The Mathematical Association of Victoria

Trial Exam 2012

FURTHER MATHEMATICS

Written Examination 2

STUDENT NAME: ....

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of Book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 33 pages, with a detachable sheet of miscellaneous formulas at the back.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the back of this book during reading time.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

This examination consists of core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer all questions with the modules selected.
You need not give numerical answer as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.
Diagrams are not to scale unless specified otherwise.

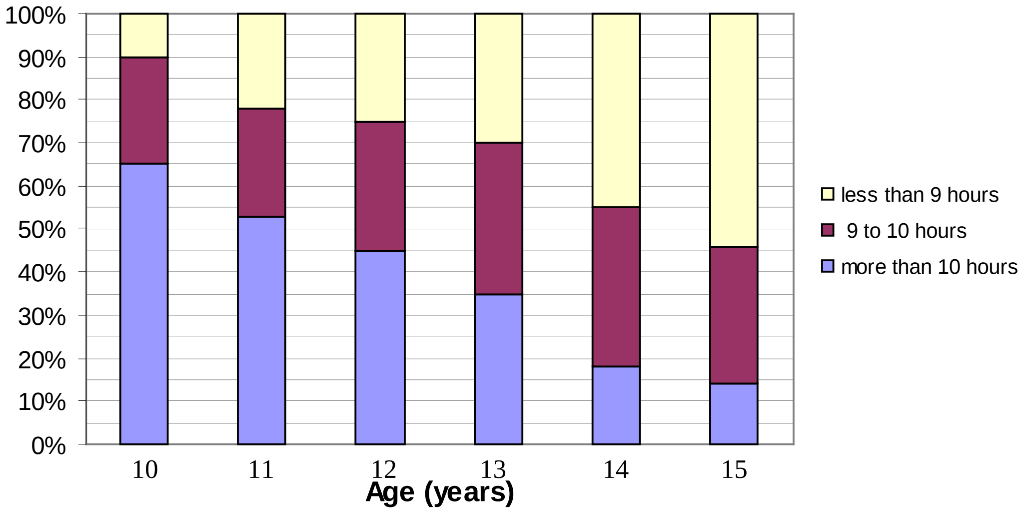
Page

TURN OVER

Core

Question 1

The distribution of sleep duration among children aged 10 to 15 years is shown in the segmented bar chart below.



a. What percentage of 13 year olds slept from 9 to 10 hours?

1 mark

b. Does the graph support the theory that sleep time is related to age? Quote percentages to support your statement.

1 mark

Core – continued

Question 2

A study investigating the relationship between sleep time and the body mass index (BMI) was conducted. The results of sixteen people who participated in the study are shown in the table below.

A person with a BMI greater that 35 is classified as overweight.

Person	Average Sleep Time (hours)	BMI
A	2.5	53
B	2	48
C	3.5	46
D	3	38
E	5	29
F	6	31
G	4	36
H	5	32
I	7	25
J	8.5	26
K	8	24
L	9	21
M	10	22
N	12	21
O	10	20
P	7	28

a. For this sample determine:

i. the mean BMI

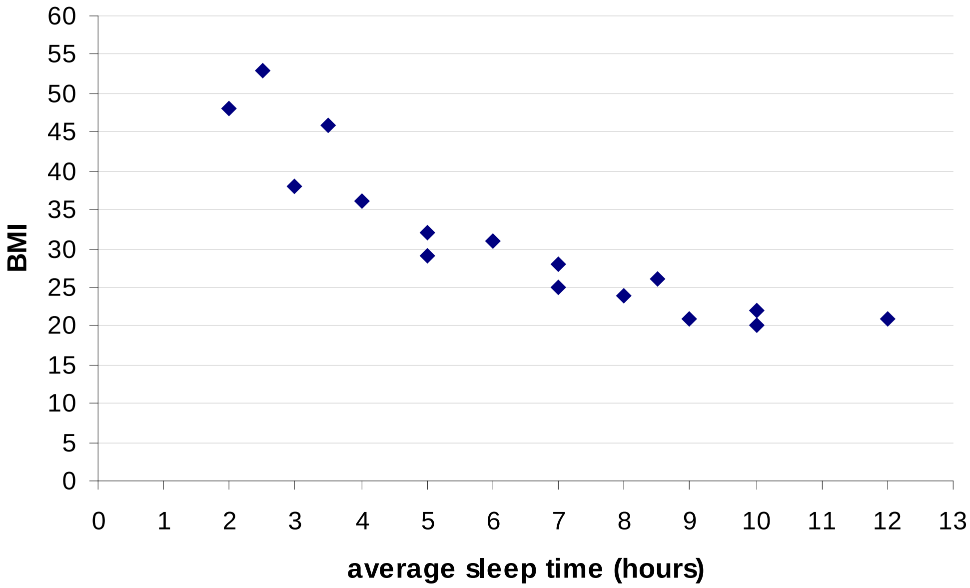
1 mark

ii. the percentage of people who are classified as overweight.

1 mark

Core – Question 2 - continued
TURN OVER

A scatterplot of BMI with the average sleep time is shown below.



b. The correlation coefficient for *BMI* and the *average sleep time* is found to be -0.90

i. Calculate the coefficient of determination for this relationship.

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ii. Interpret the coefficient of determination for this data.

1+1= 2 marks

Core – Question 2 - continued

The equation of the least squares regression line was found to be

$$BMI = 51.00 - 3.08 \times \textit{average sleep time}$$

c. According to this equation, explain how the BMI is affected by increasing the average sleep time by 2 hours.

1 mark

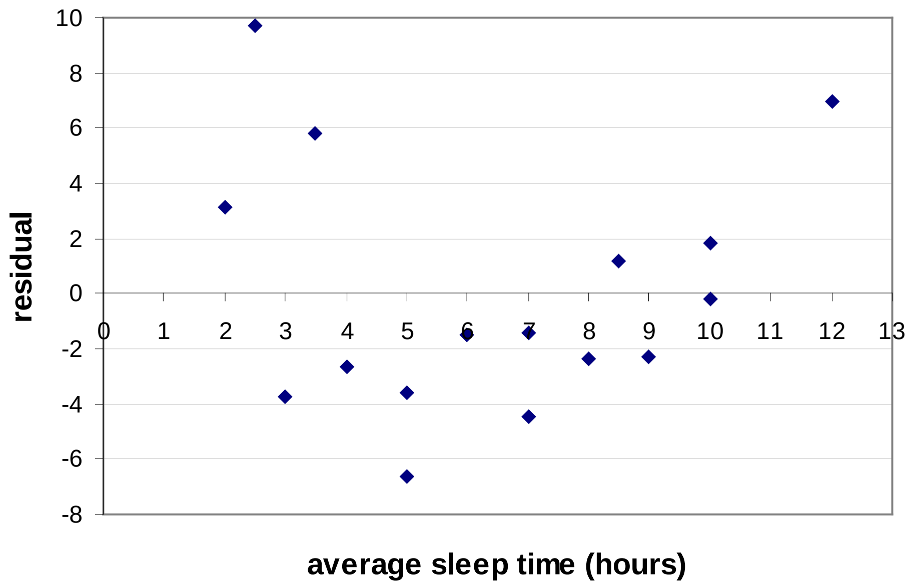
d. Sketch the graph described by the linear equation above on the scatterplot on page 6. State the coordinates of the two points that were used to produce this line.

2 marks

e. Robert has a BMI of 28 and averages 9 hours of sleep. Determine the residual BMI.
Give your answer correct to 2 decimal places.

2 marks

A residual plot of the least squares regression line is shown below.



f. Use the residual plot to comment on the suitability of the least squares regression line. Justify your answer.

1 mark

Question 3

The relationship between BMI and average sleep time is nonlinear.
A **reciprocal** transformation can be applied to the variable BMI and used to linearise the scatterplot.

a. Apply this reciprocal transformation to the original data in Question 2 and determine the least squares regression line that allows the *BMI* to be predicted from the *average sleep time*.

$$\frac{1}{\text{BMI}} = \square + \square \times \text{average sleep time}$$

Write the coefficients for this equation, correct to three decimal places, in the spaces provided.

2 marks

b. Use the transformed equation to determine the average sleep time, correct to one decimal place, for a person with a BMI of 30.

1 mark

Total 15 marks
END OF CORE

Module 1: Number Patterns

Question 1

Anna's battery on her mobile phone lasted for 144 hours the first time she charged it, 142 hours the second time she charged it and 140 hours the third time she charged it.

If this pattern continues:

a. How many hours will the battery last after the 10^{th} charge?

1 mark

b. Anna has charged her battery 10 times. What is the total amount of hours that her mobile phone has been in operation?

1 mark

The total number of hours of operation , S_n , after *n* charges can be expressed in the form $S_n = bn + cn^2$.

c. Find the value of b and c

2 marks

At some stage the battery will not work after being charged.

d. Determine the total number of hours that the battery will work.

1 mark

Module 1 – Number patterns – continued
TURN OVER

Question 2

Brendan's phone also lasts for 144 hours the first time it is charged. However, he has noticed that after each subsequent charge, the length of time the battery will last is reduced by 4%.

a. Find the time, in hours correct to 2 decimal places, for which the battery will last after the second charge.

1 mark

The difference equation that specifies the lifetime of Brendan's battery after the n^{th} charge is given by

$$B_{n+1} = kB_n \text{ , where } B_1 = 144$$

b. State the value of k.

1 mark

c. Determine the total number of hours that the battery is expected to last.

1 mark

d. The battery is deemed to be inefficient when it lasts for 30 minutes or less after charging. After which charge will the battery first be inefficient?

1 mark

Question 3

Caroline's mobile phone contains a super battery. Her phone will also last for 144 hours after the first charge and the time it will last after the n^{th} charge is specified by the difference equation shown.

$$C_{n+1} = 1.05C_n - 8.2\ ,\quad C_1 = 144$$

a. Determine the lengths of time the battery will last after the second and third charge.

1 mark

b. Show that the sequence generated by the difference equation is neither arithmetic nor geometric.

2 marks

c. After how many charges will the battery last for less than 50 hours?

1 mark

A chemical reaction occurs at some point in the life of the battery so that Caroline will always need to recharge her battery every 48 hours.

The difference equation that specifies this change is given by

$$T_{n+1} = 1.05T_n - d\ ,\quad T_1 = 48$$

d. Find the value of d

2 marks

Total 15 marks

END OF MODULE 1

TURN OVER

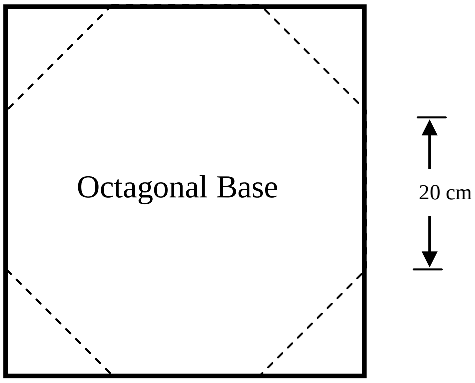
Module 2: Geometry and Trigonometry

Question 1

A manufacturer makes different sizes of fish tanks out of glass.

For the base of the fish tank four triangles are cut from each corner of a square piece of glass to create a regular octagon as shown in the diagram.

Rectangular pieces of glass are then secured to each side of the octagon to create an octagonal prism.



a. The length of each side of the octagon is 20 cm. Calculate the length of the square piece of glass. Give your answer in cm correct to 2 decimal places.

2 marks

b. Show that the interior angle of the octagon is 135^{o}

1 mark

c. **i.** Find the area of the octagonal base correct to one decimal place.

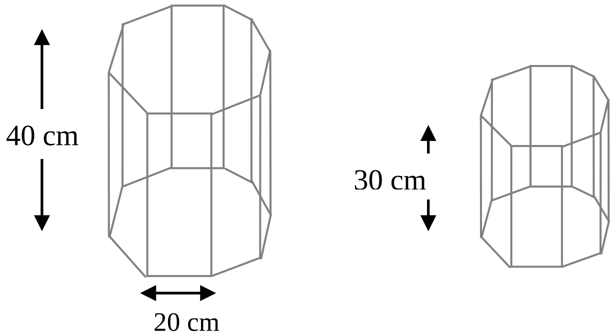
2 marks

ii. Hence find the area of glass used if the fish tank is 40 cm in height and has no lid. Give your answer correct to the nearest square centimetre.

1 mark

Module 2 – Geometry and Trigonometry – continued

d. A smaller but similar fish tank is made with a height of 30 cm.



i. Determine the side length of its octagonal base

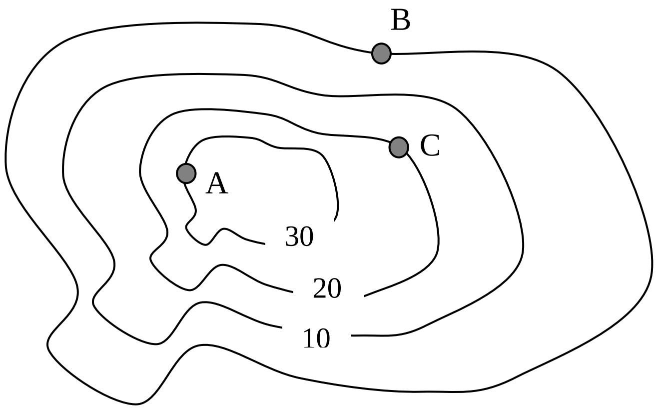
1 mark

ii. Both fish tanks are completely filled with water. What fraction of the water will the smaller tank store compared to the larger tank?

1 mark

Question 2

Peter dives in the tropical waters collecting fish for an aquarium. Below is a contour map showing the location of his boat, B, at sea level, the location of the clown fish, C, and the sea anemone, A.
This contour map has 10 metre intervals that show the distance **below** sea level.



1:1000

a. Find the angle of depression of the Clown Fish from the Boat if the horizontal distance on the map is 1.4 cm. Give your answer correct to the nearest degree.

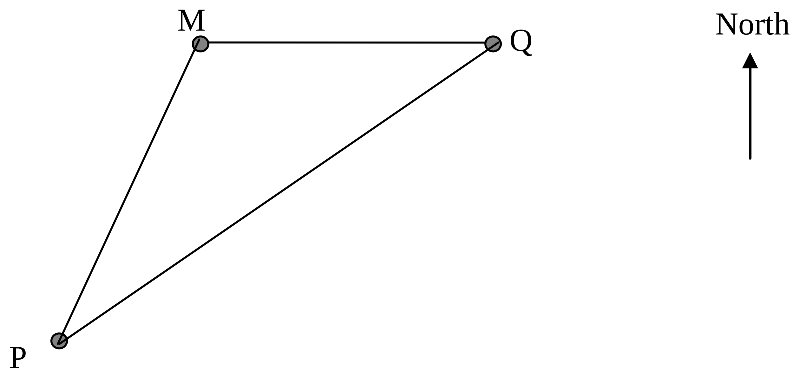
2 marks

b. Determine the direct distance from the boat to the sea anemone given that the horizontal distance is 40 metres.

1 mark

Question 3

Two boats set off from a marina, M, in search of fish. One travels in the direction 226^{o} T and stops at location P which is 640 m from the marina. The other boat travels in an easterly direction and stops at location Q which is 430 metres away from the marina.



a. Show that angle PMQ is 136^{o}

1 mark

b. The boat at P travels to location Q to join the other boat.

i. Show that the direct distance this boat will travel correct to one decimal place is equal to 995.2 m

1 mark

ii. Determine, correct to the nearest degree, the three figure bearing of Q from P.

2 marks

Total: 15 marks

END OF MODULE 2
TURN OVER

Module 3: Graphs and relations

Question 1

Sienna makes children's jewellery that is sold by the company Maccora Ltd.
She is offered the choice of two options for a particular type of bracelet that she has created.

Option 1 : $5 for every bracelet sold

Option 2 : $3 for each of the first 500 bracelets sold and then $6 for each sold over 500.

a. Determine the amount received by Sienna from Maccora Ltd., if she sells 1000 bracelets under the terms of Option 1.

1 mark

b. Determine the amount received by Sienna from Maccora Ltd., if she sells 1000 bracelets under the terms of Option 2.

1 mark

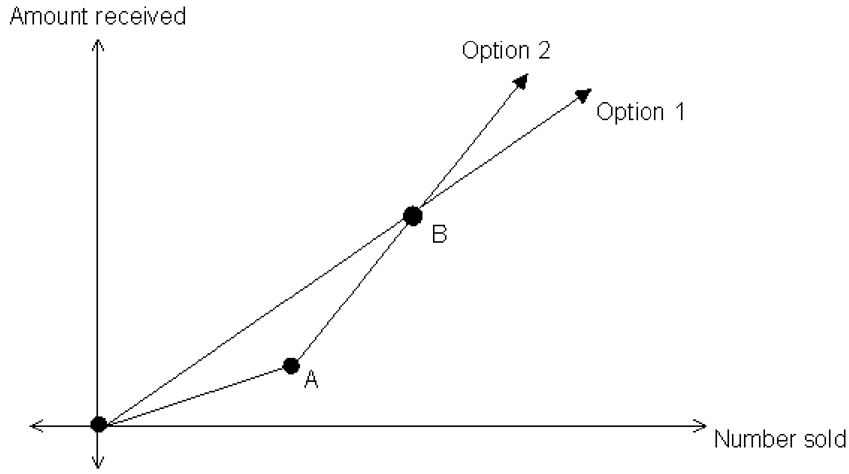
c. The amount received by Sienna when n bracelets are sold under the terms of Option 2 can be determined by a rule of the form

$$Amount\ received = \begin{cases} 3n & for\ 0 \le n \le 500 \\ kn + c & for\ n > 500 \end{cases}$$

Determine the values of k and c

2 marks

d. On the axes below are graphs that represent the amount Sienna receives under the terms of both Option 1 and Option 2. Two points are labelled as A and B. Determine the coordinates of each point.



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2 mark

e. Briefly explain when Sienna would be better off under the terms of both options.

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1 mark

Module 3 – Graphs and relations – continued
TURN OVER

Question 2

Sienna begins to sell bangles as part of her jewellery range. The number sold in each of the first four weeks can be seen in the following table.

Week	Number of bangles sold
1	150
2	1200
3	4050
4	9600

To help with future planning, Sienna attempts to find a model of the form $y = kx^n$ that passes through these four points. She plots the four points and notes the shape of the curve.

a. Write down two values of n which Sienna would need to consider for a curve of this shape.

1 mark

b. If Sienna decides to use a model of the form $y = kx^3$, determine the value of k which is appropriate for these points.

1 mark

Question 3

Sienna also makes and sells both necklaces and rings. In any particular week there are constraints where x represents the number of necklaces sold and y represents the number of rings sold.

Constraint 1 : $x \geq 0$

Constraint 2 : $y \geq 0$

Constraint 3 : $x \leq 300$

Constraint 4 : $y \leq 400$

Constraint 5 : $x + y \leq 500$

a. Briefly explain the meaning of Constraint 5 within the context of the problem.

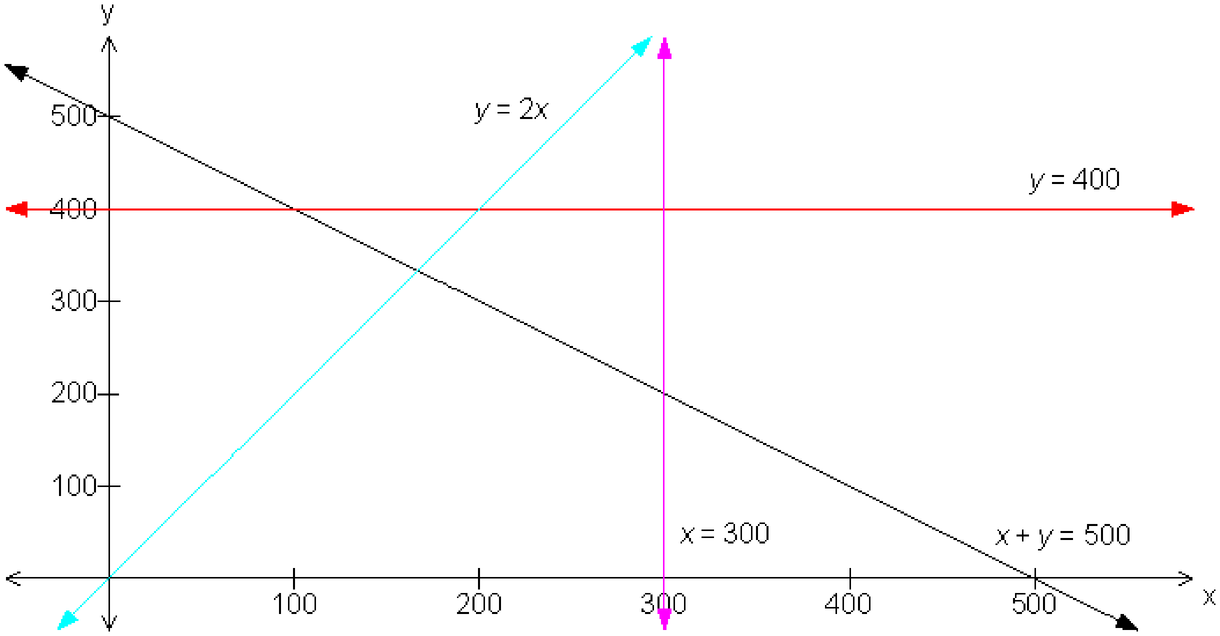
1 mark

An additional constraint is that she sells at least twice as many rings as she does necklaces.

b. Complete Constraint 6 from this information below.

1 mark

c. By considering all constraints clearly indicate the feasible region on the graph below.



1 mark

c. If the profit made on a necklace is twice that made on a ring, determine the number of each sold that will result in the maximum profit.

2 marks

e. If the profit made on a necklace is equal to that made on a ring, determine the maximum and minimum values for the number of **rings** that must be sold to achieve the maximum profit

1 mark

Total : 15 marks

END OF MODULE 3

Module 4 : Business-related mathematics

Question 1

Melissa and Shawn are about to purchase a new car for $25 000.
They will have a DVD player installed in the car after the purchase.
The DVD player will cost $1 200 plus an additional $300 for installation.

a. Determine the total cost to Melissa and Shawn once the DVD player is installed.

1 mark

b. Of the costs involved with the DVD player, what percentage represents the installation?

1 mark

Question 2

Melissa and Shawn will need to borrow the $25 000 to pay for their new car.
Finance is available through the dealer in terms of a five-year loan, compounding fortnightly.
The interest rate is 8.2% per annum on the reducing balance. Fortnightly repayments will be made.

a. Determine the fortnightly repayments required to pay off the loan correct to the nearest cent.

1 mark

b. Determine the total amount of interest that will be paid over the five-year term.

1 mark

Module 4 – Business-related mathematics –Question 2 - continued
TURN OVER

Melissa and Shawn are advised by a family member that they could save on interest by borrowing the \$25 000 on top of their existing home loan which has an interest rate of 6.7% per annum. They currently owe \$50 000 on this loan and will make fortnightly repayments for another 5 years.

c. Determine the total savings to Melissa and Shawn over the next five years if they choose this option instead of the original plan.

2 marks

Question 3

Melissa and Shawn have just had a new baby and need to buy a new pram. The model they have chosen costs \$1320 including the 10% Goods and Services Tax (GST).

a. Determine the amount of GST that is included in the total cost.

1 mark

Melissa and Shawn enter into a time-payment (hire purchase) plan for the \$1320 pram. The terms of the sale are \$250 deposit and monthly repayments of \$190 for six months.

b. What is the total cost of the pram on these terms?

1 mark

c. Determine the annual flat rate of interest charged. Give your answer correct to one decimal place.

2 marks

d. The effective interest rate for this time-payment plan is approximately 22.5%. Briefly describe why this is a higher rate than the flat rate of interest calculated in Q3**c.**

1 mark

Melissa and Shawn believe the original $1320 value of the pram will depreciate to $330 after three years.

e. Determine the annual depreciation rate if calculations are made using flat rate depreciation.

2 marks

f. Determine the annual depreciation rate if calculations are made using reducing balance depreciation. Give your answer correct to the nearest whole number.

2 marks

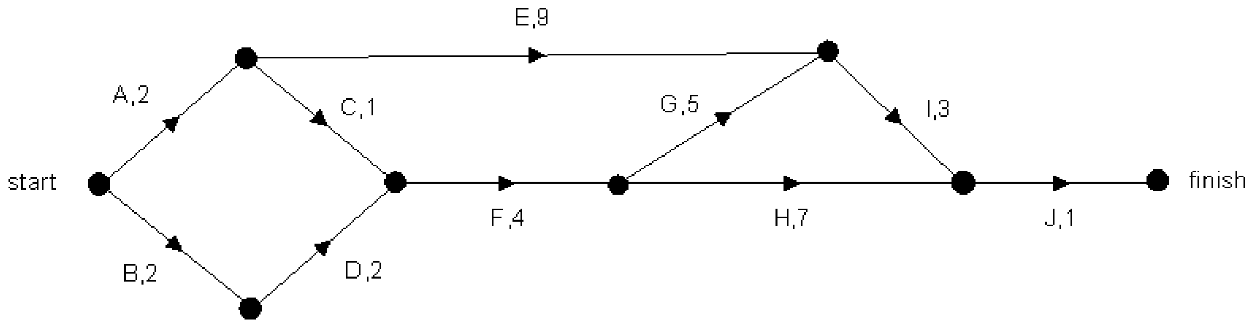
Total: 15 marks

END OF MODULE 4

Module 5 : Networks and decision mathematics

Question 1

A large school has built a new senior campus for its VCE students.
Fitting out of classrooms is a project which is about to begin.
There are ten activities that must be completed for this project.
The directed network below shows the activities, A-J, and their completion times in weeks.



a. How many paths are there from start to finish for this project?

1 mark

b. One path from start to finish is the critical path for this project. Write down the critical path and hence state the minimum time, in weeks, for the completion of the project.

2 marks

c. Activity H begins but actually takes one week longer than originally planned.
Briefly explain what effect that will have on the minimum completion time for the project.

1 mark

Module 5 – Networks and decision mathematics –Question 1 - continued

d. One of the ten activities has a float time of 2 weeks. Determine this activity.

1 mark

Question 2

Four of the builders are named Hunter, Max, Archer and Knox.
The supervisor asks each builder to perform one of four tasks on a particular day.
Experience has shown that the time, in minutes, each builder takes to complete each task can be summarised in table 1.

	Task 1	**Task 2**	**Task 3**	**Task 4**
Hunter	15	16	21	12
Max	9	11	27	9
Archer	15	10	23	13
Knox	14	9	20	12

The Hungarian Algorithm is to be used to find the minimum time overall for the tasks to be completed. After several steps of the algorithm have been completed the information in table 2 below can be used to allocate each builder to one of the tasks

	Task 1	**Task 2**	**Task 3**	**Task 4**
Hunter	3	6	0	0
Max	0	4	9	0
Archer	3	0	2	1
Knox	3	0	0	1

a. Explain what the two zeros in Knox's row indicate in the context of the problem.

1 mark

b. Use the information in table 2 to draw a bipartite graph linking each individual with the tasks they could perform.

Hunter Task 1

Max Task 2

Archer Task 3

Knox Task 4

2 marks

c. Complete the table indicating the allocation of tasks for minimum time.

	Task
Hunter	
Max	
Archer	
Knox	

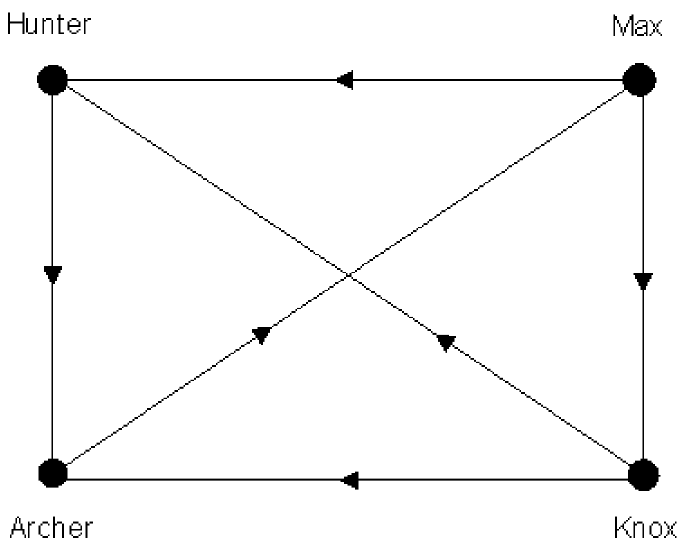
1 mark

d. Determine the minimum overall time, in minutes, for the completion of the four tasks.

1 mark

Question 3

The four builders are all enthusiastic wrestlers.
They engage in a friendly round-robin tournament where each builder will compete against each other.
In each contest there is a winner and a loser.
The directed graph illustrates the results where an arrow from Hunter to Archer shows that Hunter defeated Archer.



Tables of one-step and two-step dominances can be completed from the graph.

a. Complete the missing values in the table of one-step dominances.

	One step dominance total
Hunter	1
Max	2
Archer	
Knox	

1 mark

b. Explain why Knox has a value of 2 for two-step dominance.

1 mark

c. The table of one-step dominances did not allow the builders to be ranked from first to fourth. The totals of one-step and two step-dominances for each person will decide the final order. Complete the table for the final order of dominance.

Final dominance order	Person
1^{st}	
2^{nd}	
3^{rd}	
4^{th}	

1 mark

d. With these four competitors the total of one-step dominances will always be 6 regardless of the results. The total of two-step dominances, however, can vary depending on the original results. Determine the maximum and minimum totals for two-step dominances that can possibly occur.

2 marks

Total : 15 marks

END OF MODULE 5

Module 6 : Matrices

Question 1

Arcade city is a complex, primarily for children, containing various forms of entertainment.
One of the attractions at Arcade city is ten-pin bowling.
The cost to play is $8 for an adult, $6 for a concession and $4 for a child.

a. Form a 1×3 cost matrix, C to represent this information.

$C =$

1 mark

b. A large group of 3 adults, 2 concession cardholders and 30 children arrive for a game of bowling. A matrix N is to be formed that when multiplied by matrix C gives the total cost for all 35 players. Write down the order of matrix N.

1 mark

c. Write down the appropriate matrix product and hence determine the total cost for the group.

1 mark

Module 6 – Matices – continued
TURN OVER

Question 2

At Arcade city, children enjoy hot dogs and slurpee drinks. One family purchased 5 hot dogs and 4 slurpees at a total cost of $33.50 whereas another family purchased 3 hot dogs and 6 slurpees at a total cost of $32.70.
Matrices can be used to determine the individual cost of each hot dog and each slurpee.

a. Complete the following simultaneous equations in matrix form where h represents hot dogs and s represents slurpees.

$$\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}\begin{bmatrix} h \\ s \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

1 mark

b. Explain briefly why there will be a unique solution to these simultaneous equations.

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1 mark

c. Complete the matrices below which when multiplied give the individual cost of each hot dog and each slurpee. Use fractions where appropriate.

$$\begin{bmatrix} h \\ s \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}\begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

1 mark

d. Jamie's father bought 3 hot dogs and 2 slurpees for his family. How much was he charged?

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1 mark

Module 6 – Matices – continued

Question 3

One of the video games at Arcade city is called "Monster's Ink".
Part of the game involves four monsters Red (R), Blue (B), Green (G) and Yellow (Y) that aim to destroy each other with deadly ink. Not all monsters are capable of destroying every other monster.

In the following matrix D a '1' represents "can destroy" and a '0' represents "cannot destroy".

$$\begin{matrix} & \begin{matrix} R & B & G & Y \end{matrix} & \\ D = & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} & \begin{matrix} R \\ B \\ G \\ Y \end{matrix} \end{matrix}$$

a. Which two monsters are the most capable in terms of destroying others?

1 mark

b. Matrix D contains one column of zeros. What does this tell us?

1 mark

c. What feature of Matrix D suggests that none of the four monsters can destroy itself?

1 mark

In addition to Monster's Ink, *M*, another popular video game is Devil's disguise, *D*.
The following transition matrix, T, is used to predict the number playing each game on a visit-to-visit basis.

$$\begin{matrix} & \textit{this visit} \\ & \begin{matrix} M & D \end{matrix} \\ T = & \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix} \begin{matrix} M \\ D \end{matrix} \ \textit{next visit} \end{matrix}$$

d. Briefly explain what the figure of 0.9 in the matrix tells us in the context of the problem.

1 mark

The number playing Monster's Ink and Devil's disguise respectively on the first visit can be represented by matrix S_1 where

$$S_1 = \begin{bmatrix} 420 \\ 270 \end{bmatrix}$$

e. Determine the number expected to play each game on the second visit.

1 mark

f. In the long run, what **proportion** are expected to play Monster's Ink?

1 mark

Module 6 – Matices – continued
TURN OVER

Question 4

A second entertainment complex for children, Leisure heaven, has opened.

Children in the area generally attend one of the two venues each week but may vary their choice from week to week. The transition matrix that is used to predict the proportions attending each venue from one week to the next is given by

$$\begin{bmatrix} x & y \\ 0.3 & z \end{bmatrix}$$

It is known that in the long run 50% of children will attend each venue.
Determine the values of x, y and z in the transition matrix.

$x =$ ☐ $y =$ ☐ $z =$ ☐

2 marks

Total : 15 marks

END OF QUESTION AND ANSWER BOOKLET

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET