The Mathematical Association of Victoria
FURTHER MATHEMATICS

SOLUTIONS: Trial Exam 2013

Written Examination 2

SECTION A: Core--Data analysis

Question 1

a. Seasonal index $= 4 - 1.31 - 0.72 - 1.09 = 0.88$

A1

b. Sales in Winter are 28% below the seasonal average A1

c. Deseasonalised sales = $\dfrac{actual\ sales}{seasonal\ index}$

$= \dfrac{654}{1.09} = 600$ A1

d. Time period for Spring 2012 is $t = 12$

Substituting gives Deseasonalised sales = $540.1 + 4.25 \times 12 = 591.1$ M1

Residual = actual deseasonalised value – predicted deseasonalised value

$600 - 591.1 = 8.9$ A1

e. Time period for Winter 2014 is $t = 19$

Deseasonalised sales = $540.1 + 4.25 \times 19 = 620.85$ M1

Actual sales will equal $620.85 \times 0.72 = 447$ bikes (to nearest whole number) A1

Question 2

a. numerical

categorical A1

b. shape – Mountain Climber symmetric whereas Easy Rider is positively skewed

centre – Mountain Climber has a higher median than Easy Rider

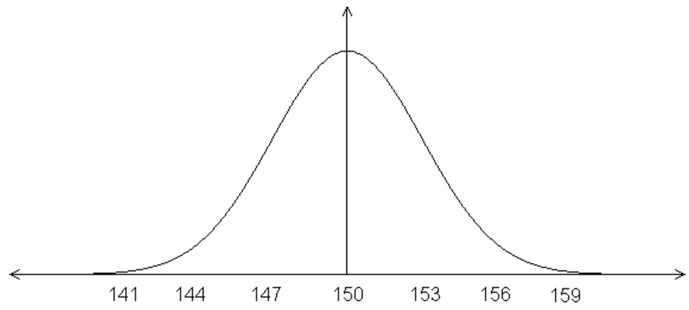
spread – Both range and IQR are greater for Mountain Climber

outliers - Mountain Climber has no outliers, Easy Rider has one at the upper end

4 x ½ marks (round down) A2

Question 3

a.



A1

b. 153 is one standard deviation above the mean

16% are slower therefore 84% of riders finished ahead of Hunter A1

c. $-1.2 = \dfrac{x-150}{3}$

$x = 146.4$ therefore 146.4 minutes A1

d. 2.5% are faster therefore $z = -2$ M1

Solve $-2 = \dfrac{116.4-120}{\sigma}$

$\sigma = 1.8$ A1

SECTION B Module 1: Number Patterns

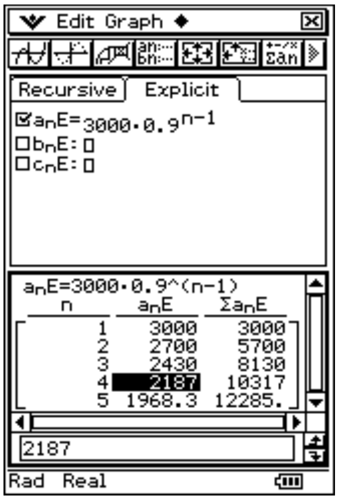
Question 1

a. $\frac{t_3}{t_2} = \frac{2430}{2700} = 0.9$ and $\frac{t_2}{t_1} = \frac{2700}{3000} = 0.9$ (must show both calculations) A1

since $\frac{t_3}{t_2} = \frac{t_2}{t_1} = 0.9$ therefore the sequence is geometric

b. $t_4 = ar^3 = 3000 \times 0.9^3 = 2187$ A1

Alternatively the sequence can be generated on the calculator by using the general rule $t_n = 3000 \times 0.9^{n-1}$

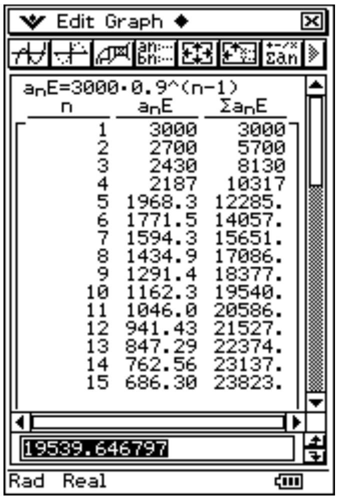


c. 19539 tickets were sold A1

$$S_n = \frac{a(1-r^n)}{1-r}$$

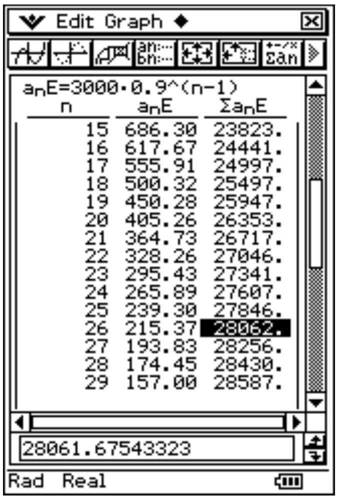
$$S_{10} = \frac{3000(1-0.9^{10})}{1-0.9}$$

$$= 19539.65$$



d. During the 26th minute A1

The crowd is first over 28 000 after the 26th minute so it will reach capacity during the 26th minute



Question 2

a. $200 + 400 = 600$ people A1

b. $200 + 10 \times 400 = 4200$ A1

c. $b = 400 \quad c = 200$ A2

d. $400n + 200 = 28000$

$n = 69.5$ minutes A1

Question 3

a. 737 spaces A1

$750 + 2\% \times 750 - 28 = 750 + 0.02 \times 750 - 28 = 737$

b. $P_{n+1} = 1.02P_n - 28, \quad P_0 = 750$

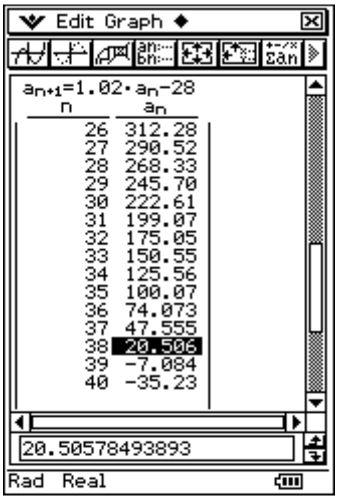
$b = 1.02$ A1

$c = -28$ A1

$a = 750$ A1

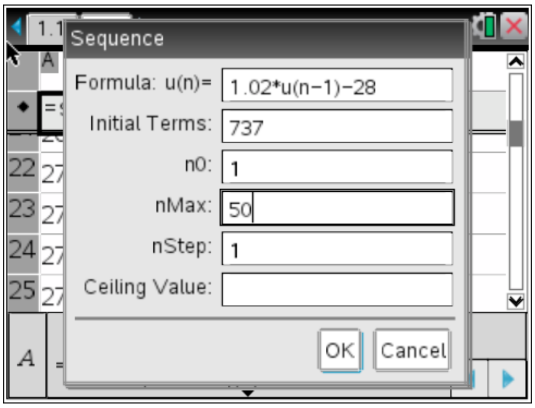
c. 39th minute 5.39 pm A1

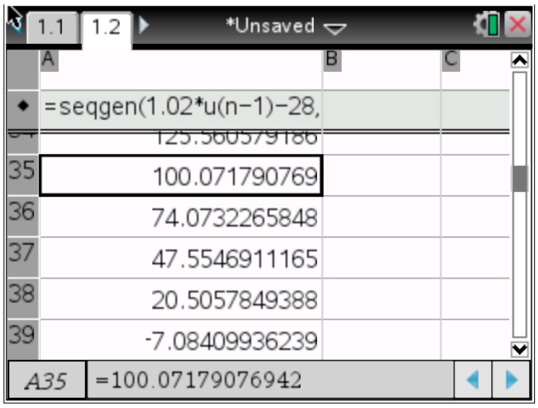
The recursive equation is entered in the calculator



$$P_{n+1} = 1.02P_n - 28, \quad P_0 = 750$$

to reveal that after 38 minutes only 20 car spaces are left so the car park will be full during the 39th minute.



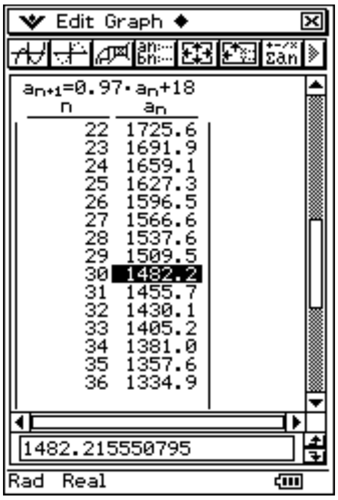


d. 1482 cars A1

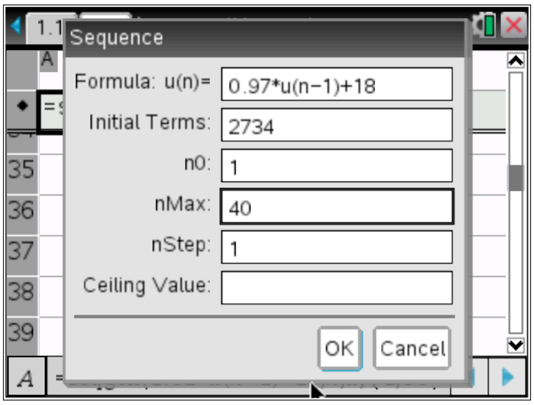
The recursive equation is entered in the calculator

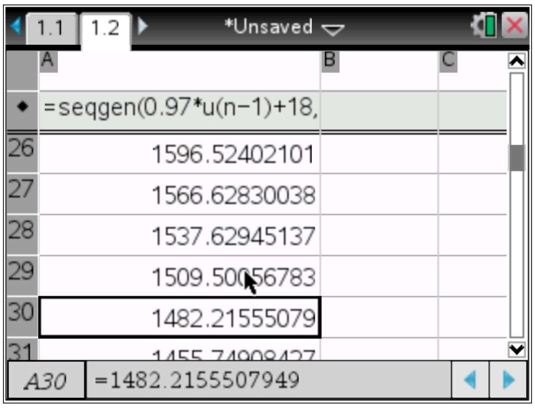
$C_{n+1} = 0.97C_n + 18, \quad C_0 = 2800$

After 30 minutes there are 1482 cars



C(1) = 0.97 × 2800 + 18 = 2734.
Find C(30)





e. When $C_n = 600$ then $C_{n+1} = 0.97 \times 600 + 18 = 600$ A1

The number of cars in the car park is reduced by 3%

When there are 600 cars in the car park 3% leave, that is,

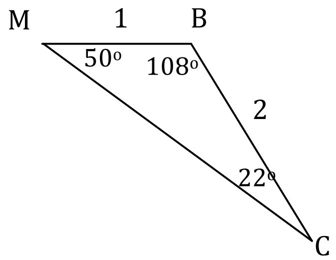
$3\% \times 600 = 18$ cars leave per minute = 18 cars arriving per minute to fill a car space

SECTION B Module 2: Geometry and Trigonometry

Question 1

a. The interior angle of the pentagon $\angle MBC = 180 - \frac{360}{5} = 180 - 72 = 108^o$

This means that $\angle BCM = 180 - 108 - 50 = 22^o$ M1



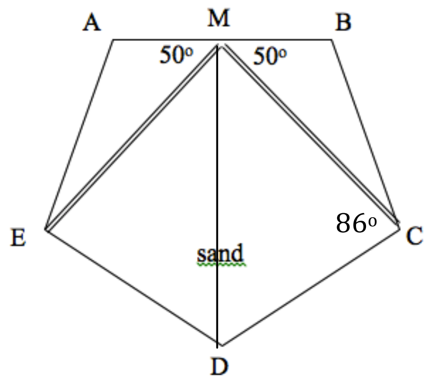
b.

Using the cosine rule

distance PQ = $\sqrt{1^2 + 2^2 - 2 \times 1 \times 1 \times \cos 108°}$

= 2.5 m A1

c.i.



The quadrilateral can be divided into two triangles.

Triangle MCD has the same area as triangle MDE

Angle MCD = $108^o - 22^o = 86^o$

Length CD = 2 m

Length MC = 2.5 m (from part b)

The Area of triangle MCD = $\frac{1}{2} \times MC \times CD \times \sin C$

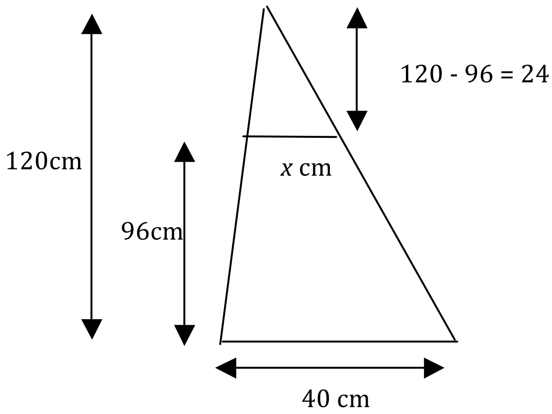
$= \frac{1}{2} \times 2.5 \times 2 \times \sin 86^o$ M1

Area of MCDE = $2 \times \frac{1}{2} \times 2.5 \times 2 \times \sin 86^o = 4.9878... = 5 \text{ m}^2$ A1

c.ii. Volume = $5 \times 0.45 = 2.25 \text{ m}^3$ H1

Question 2

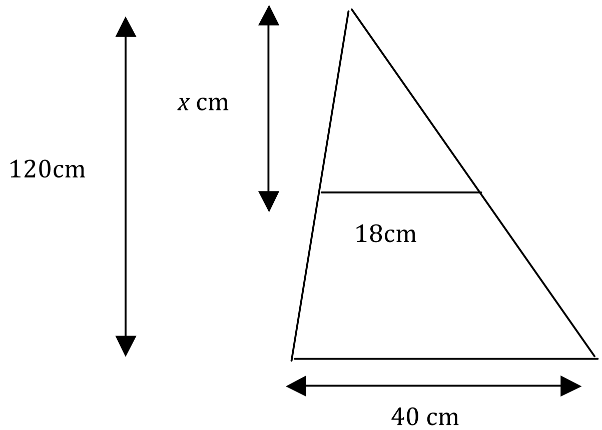
a. Using similar triangles



$$\frac{base}{height} = \frac{x}{24} = \frac{40}{120}$$

$$x = \frac{40}{120} \times 24 = 8 \text{ cm}$$ A1

b. Using similar triangles

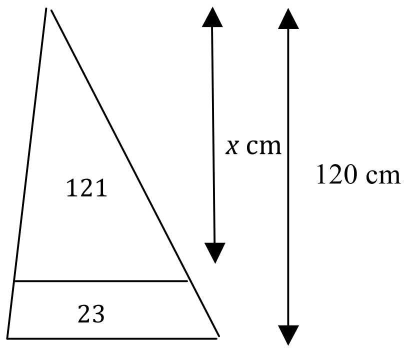


$$\frac{height}{base} = \frac{x}{18} = \frac{120}{40}$$

$$x = \frac{120}{40} \times 18 = 54 \text{cm}$$

so the middle shclf is $120 - 54 = 66$ cm above the ground A1

c.



Area ratio of the similar triangles

= 121: (23+121) = 121: 144

The height ratio = $\sqrt{121} : \sqrt{144} = 11:12$ Attempt at finding length ratio M1

$$\frac{x}{120} = \frac{11}{12} \quad \text{so } x = \frac{11}{12} \times 120 = 110$$

The height above the bottom shelf is 110 cm

The bottom shelf is 120-110 = 10 cm above the ground. A1

Question 3

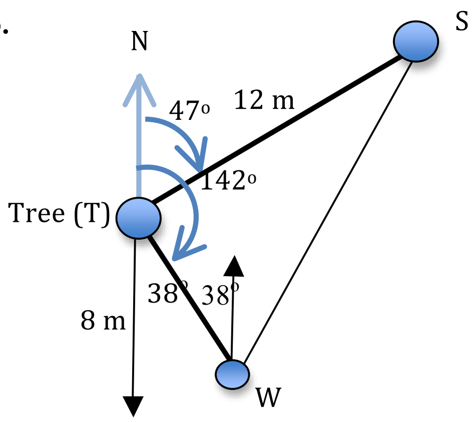
a. The angle STW = 142 – 47 = 95°

Using the cosine rule

distance SW = $\sqrt{12^2 + 8^2 - 2 \times 12 \times 8 \times \cos 95°}$ M1

= 15 metres

b.



To find Angle TWS use sine rule

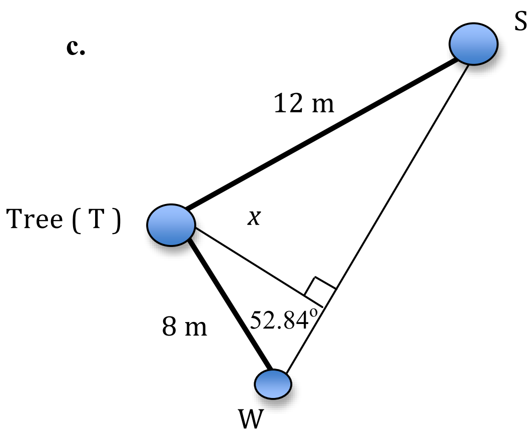
$\sin W = \dfrac{12 \sin 95^o}{15}$ M1

$W = \sin^{-1}(\dfrac{12 \sin 95^o}{15}) = 52.84^o$ M1

Bearing = 52.84° – 38° = 14.84

= 015° to the nearest degree A1

c.



The shortest distance from the tree is the line perpendicular to SW from T.

Using triangle TWB

$\sin 52.84 = \dfrac{x}{8}$

$x = 8 \sin 52.84 = 6.4$ m A1

The tree is 6 metres high and the shortest distance from the tree to the bench is 6.4 metres. This means that the tree will not hit the bench because its height is less than the shortest distance to the bench.

A1

Module 3: Graphs and relations

Question 1

a. $30 \times \$1.15 + 18 \times 1.25 = \$57$ A1

b. $\frac{45}{1.05} = 43$ or $\frac{45}{1.15} = 39$

The distributor can purchase 43 kg of tomatoes at $1.05 per kg.
or the distributor can purchase 39 kg of tomatoes at $1.15 per kg. A1

Question 2

a. Find gradient of (260, 32.80) and (315, 75.70)

$$\frac{75.70 - 32.80}{315 - 260} = 0.78$$

78 cents per kilogram A1

b. Find the y-intercept

$$y = mx + c$$
$$32.80 = 0.78 \times 260 + c$$
$$c = -170$$

A loss of $170 A1

c. Break even occurs when Profit = 0

$$Profit = 0.78x - 170$$
$$0 = 0.78x - 170$$
$$x = 217.9487$$

To break even 218 kg of apples must be sold. A1

Question 3

a.

$$38 + 2y \geq 140$$
$$2y \geq 102$$
$$y \geq 51$$

The minimum number of hectares of apples trimmed is 51. A1

b. $0.3x + 0.1y \le 30$ or $3x + y \le 300$ A1

c. 150 hectares of land is available to grow strawberries and apples. A1

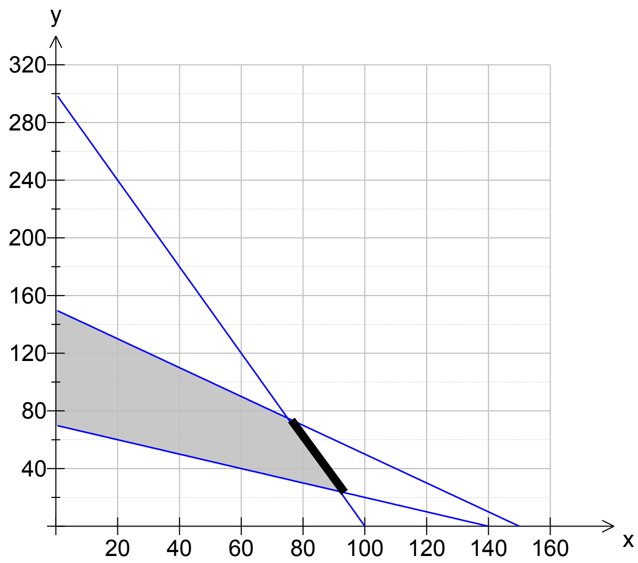
d. Line A : $3x + y = 300$

Line B : $x + y = 150$

Line C : $x + 2y = 140$

(One correct A1)
All correct A2

e.



Feasible region is shaded A1

f.

$\begin{cases} x+y=150 \\ 3x+y=300 \end{cases} \Big| x,y$ $\{x=75, y=75\}$

$\begin{cases} x+2y=140 \\ 3x+y=300 \end{cases} \Big| x,y$ $\{x=92, y=24\}$

Extreme Points	Profit= $345x + 115y$
(0 , 70)	$115 \times 70 = \$8050$
(0 , 150)	$115 \times 150 = \$17250$
(75 , 75)	$345 \times 75 + 115 \times 75 = \$34500$
(92 , 24)	$345 \times 92 + 115 \times 24 = \$34500$

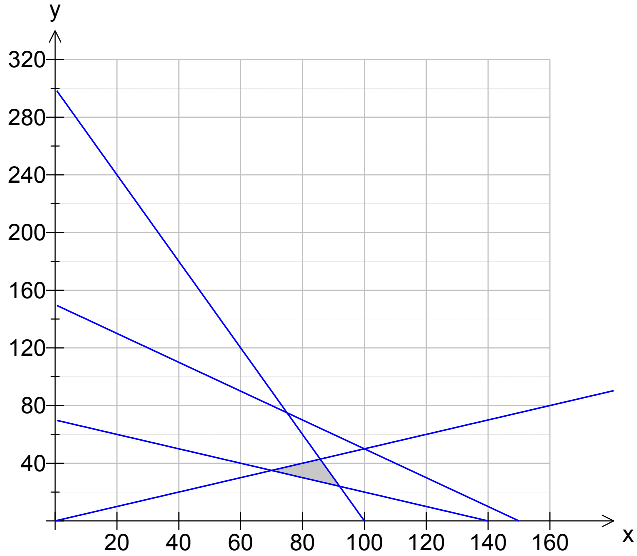
Since the points line (75, 75) and (92, 24) both give a maximum profit, then

The maximum profit occurs for all points on the line joining the line (75, 75) and (92, 24) inclusive.

(Award A1 if end points given only) A2

g. **i.** $x \geq 2y$ **or** $y \leq \dfrac{x}{2}$ A1

ii.



$$\begin{cases} x+2y=140 \\ y=x/2 \end{cases}\Bigg|_{x,y} \quad \{x=70, y=35\}$$

$$\begin{cases} x+2y=140 \\ 3x+y=300 \end{cases}\Bigg|_{x,y} \quad \{x=92, y=24\}$$

$$\begin{cases} 3x+y=300 \\ y=x/2 \end{cases}\Bigg|_{x,y} \quad \{x=85.71428571, y=42.857\blacktriangleright$$

The maximum now occurs along the line joining the points (600/7, 300/7) and (92, 24) inclusive.

Therefore the least number of hectares of strawberries is $\dfrac{600}{7} = 85.7$ A1

Module 4 Business related mathematics

Question 1

a. $\frac{2.4\%}{12} = 0.2\%$ A1

b. 0.2% of January balance = 2.70.

Solving $\frac{0.2}{100} \times x = 2.70$ gives $x = 1350$ M1

Therefore he withdrew $1525.50 - 1350 = \$175.50$ A1

c. Balances \$1350, \$1850, \$1500, \$1100, \$2100 A1

d. January: \$2.70 February: $0.2\% \times 1350 = \$2.70$ March $0.2\% \times 1100 = \$2.20$ M1

Interest earned = 2.70 + 2.70 + 2.20 = \$7.60 A1

Question 2

a. Finance Solver

$\text{N} = 12$

$\text{I}(\%) = 2.4$

$\text{PV} = -2500$

$\text{Pmt} = 0$

$\text{FV} = ?$

$\text{PpY} = 12$

$\text{CpY} = 12$

Future value is \$2560.66 therefore interest earned is $2560.66 - 2500 = \$60.66$ A1

b. $R = 1 + \frac{2.4}{1200} = 1.002$ A1

$n = 12$ A1

c. Finance Solver

$\text{N} = 12$

$\text{I}(\%) = 2.4$

$\text{PV} = -2500$

$\text{Pmt} = -100$

$\text{FV} = ?$

$\text{PpY} = 12$

$\text{CpY} = 12$

Future value is \$3773.95 A1

Question 3

a. Depreciation per annum $= \dfrac{5000-400}{5} = 920$

Annual depreciation rate = $\dfrac{920}{5000} \times 100 = 18.4\%$ A1

b. $\dfrac{4600}{5000} \times 100 = 92\%$ A1

c. $\dfrac{4600}{0.50} = 9200$ hours

$\dfrac{9200}{5} = 1840$ hours A1

d. Depreciated value = Purchase price $\times (1 - \dfrac{\textit{depreciation rate}}{100})^{\textit{number of years}}$

$400 = 5000(1 - \dfrac{d}{100})^5$ M1

$1 - \dfrac{d}{100} = \sqrt[5]{0.08}$

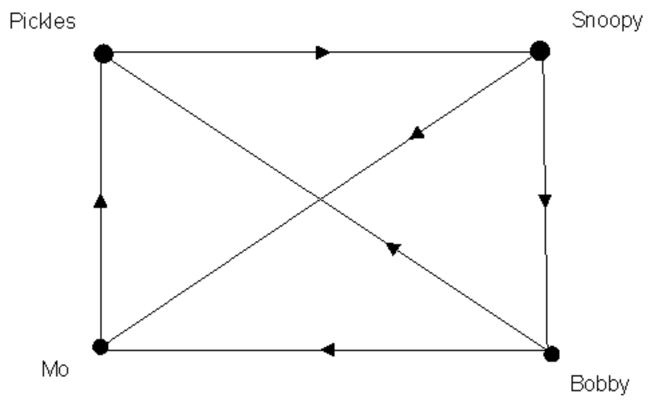
$\dfrac{d}{100} = 0.39658$

depreciation rate =39.7% A1

Module 5: Networks and decision mathematics

Question 1

a.



A1

b. Bobby has dominance over Pickles who in turn has dominance over Snoopy A1

c.

Dominance order	Total
Snoopy	5 (2 + 3)
Bobby	4 (2 + 2)
Pickles	3 (1 + 2)
Mo	2 (1 + 1)

Any 2 correct A1
All 4 correct A1

Question 2

a. Since this would be an Euler circuit and this only exists if the degree of every vertex is even. The degrees of vertices B, C, D and E are all odd.
A1

b. **i.** Any 2 of vertices B, C, D and E. A1

ii. Any appropriate path beginning and ending at two of B, C, D and E A1

c. 3550 metres (along path CBAFEDG) A1

Question 3

a. ACFJ and BHIJ (both have duration of 14 hours) A1

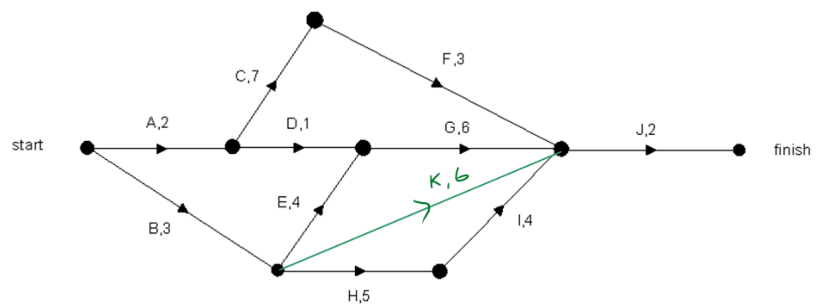
b. 15 hours (along path BEGJ) A1

c. A, C, D, F, H and I A1

d. F has an earliest start time of 9 and a latest start time of 10
Float time = latest start time – earliest start time = 10-9 =1 M1

e. 5 (A, C, H, I and F) A1

f. Arrow as marked on diagram



A1

g. Initially K does not affect the critical path and has a float time of 4.
If K takes 5 hours longer it will add 1 to the minimum completion time.
Answer equals 16 hours (with new critical path BKJ) A1

Module 6: Matrices

Question 1

a. $2\times\begin{bmatrix}20\\15\\10\\12\end{bmatrix}+\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$ A1

b. $2\times\begin{bmatrix}20\\15\\10\\12\end{bmatrix}+\begin{bmatrix}1\\1\\1\\1\end{bmatrix}=\begin{bmatrix}41\\31\\21\\25\end{bmatrix}$ 41+31+21+25= 118 staff A1

Question 2

a. $\begin{bmatrix}1 & 1\\250 & 150\end{bmatrix}\begin{bmatrix}P\\S\end{bmatrix}=\begin{bmatrix}5500\\975000\end{bmatrix}$

Using matrices of correct order M1

All correct A1

b. The determinant is non zero (can be 100 or – 100 depending on the row in which equations are entered) therefore there is a unique solution.

A1

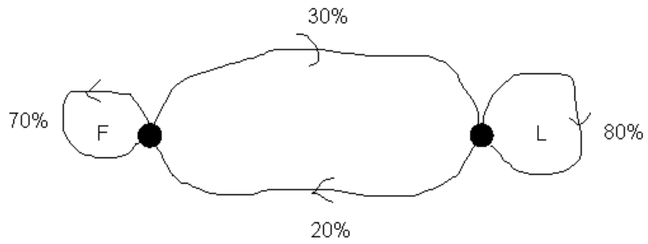
c.

$\begin{bmatrix}P\\S\end{bmatrix}=\begin{bmatrix}-1.5 & 0.01\\2.5 & -0.01\end{bmatrix}\begin{bmatrix}5500\\975000\end{bmatrix}$ A1

d. Solving from **c.** gives P = 1500 and S = 4000 A1
1500 Platinum tickets are sold and 4000 Standard tickets are sold.

Question 3

a.



A1

b. When he plays Like a Cyclone to open a concert, 80% of the time he will open the next concert with Like a Cyclone A1

c. $S_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ A1

d. $S_4 = T^3 S_1 = \begin{bmatrix} 0.475 \\ 0.525 \end{bmatrix}$ therefore 52.5% chance A1

e. Using two consecutive large values of n, the steady state can be found e.g.

$T^{99} \times S_1 = T^{100} \times S_1 = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$ M1

Chance of opening with Fingerpowder is 40%

f. No effect as the steady state is independent of the initial state.

Or demonstrate as in e. A1

g. Let the transition matrix be $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

For this to occur $\frac{b}{b+c} = 0.75$ and $\frac{c}{b+c} = 0.25$ M1

Solve to give $b = 0.75$ and $c = 0.25$ therefore matrix is $\begin{bmatrix} 0.75 & 0.75 \\ 0.25 & 0.25 \end{bmatrix}$ A1