VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2012

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

									Letter
Figures									
Words									

FURTHER MATHEMATICS

Written examination 2

Monday 5 November 2012

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 31 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

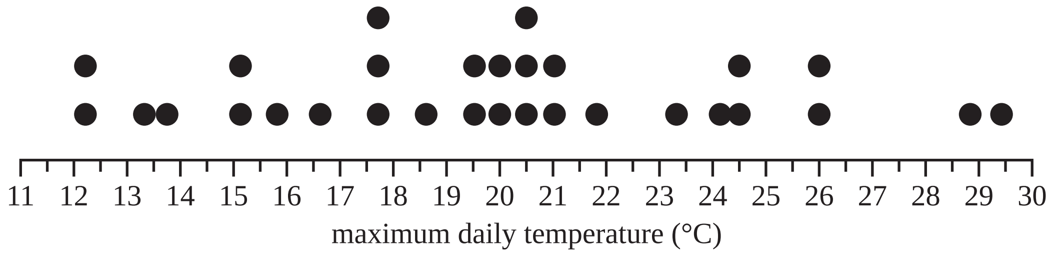
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Diagrams are not to scale unless specified otherwise.

Core

Question 1

The dot plot below displays the maximum daily temperature (in °C) recorded at a weather station on each of the 30 days in November 2011.



a. From this dot plot, determine

i. the median maximum daily temperature, correct to the nearest degree

ii. the percentage of days on which the maximum temperature was less than 16 °C.

Write your answer, correct to one decimal place.

1 + 1 = 2 marks

Records show that the **minimum** daily temperature for November at this weather station is approximately normally distributed with a mean of 9.5 °C and a standard deviation of 2.25 °C.

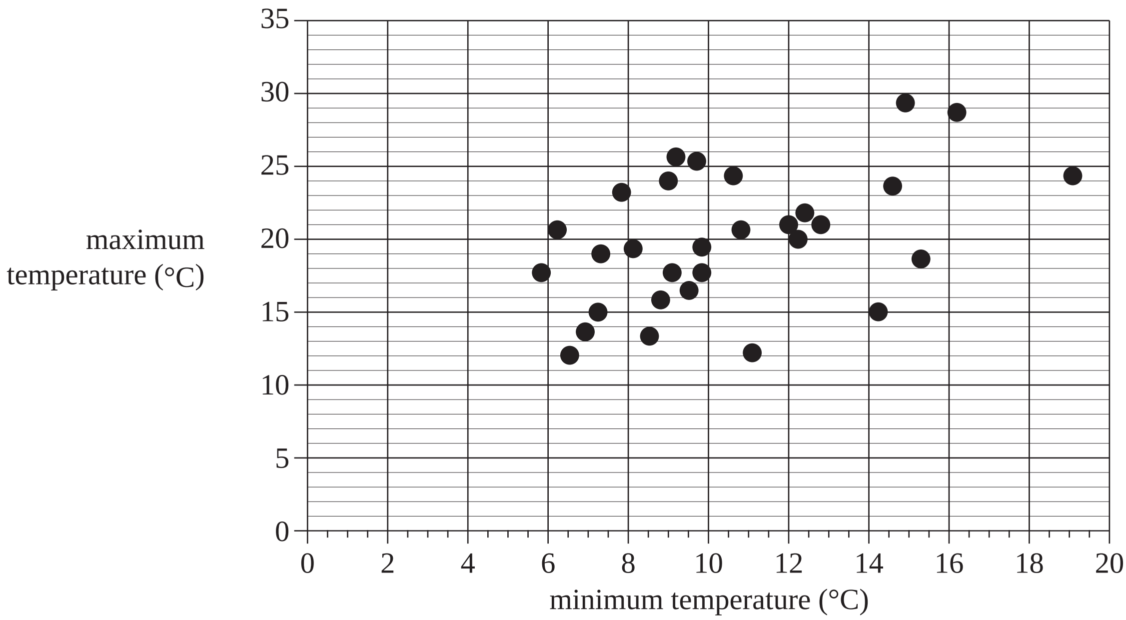
b. Determine the percentage of days in November that are expected to have a minimum daily temperature less than 14 °C at this weather station.

Write your answer, correct to one decimal place.

1 mark

Question 2

The maximum temperature and the minimum temperature at this weather station on each of the 30 days in November 2011 are displayed in the scatterplot below.



The correlation coefficient for this data set is $r = 0.630$.

The equation of the least squares regression line for this data set is

$$\textit{maximum temperature} = 13 + 0.67 \times \textit{minimum temperature}$$

a. **Draw** this least squares regression line on the scatterplot above.

1 mark

b. Interpret the vertical intercept of the least squares regression line in terms of maximum temperature and minimum temperature.

1 mark

c. Describe the relationship between the maximum temperature and the minimum temperature in terms of strength and direction.

1 mark

d. Interpret the slope of the least squares regression line in terms of maximum temperature and minimum temperature.

1 mark

e. Determine the percentage of variation in the maximum temperature that may be explained by the variation in the minimum temperature.

Write your answer, correct to the nearest percentage.

1 mark

On the day that the minimum temperature was 11.1 °C, the actual maximum temperature was 12.2 °C.

f. Determine the residual value for this day if the least squares regression line is used to predict the maximum temperature.

Write your answer, correct to the nearest degree.

