$8 775 in interest.

- a. Calculate the annual interest amount in dollars. 1 mark

- b. Calculate the annual interest rate as a percentage. Write your answer correct to one decimal place. 1 mark

Holly considered investing the \$65 000 as a compound interest investment. Interest is paid at 4.1% per annum, compounding quarterly.

- c. Calculate the amount of interest earned during the third year. Write your answer correct to the nearest dollar. 2 marks

Question 4 (2 marks)

Holly investigated a reducing balance loan of \$48 000 for landscaping at the hospital. Interest is charged monthly at 5.3% per annum. Holly considers two options to fully repay this loan.

Option 1

Equal monthly repayments of \$1 450 and one final repayment of less than \$1 450.

Option 2

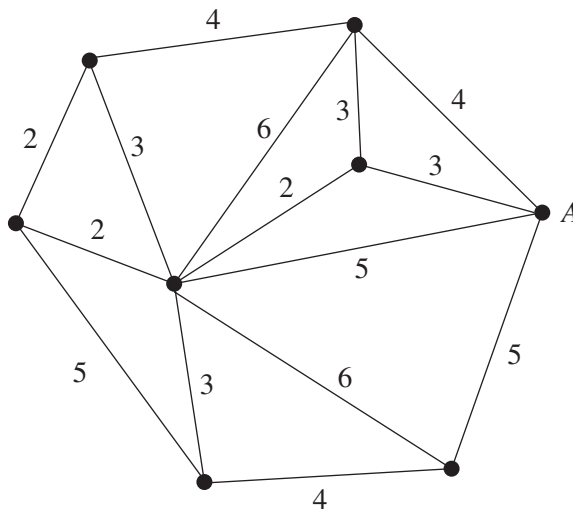
Equal monthly repayments of \$1 700 and one final repayment of less than \$1 700.

Calculate the difference in the number of repayments required for option 1 and the number of repayments required for option 2.

END OF MODULE 4

Module 5: Networks and decision mathematics

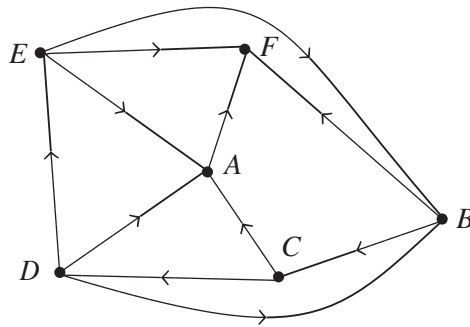
Question 1 (2 marks)



- a. Indicate the minimum spanning tree on the diagram above. 1 mark
- b. If this diagram was to be made into a complete graph, how many edges would need to be added to the vertex labelled A? 1 mark

Question 2 (5 marks)

The directed diagram illustrates six teams, A, B, C, D, E and F . Each team must play each other once. An arrow leading away from a team indicates a win. For example, D beat E .



- a. Which two teams must still play **two** more games? 1 mark

- b. Represent the directed network in the matrix below. Where no game has been played, record 0 for both teams. 2 marks

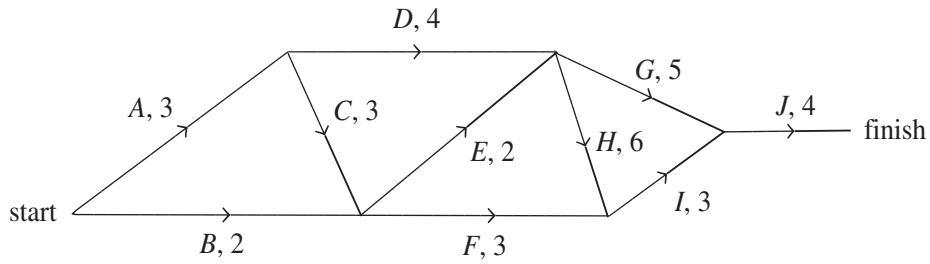
	A	B	C	D	E	F
A	<div style="display: flex; align-items: center; justify-content: center; height: 100px;"> <div style="border-right: 1px solid black; padding-right: 5px; margin-right: 5px;"> $\left[\begin{array}{cccccc} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{array} \right]$ </div> </div>					
B						
C						
D						
E						
F						

- c. What does the sum of the top row of the matrix represent? 1 mark

- d. What kind of graph is represented once all games are played and represented on the diagram? 1 mark

Question 3 (4 marks)

A project consist of the 10 activities given, the time in days listed next to each activity.



- a. Name the activities which are the immediate predecessors of activity *H*. 1 mark

- b. What is the critical path for the directed network and what is the minimum completion time for the project? 2 marks

- c. What is the slack time for the activity *F*? 1 mark

Question 4

Four chefs working at ‘Joshua’s’, a five star restaurant, have recorded the time (in minutes) it takes to prepare four dishes.

	Four cheese gnocchi	Salt and pepper calamari	Veal scaloppini	Oysters kilpatrick
Ian	2.3 minutes	3.2 minutes	3.8 minutes	5.8 minutes
Abel	1.9 minutes	2.8 minutes	4.3 minutes	6.2 minutes
Rob	2.1 minutes	3.1 minutes	4.2 minutes	6.1 minutes
Francis	2.4 minutes	2.7 minutes	3.9 minutes	5.7 minutes

A matrix is set up to represent this information, and it is suggested that the Hungarian algorithm should be used to allocate one Chef to the task of producing one type of dish for optimum efficiency.

$$\begin{bmatrix} 2.3 & 3.2 & 3.8 & 5.8 \\ 1.9 & 2.8 & 4.3 & 6.2 \\ 2.1 & 3.1 & 4.2 & 6.1 \\ 2.4 & 2.7 & 3.9 & 5.7 \end{bmatrix}$$

- a. Complete the first and second step of the Hungarian algorithm in the matrices provided. 2 marks

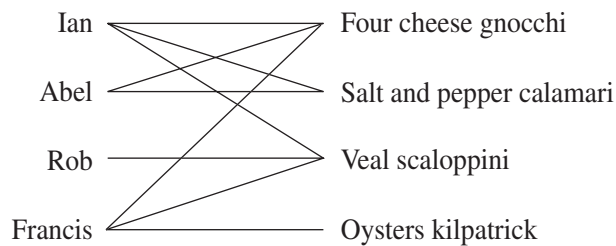
Step 1

$$\left[\begin{array}{cccc} & & & \\ & & & \\ & & & \\ & & & \end{array} \right]$$

Step 2

$$\left[\begin{array}{cccc} & & & \\ & & & \\ & & & \\ & & & \end{array} \right]$$

When the Hungarian algorithm is completed the following bipartite graph is presented to Joshua himself to allocate the specific tasks to the chefs.



- b. Identify a possible solution for who should be preparing each meal. 1 mark

- c. Find the quickest total amount of time it would take to prepare one of each meal. 1 mark

END OF MODULE 5

Module 6: Matrices

Question 1 (8 marks)

Students are entered into an athletics competition each year and may be awarded state selection, achievement or participation certificate.

When the students are in Year n the number of selection, achievement and participation certificates is denoted s_n, a_n, p_n respectively.

Performance in successive years can be determined using a transition matrix so that

$$\begin{bmatrix} s_{n+1} \\ a_{n+1} \\ p_{n+1} \end{bmatrix} = \begin{bmatrix} 0.15 & 0.05 & 0.80 \\ 0.35 & 0.80 & 0.10 \\ 0.50 & 0.15 & 0.10 \end{bmatrix} \begin{bmatrix} s_n \\ a_n \\ p_n \end{bmatrix}$$

- a. What percentage of those who achieve state selection in any year achieve achievement award in the next also? 1 mark

In year 9, there are 41 state selectees, 50 achieve achievement awards and 25 are noted for their participation.

- b. Predict the number in each of these categories in year 12. 2 marks

- c. How many awards were given in each of the three categories in grade 8? 3 marks

- d. Is it possible that this transition matrix was valid and accurate for the previous 8 years? Justify your answer with calculations. 2 marks

Question 2 (7 marks)

The cost of the athletic awards at Novum Hill athletics club needs to be considered. Each award for a student gaining state selection costs \$25 while those for achievement and participation certificates are \$10 and \$5 respectively.

Thus the cost matrix $C = \begin{bmatrix} 25 \\ 10 \\ 5 \end{bmatrix}$. There will be 10 state selection, 25 achievement and 30 participation awards required.

- a. Write an awards matrix, A , so that either AC or CA gives the total cost of awards as a 1×1 matrix. State whether AC or CA should be used. 2 marks

A discount system is to be instigated for clubs ordering awards. One possible discount involves 10% discount on state selection, 5% on achievement awards and 15% on participation awards.

- b. Write a matrix, D , that achieves this so that DC gives the new 3×1 cost matrix. Also give the resulting new cost matrix. 2 marks

A second discount system operates as follows.

- Participation awards price reduced by 10% of achievement award price.
 - State selection award price is reduced by 5% of both other award prices.
 - Achievement award price reduced by 8%.
- c. Write a matrix, N , that achieves this so that NC gives the new 3×1 cost matrix. 2 marks

- d. Thus give the new cost matrix. 1 mark

END OF QUESTION AND ANSWER BOOKLET