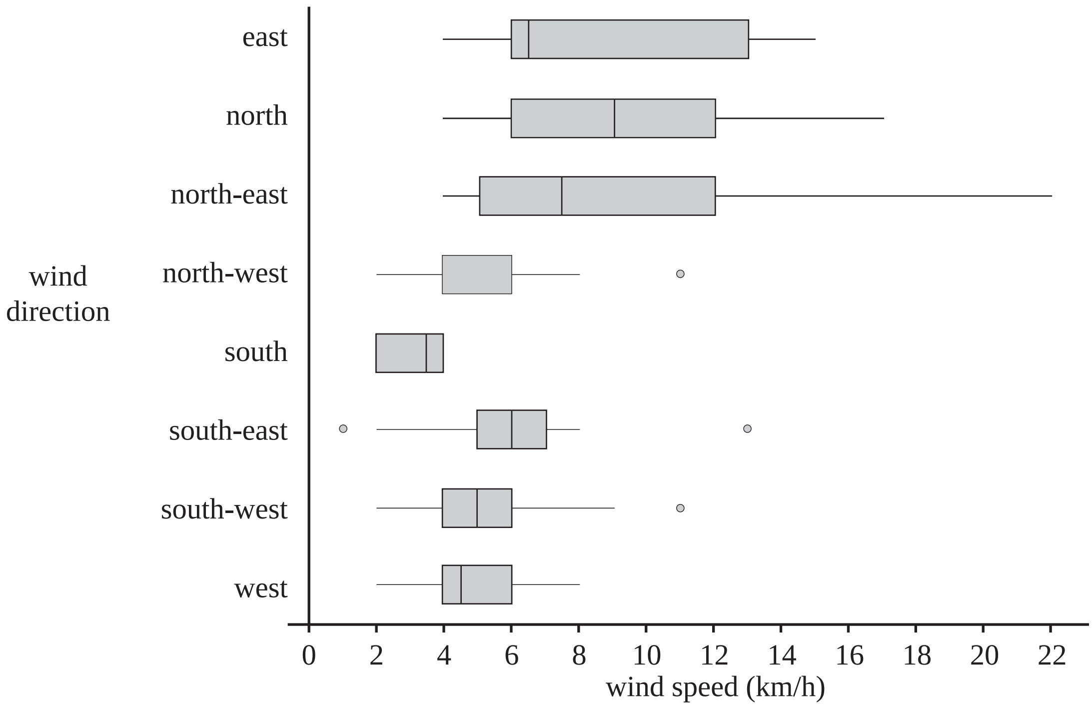
2 marks

Question 3

A weather station records the wind speed and the wind direction each day at 9.00 am.

The wind speed is recorded, correct to the nearest whole number.

The parallel boxplots below have been constructed from data that was collected on the 214 days from June to December in 2011.



a. Complete the following statements.

The wind direction with the lowest recorded wind speed was ______.

The wind direction with the largest range of recorded wind speeds was ______.

1 mark

b. The wind blew from the south on eight days.

Reading from the parallel boxplots above we know that, for these eight wind speeds, the

first quartile	$Q_1 = 2$ km/h
median	$M = 3.5$ km/h
third quartile	$Q_3 = 4$ km/h

Given that the eight wind speeds were recorded to the nearest whole number, write down the eight wind speeds.

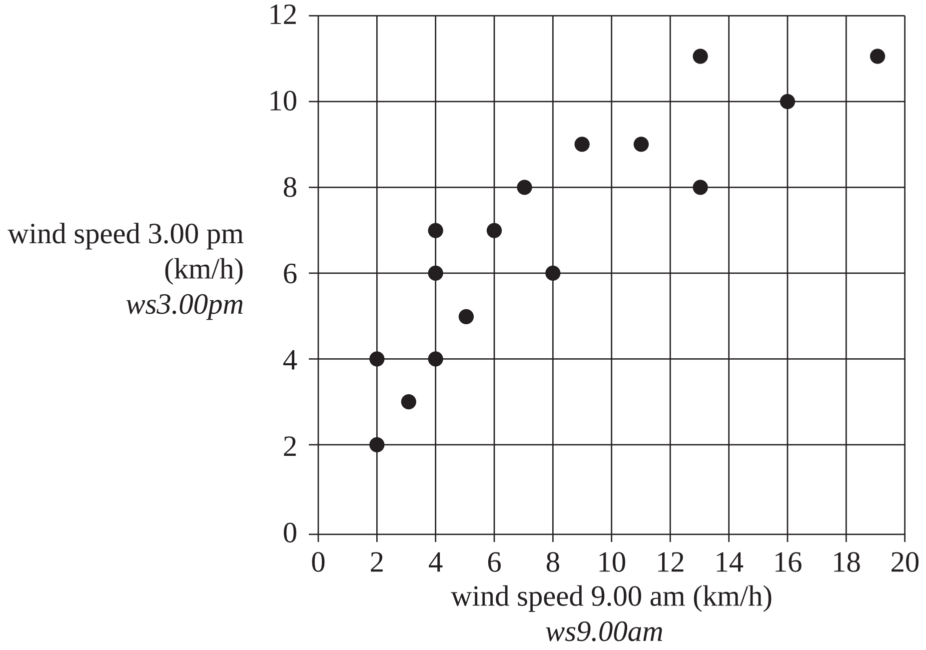
___

1 mark

Question 4

The wind speeds (in km/h) that were recorded at the weather station at 9.00 am and 3.00 pm respectively on 18 days in November are given in the table below. A scatterplot has been constructed from this data set.

Wind speed (km/h)	
9.00 am	**3.00 pm**
2	2
4	6
4	7
4	4
13	11
6	7
3	3
16	10
6	7
13	8
11	9
2	4
7	8
5	5
8	6
6	7
19	11
9	9



Let the wind speed at 9.00 am be represented by the variable *ws9.00am* and the wind speed at 3.00 pm be represented by the variable *ws3.00pm*.

The relationship between *ws9.00am* and *ws3.00pm* shown in the scatterplot above is nonlinear.

A **squared transformation** can be applied to the variable *ws3.00pm* to linearise the data in the scatterplot.

a. Apply the squared transformation to the variable *ws3.00pm* and determine the equation of the least squares regression line that allows $(ws3.00pm)^2$ to be predicted from *ws9.00am*.

In the boxes provided, write the coefficients for this equation, correct to one decimal place.

$(ws3.00pm)^2 = \boxed{} + \boxed{} \times ws9.00am$

2 marks

b. Use this equation to predict the wind speed at 3.00 pm on a day when the wind speed at 9.00 am is 24 km/h.

Write your answer, correct to the nearest whole number.

1 mark

END OF CORE
TURN OVER

Module 1: Number patterns

Question 1

In the first month of land sales in a new housing estate, 168 blocks of land were sold.
In the second month of land sales, 162 blocks of land were sold.
In the third month of land sales, 156 blocks of land were sold.
The land sales continued in this pattern until all the blocks of land were sold.
The number of blocks of land sold each month forms the terms of an arithmetic sequence.

$$168, 162, 156, \ldots$$

a. Show that the common difference for this sequence is –6.

1 mark

b. How many blocks of land were sold in the sixth month of land sales?

1 mark

c. How many **more** blocks of land were sold in the eighth month than in the tenth month?

1 mark

d. **i.** How many blocks of land were sold, in total, in the first 18 months?

ii. How many blocks of land had **not** been sold after 18 months?

1 + 2 = 3 marks

Question 2

In the first year after land sales began, 16 building applications were approved by the council.

The number of building applications that the council approved each year after that formed the terms of a geometric sequence with a common ratio of 1.5.

a. How many building applications were approved by the council in the third year after land sales began?

1 mark

b. In which year after land sales began did the number of building applications approved by the council each year first exceed 100?

1 mark

c. How many building applications were approved, in total, by the council, in the first five years after land sales began?

1 mark

d. In which year after land sales began did the total number of building applications approved by the council first exceed 1000?

1 mark

The sequence that models the number of building applications approved by the council in the *n*th year after land sales began can be written as the difference equation

$$t_{n+1} = a \times t_n + b \qquad t_1 = c$$

where t_n represents the number of building applications approved by the council in the *n*th year after land sales began, and a, b and c are constants.

e. Write down the values of a, b and c.

a =

b =

c =

2 marks

Question 3

Town planners use the following difference equation to model the population, P_n, of the housing estate at the end of the nth year after land sales began.

$$P_{n+1} = 0.96\,P_n + 500 \qquad P_1 = 50$$

a. What is the population of the housing estate at the end of the third year after land sales began?

Write your answer, correct to the nearest whole number.

1 mark

b. At the end of which year after land sales began will the population of the housing estate exceed 4000 people?

1 mark

c. According to this mathematical model, what is the greatest possible population of the housing estate?

Round your answer, correct to the nearest person.

1 mark

END OF MODULE 1

CONTINUES OVER PAGE

Module 2: Geometry and trigonometry

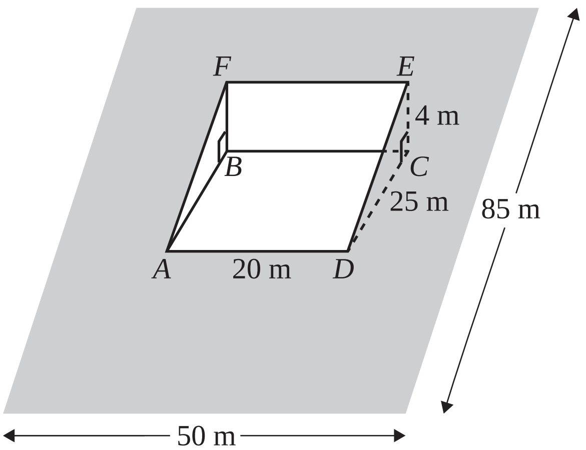
Question 1

A rectangular block of land has width 50 metres and length 85 metres.

a. Calculate the area of this block of land.
Write your answer in m^2.

1 mark

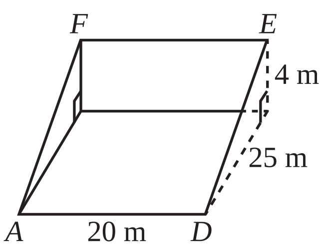
In order to build a house, the builders dig a hole in the block of land.
The hole has the shape of a right-triangular prism, *ABCDEF*.
The width *AD* = 20 m, length *DC* = 25 m and height *EC* = 4 m are shown in the diagram below.



b. Calculate the volume of the right-triangular prism, *ABCDEF*.
Write your answer in m^3.

1 mark

Once the right-triangular prism shape has been dug, a fence will be placed along the two sloping edges, *AF* and *DE*, and along the edges *AD* and *FE*.



c. Calculate the total length of fencing that will be required.
Write your answer, in metres, correct to one decimal place.

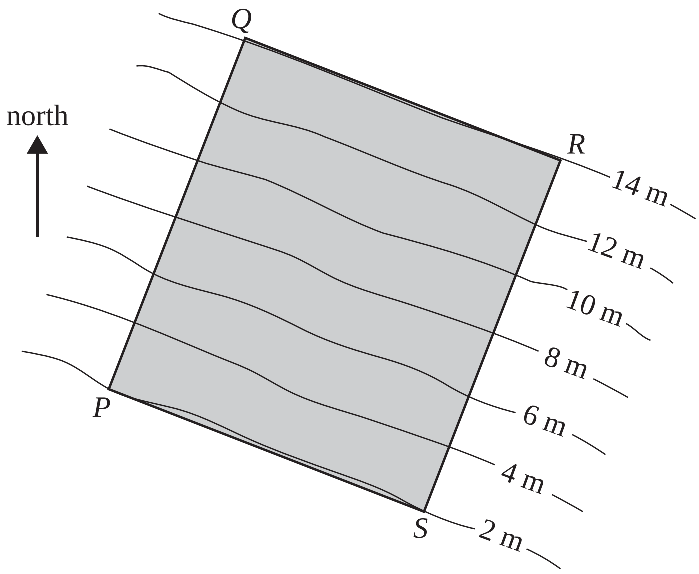
1 mark

Question 2

A contour map for the rectangular block of land, labelled *PQRS*, is shown below.

The boundary, *PS*, of the land is 2 metres above sea level.

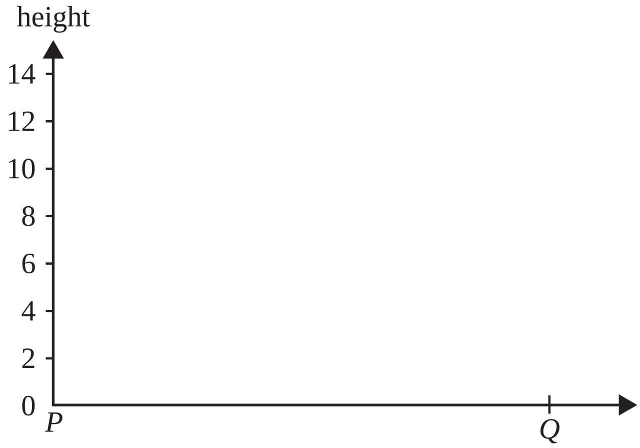
The contour interval on the map is 2 metres.



a. Determine the difference in height between point *P* and point *Q*.
Write your answer in metres.

1 mark

b. Sketch the cross-section of the block of land along the horizontal axis, *PQ*, below.



1 mark

c. The bearing of point *Q* from point *P* is 042°.
Determine the bearing of

i. point *S* from point *P*

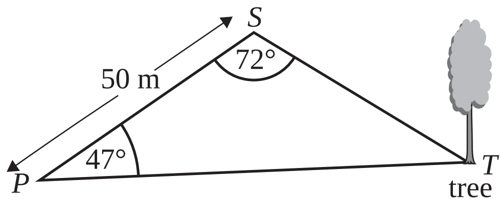
ii. point *S* from point *R*.

1 + 1 = 2 marks

Question 3

A tree is growing near the block of land.

The base of the tree, *T*, is at the same level as the corners, *P* and *S*, of the block of land.



a. Show that, correct to two decimal places, distance *ST* is 41.81 metres.

1 mark

b. From point *S,* the angle of elevation to the top of the tree is 22°.

Calculate the height of the tree.

Write your answer, in metres, correct to one decimal place.

1 mark

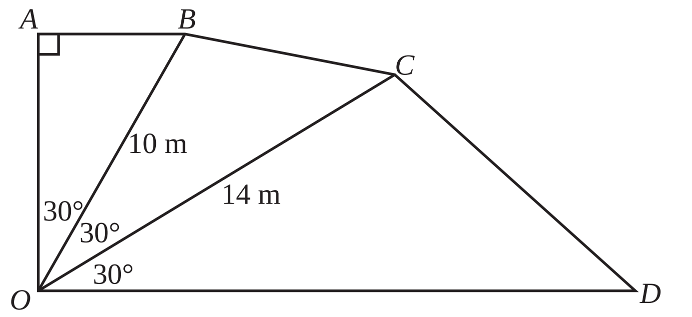
Question 4

OABCD has three triangular sections, as shown in the diagram below.

Triangle *OAB* is a right-angled triangle.

Length *OB* is 10 m and length *OC* is 14 m.

Angle *AOB* = angle *BOC* = angle *COD* = 30°



a. Calculate the length, *OA*.
Write your answer, in metres, correct to two decimal places.

1 mark

b. Determine the area of triangle *OAB*.
Write your answer, in m^2, correct to one decimal place.

1 mark

c. Triangles *OBC* and *OCD* are similar.
The area of triangle *OBC* is 35 m^2.
Find the area of triangle *OCD*, in m^2.

2 marks

d. Determine angle *CDO*.
Write your answer, correct to the nearest degree.

2 marks

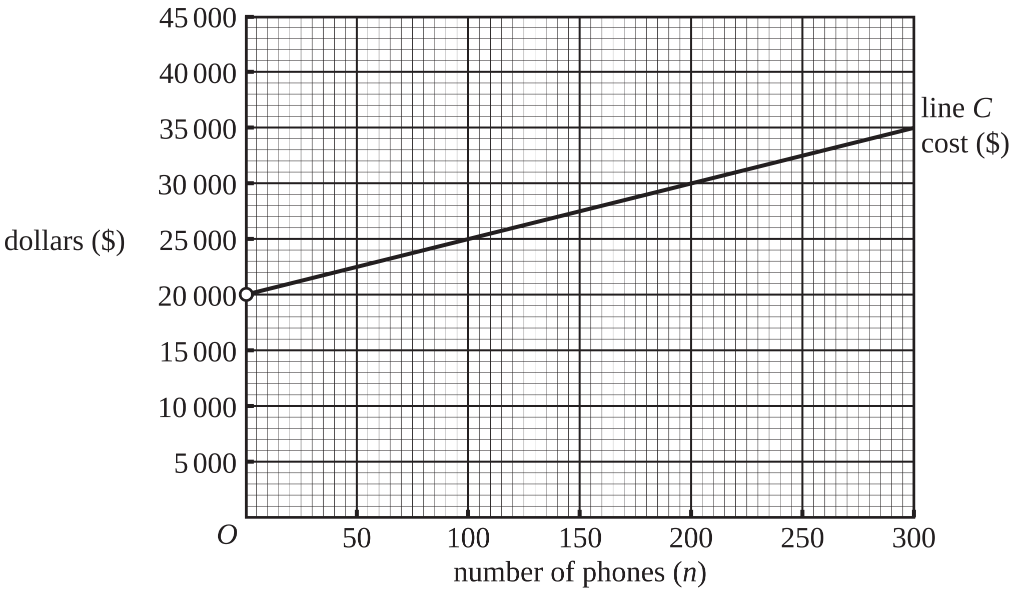
END OF MODULE 2

TURN OVER

Module 3: Graphs and relations

Question 1

The cost, C, in dollars, of making n phones, is shown by the line in the graph below.



a. **i.** Calculate the gradient of the line, C, drawn above.

---

ii. Write an equation for the cost, C, in dollars, of making n phones.

$C =$

1 + 1 = 2 marks

b. The revenue, R, in dollars, obtained from selling n phones is given by $R = 150n$.

i. Draw this line on the graph above.

ii. How many phones would need to be sold to obtain $54 000 in revenue?

---

1 + 1 = 2 marks

c. Determine the number of phones that would need to be made and sold to break even.

---

1 mark

Question 2

The cost, C, and revenue, R, in dollars, for making and selling n laptops respectively is given by

cost	$C = 320n + 125\,000$
revenue	$R = 600n$

a. What is the minimum number of laptops that should be made and sold in order to obtain a profit?

1 mark

The cost of making each laptop increases by \$50.

b. The selling price of each laptop will need to increase to offset this cost increase.

Find the new selling price of each laptop so that the break-even point occurs when 400 laptops are made and sold.

1 mark

Question 3

A company repairs phones and laptops.

Let x be the number of phones repaired each day
y be the number of laptops repaired each day.

It takes 35 minutes to repair a phone and 50 minutes to repair a laptop.
The constraints on the company are as follows.

Constraint 1 $x \geq 0$

Constraint 2 $y \geq 0$

Constraint 3 $35x + 50y \leq 1750$

Constraint 4 $y \leq \frac{4}{5}x$

a. Explain the meaning of Constraint 3 in terms of the time available to repair phones and laptops.

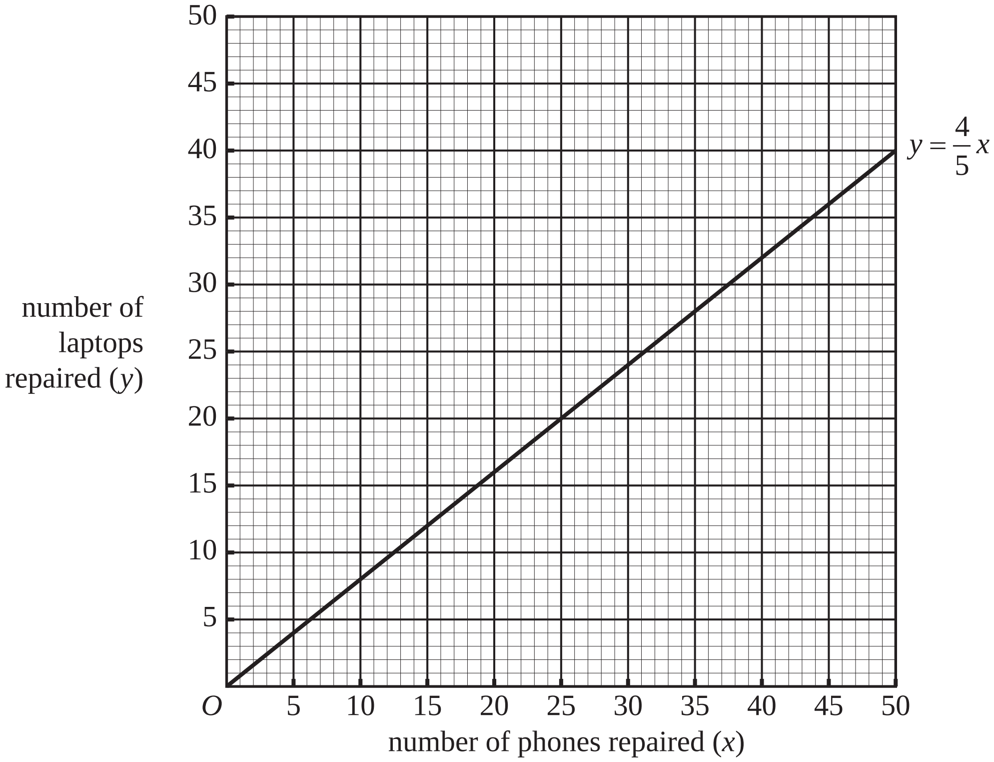
1 mark

b. Constraint 4 describes the maximum number of phones that may be repaired relative to the number of laptops repaired.
Use this constraint to complete the following sentence.

For every ten phones repaired, at most $\boxed{}$ laptops may be repaired.

1 mark

The line $y = \frac{4}{5}x$ is drawn on the graph below.



c. Draw the line $35x + 50y = 1750$ on the graph.

1 mark

d. Within Constraints 1 to 4, what is the maximum number of laptops that can be repaired each day?

1 mark

e. On a day in which exactly nine laptops are repaired, what is the maximum number of phones that can be repaired?

1 mark

The profit from repairing one phone is \$60 and the profit from repairing one laptop is \$100.

f. **i.** Determine the number of phones and the number of laptops that should be repaired each day in order to maximise the total profit.

ii. What is the maximum total profit per day that the company can obtain from repairing phones and laptops?

2 + 1 = 3 marks

END OF MODULE 3
TURN OVER

Module 4: Business-related mathematics

Question 1

A club purchased new equipment priced at $8360. A 15% deposit was paid.

a. Calculate the deposit.

1 mark

b. **i.** Determine the amount of money that the club still owes on the equipment after the deposit is paid.

The amount owing will be fully repaid in 12 instalments of $650.

ii. Determine the total interest paid.

1 + 1 = 2 marks

c. The price, $8360, included 10% GST (Goods and Services Tax).

Calculate the price of the equipment before the GST was added.

1 mark

Question 2

The value of the equipment will be depreciated using the unit cost method.

The initial value of the equipment is \$8360. It will depreciate by 22 cents per hour of use.

On average, the equipment will be used for 3800 hours each year.

a. Calculate the depreciated value of the equipment after three years.

1 mark

b. Show that, in any one year, the flat rate method of depreciation with a depreciation rate of 10% per annum will give the same annual depreciation as the unit cost method.

1 mark

c. After how many years will equipment be written off with a depreciated value of \$0?

1 mark

d. Suppose the reducing balance method is used to depreciate the equipment instead of the unit cost method.

The initial value of the equipment is \$8360. It will depreciate at a rate of 14% per annum of the reducing balance.

Find, correct to the nearest dollar, the depreciated value of the equipment after ten years.

1 mark

Question 3

An area of the club needs to be refurbished.

\$40 000 is borrowed at an interest rate of 7.8% per annum.

Interest on the unpaid balance is charged to the loan account monthly.

Suppose the \$40 000 loan is to be fully repaid in equal monthly instalments over five years.

a. Determine the monthly payment, correct to the nearest cent.

1 mark

b. If, instead, the monthly payment was \$1000, how many months will it take to fully repay the \$40 000?

1 mark

c. Suppose no payments are made on the loan in the first 12 months.

i. Write down a calculation that shows that the balance of the loan account after the first 12 months will be \$43 234, correct to the nearest dollar.

ii. After the first 12 months, only the interest on the loan is paid each month.

Determine the monthly interest payment, correct to the nearest cent.

1 + 1 = 2 marks

Question 4

Arthur invested \$80 000 in a perpetuity that returns \$1260 per quarter. Interest is calculated quarterly.

a. Calculate the annual interest rate of Arthur's investment.

1 mark

b. After Arthur has received 20 quarterly payments, how much money remains invested in the perpetuity?

1 mark

c. Arthur's wife, Martha, invested a sum of money at an interest rate of 9.4% per annum, compounding quarterly.

She will be paid \$1260 per quarter from her investment.

After ten years, the balance of Martha's investment will have reduced to \$7000.

Determine the initial sum of money Martha invested.

Write your answer, correct to the nearest dollar.

1 mark

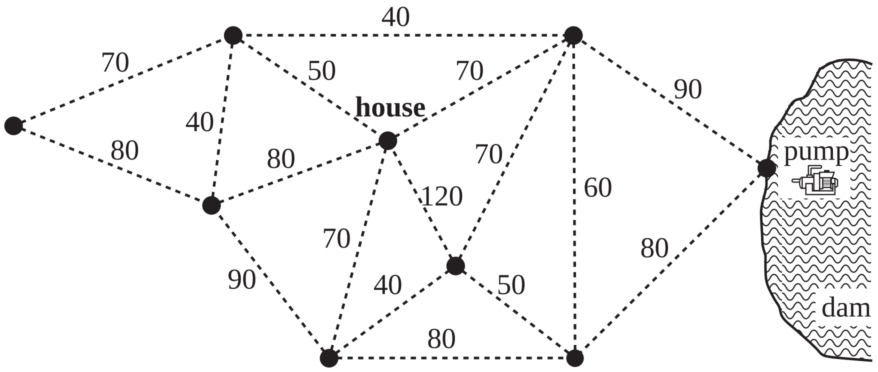
Module 5: Networks and decision mathematics

Question 1

Water will be pumped from a dam to eight locations on a farm.

The pump and the eight locations (including the house) are shown as vertices in the network diagram below.

The numbers on the edges joining the vertices give the shortest distances, in metres, between locations.



a. **i.** Determine the shortest distance between the house and the pump.

____

ii. How many vertices on the network diagram have an odd degree?

____

iii. The total length of all edges in the network is 1180 metres.
A journey starts and finishes at the house and travels along every edge in the network.
Determine the shortest distance travelled.

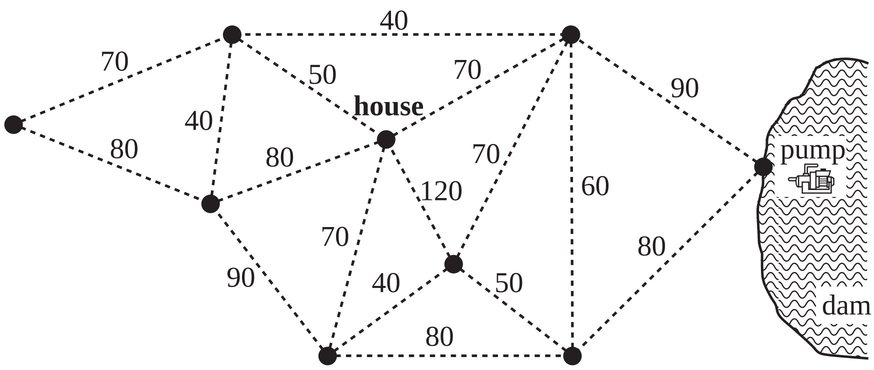
____

1 + 1 + 1 = 3 marks

The total length of pipe that supplies water from the pump to the eight locations on the farm is a minimum.

This minimum length of pipe is laid along some of the edges in the network.

b. **i.** On the diagram below, **draw** the minimum length of pipe that is needed to supply water to all locations on the farm.



ii. What is the mathematical term that is used to describe this minimum length of pipe in **part i.**?

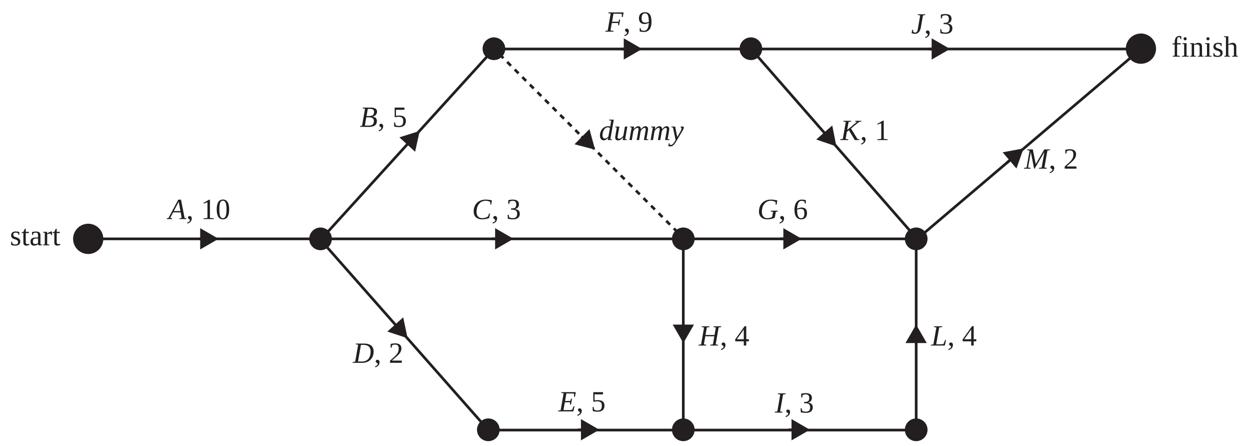
____

1 + 1 = 2 marks

Question 2

Thirteen activities must be completed before the produce grown on a farm can be harvested.

The directed network below shows these activities and their completion times in days.



a. Determine the earliest starting time, in days, for activity *E*.

1 mark

b. A *dummy* activity starts at the end of activity *B*.
Explain why this *dummy* activity is used on the network diagram.

1 mark

c. Determine the earliest starting time, in days, for activity *H*.

1 mark

d. In order, list the activities on the critical path.

1 mark

e. Determine the latest starting time, in days, for activity *J*.

1 mark

Question 3

Four tasks, *W*, *X*, *Y* and *Z*, must be completed.

Four workers, Julia, Ken, Lana and Max, will each do one task.

Table 1 shows the time, in minutes, that each person would take to complete each of the four tasks.

Table 1

	Worker			
Task	**Julia**	**Ken**	**Lana**	**Max**
W	26	21	22	25
X	31	26	21	38
Y	29	26	20	27
Z	38	26	26	35

The tasks will be allocated so that the total time of completing the four tasks is a minimum.

The Hungarian method will be used to find the optimal allocation of tasks.

Step 1 of the Hungarian method is to subtract the minimum entry in each row from each element in the row.

Table 2

	Worker			
Task	**Julia**	**Ken**	**Lana**	**Max**
W	5	0	1	4
X	10	5	0	
Y	9	6	0	7
Z	12	0	0	9

a. Complete step 1 for task *X* by writing down the number missing from the shaded cell in Table 2.

1 mark

The second step of the Hungarian method ensures that all columns have at least one zero.

The numbers that result from this step are shown in Table 3 below.

Table 3

	Worker			
Task	**Julia**	**Ken**	**Lana**	**Max**
W	**0**	**0**	**1**	**0**
X	**5**	**5**	**0**	**13**
Y	**4**	**6**	**0**	**3**
Z	**7**	**0**	**0**	**5**

b. Following the Hungarian method, the smallest number of lines that can be drawn to cover the zeros is shown dashed in Table 3.

These dashed lines indicate that an optimal allocation cannot be made yet.

Give a reason why.

1 mark

c. Complete the steps of the Hungarian method to produce a table from which the optimal allocation of tasks can be made.

Two blank tables have been provided for working if needed.

Worker

Task	**Julia**	**Ken**	**Lana**	**Max**
W				
X				
Y				
Z				

Worker

Task	**Julia**	**Ken**	**Lana**	**Max**
W				
X				
Y				
Z				

1 mark

d. Write the name of the task that each person should do for the optimal allocation of tasks.

Worker	**Task**
Julia	
Ken	
Lana	
Max	

2 marks

Module 6: Matrices

Question 1

Matrix F below shows the flight connections for an airline that serves four cities, Anvil (A), Berga (B), Cantor (C) and Dantel (D).

$$F = \begin{array}{c} \quad\quad \textit{from} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ A \\ B \\ C \\ D \end{array} \; \textit{to}$$

In this matrix, the '1' in column C row B, for example, indicates that, using this airline, you can fly directly from Cantor to Berga. The '0' in column C row D, for example, indicates that you cannot fly directly from Cantor to Dantel.

a. Complete the following sentence.

On this airline, you can fly directly from Berga to ☐ and ☐ .

1 mark

b. List the route that you must follow to fly from Anvil to Cantor.

1 mark

c. Evaluate the matrix product $G = KF$, where $K = [1 \;\; 1 \;\; 1 \;\; 1]$.

$G =$

1 mark

d. In the context of the problem, what information does matrix G contain?

1 mark

Question 2

Rosa uses the following six-digit pin number for her bank account: 216342

With her knowledge of matrices, she decides to use matrix multiplication to disguise this pin number.

First she writes the six digits in the 2×3 matrix A.

$$A = \begin{bmatrix} 2 & 6 & 4 \\ 1 & 3 & 2 \end{bmatrix}$$

Next she creates a new matrix by forming the matrix product, $C = BA$,

where $B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$.

a. **i.** Determine the matrix $C = BA$.

$C =$

ii. From the matrix C, Rosa is able to write down a six-digit number that disguises her original pin number. She uses the same pattern that she used to create matrix A from the digits 216342.

Write down the new six-digit number that Rosa uses to disguise her pin number.

1 + 1 = 2 marks

b. Show how the original matrix A can be regenerated from matrix C.

1 mark

Question 3

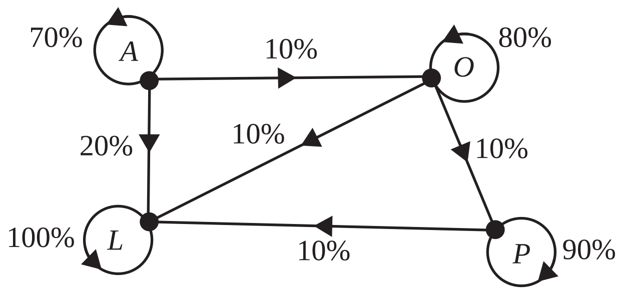
When a new industrial site was established at the beginning of 2011, there were 350 staff at the site.

The staff comprised 100 apprentices (*A*), 200 operators (*O*) and 50 professionals (*P*).

At the beginning of each year, staff can choose to stay in the same job, move to a different job at the site or leave the site (*L*).

The number of staff in each category at the beginning of 2011 is given in the matrix $S_{2011} = \begin{bmatrix} 100 \\ 200 \\ 50 \\ 0 \end{bmatrix} \begin{matrix} A \\ O \\ P \\ L \end{matrix}$

The transition diagram below shows the way in which staff are expected to change their jobs at the site each year.



a. How many staff at the site are expected to be working in their same jobs after one year?

1 mark

The information in the transition diagram has been used to write the transition matrix *T*.

$$\begin{array}{cc} & \textit{this year} \\ & \begin{matrix} A \quad & O \quad & P \quad & L \end{matrix} \\ T = & \begin{bmatrix} 0.70 & 0 & 0 & 0 \\ 0.10 & 0.80 & 0 & 0 \\ 0 & 0.10 & 0.90 & 0 \\ 0.20 & 0.10 & 0.10 & 1.00 \end{bmatrix} \begin{matrix} A \\ O \\ P \\ L \end{matrix} \quad \textit{next year} \end{array}$$

b. Explain the meaning of the entry in the fourth row and fourth column of transition matrix *T*.

1 mark

If staff at the site continue to change their jobs in this way, the matrix S_n will contain the number of apprentices (A), operators (O), professionals (P) and staff who leave the site (L) at the beginning of the nth year.

c. Use the rule $S_{n+1} = T\,S_n$ to find

i. S_{2012}

ii. the expected number of operators at the site at the beginning of 2013

iii. the beginning of which year the number of operators at the site first drops below 30

iv. the total number of staff at the site in the longer term.

1 + 1 + 1 + 1 = 4 marks

Suppose the manager decides to bring 30 new apprentices, 20 new operators and 10 new professionals to the site at the beginning of each year.

The matrix S_{n+1} will then be given by

$$S_{n+1} = T\,S_n + A \quad \text{where} \quad S_{2011} = \begin{array}{c} \begin{bmatrix} 100 \\ 200 \\ 50 \\ 0 \end{bmatrix} \end{array} \begin{array}{l} A \\ O \\ P \\ L \end{array} \quad \text{and} \quad A = \begin{bmatrix} 30 \\ 20 \\ 10 \\ 0 \end{bmatrix} \begin{array}{l} A \\ O \\ P \\ L \end{array}$$

d. Find the expected number of operators at the site at the beginning of 2013.

2 marks

END OF QUESTION AND ANSWER BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Further Mathematics formulas

Core: Data analysis

standardised score:	$z = \dfrac{x - \bar{x}}{s_x}$
least squares line:	$y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value:	residual value = actual value – predicted value
seasonal index:	seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series:	$a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}[2a + (n-1)d] = \dfrac{n}{2}(a + l)$
geometric series:	$a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1-r^n)}{1-r}, r \neq 1$
infinite geometric series:	$a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1-r}, \lvert r \rvert < 1$

Module 2: Geometry and trigonometry

area of a triangle:	$\dfrac{1}{2}bc\sin A$
Heron's formula:	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a+b+c)$
circumference of a circle:	$2\pi r$
area of a circle:	πr^2
volume of a sphere:	$\dfrac{4}{3}\pi r^3$
surface area of a sphere:	$4\pi r^2$
volume of a cone:	$\dfrac{1}{3}\pi r^2 h$
volume of a cylinder:	$\pi r^2 h$
volume of a prism:	area of base × height
volume of a pyramid:	$\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET