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Units 3 and 4 Further Maths: Exam 2

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 2 hours writing time

Structure of book:

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Number of marks
A	4	4			15
B			6	3	45
				Total	61

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied:
- This question and answer booklet of 37 pages, including a sheet of miscellaneous formulas.

Instructions:
- Detach the formula sheet from this book during reading time.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.
- Write all your answers in the spaces provided in this booklet.

Instructions

This examination consists of a core section and six modules. Students should answer all questions in the core section then select three modules and answer all questions within the modules selected.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.
Diagrams are not to scale unless specified otherwise.

Core: Data Analysis

Question 1

The Woopwoop Wombats and Poowoomba Pandas have just held their annual football grudge match. The number of possessions gathered by each player from each team was recorded by the game day statisticians. The data is displayed below.

Player Number	Wombats	Pandas
1	42	25
2	34	24
3	32	23
4	28	22
5	27	20
6	18	19
7	16	16
8	16	16
9	16	16
10	15	15
11	15	15
12	14	14
13	13	14
14	13	13
15	12	13
16	12	11
17	11	11
18	10	10
19	10	7
20	9	7
21	5	7
22	4	7

a. Write the 5 number summary for each team

2 marks

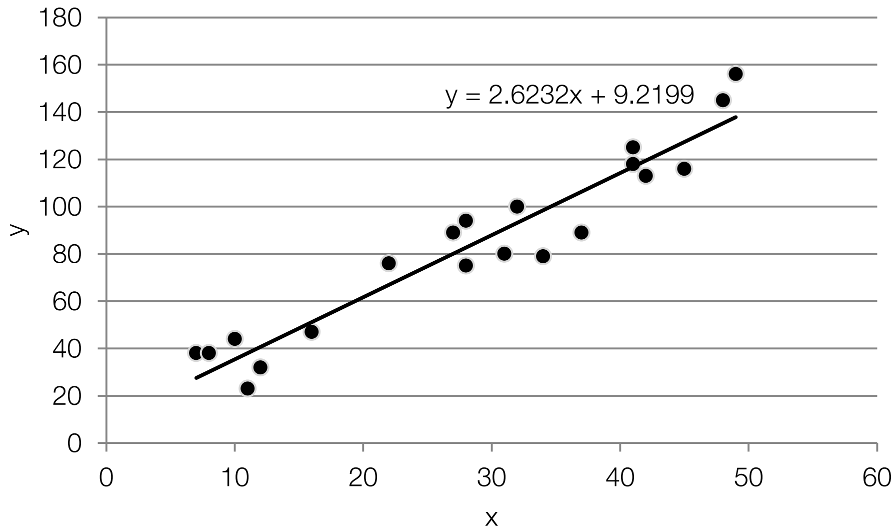
b. If there are any outliers in the data, identify them and give their values.

1 mark

Total: 3 marks

Question 2

From the weekends game a survey was conducted using a small sample of the spectators. The age of the spectators and the amount of cash, to the nearest dollar, that they had in their wallet was recorded. The results of the survey are plotted below, along with the regression line for the data.



a. Correctly identify the independent and dependent variables. Hence rewrite the regression equation using words instead of x and y.

1 mark

b. If Andrew is 15 years old, and his sister Vanessa is 18 months younger than him. From the regression line, how much more cash is Andrew expected to have in his wallet? Answer to the nearest 5 cents.

1 mark

c. If the correlation coefficient for this data is $r = 0.9556$. State the coefficient of determination and comment on what it means for this data.

2 marks

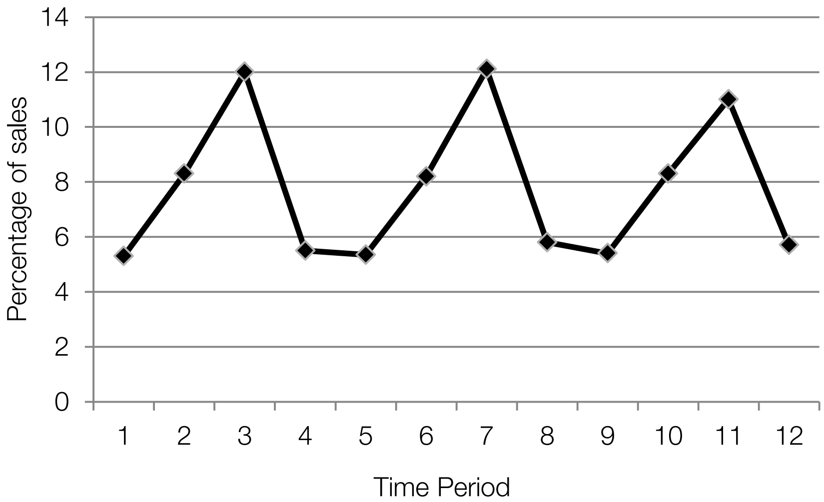
Total: 4 marks

Question 3

The Woopwoop Wombats football club is based at the Woopwoop city sports complex. The complex also hosts cricket and netball matches. To cater for the spectators, and hungry competitors the complex has a canteen which provides various refreshments.

The canteen sells hot sausage rolls among other items on its menu. Below is a table with has been collected over 3 years, showing the percentage of total sales that sausage rolls make up for the canteen. Also included is the corresponding time series graph.

	2010	2011	2012
Summer	5.3	5.35	5.4
Autumn	8.3	8.2	8.3
Winter	12	11.5	11
Spring	5.5	6.4	5.7



a. Describe the shape and any general trends of the time series graph.

____

____

1 mark

b. Find the yearly average over the four seasons for each year. Answer to 3 decimal places.

Year	2010	2011	2012
Average			

1 mark

The following table contains the result of each original term in the time series divided by its yearly average

	2010	2011	2012
Summer	0.68167203	0.712146	0.710526
Autumn	1.06752412	1.091514	1.092105
Winter	1.54340836	1.530782	1.447368
Spring	0.7073955	0.665557	0.75

c. Hence calculate the seasonal index for each season. Give your answer to 3 decimal places.

Season	Summer	Autumn	Winter	Spring
Seasonal Index				

1 mark

d. What is the deseasonalised figure for time period 11?

1 mark

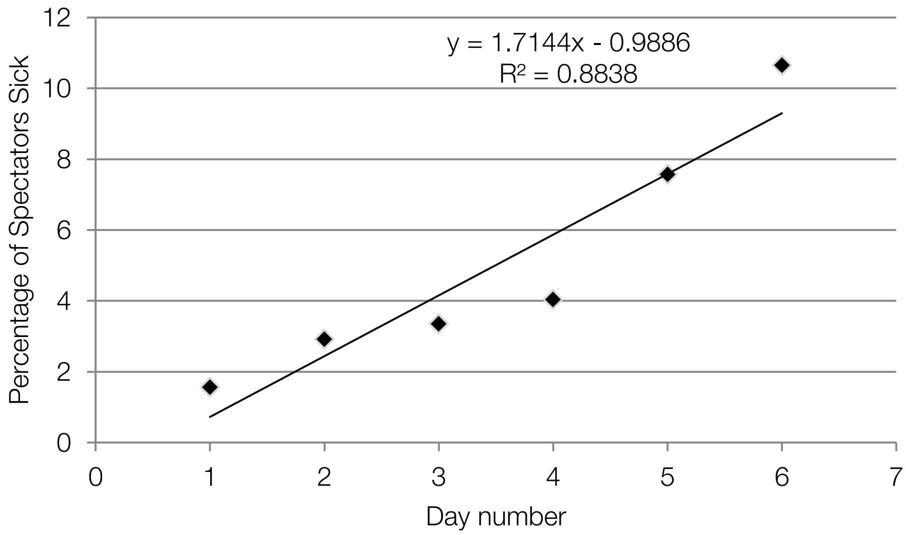
e. The owner of the canteen deseasonalised all the figures. Unfortunately he got some of his time codes mixed up. If he knows that the deseasonalised figure is 7.7722 and that this figure belongs to spring. Which time code, and hence year does this figure belong to?

1 mark

Total: 5 marks

Question 4

Unfortunately, after the big match between the Tigers and Wombats, spectators reported feeling ill with food poisoning. Below is a graph displaying the percentage of spectators who were ill days after the match. Also included is a linear regression line, and the coefficient of determination.



a. Although the coefficient of determination is 88.38%, explain why a linear model would not necessarily be the most correct model to apply to this data.

1 mark

b. If a transformation had to be made on the number of days. Which is the most appropriate transformation?

1 mark

c. The transformed data produces a regression line of the form
$percentage\ of\ spectators\ ill = 1.7016 + 0.2025 \times (transformed\ variable)$.
What is the residual for this regression line given that on the 2nd day 2.44% of spectators reported being sick? Give your answer to 4 decimal places.

1 mark

Total: 3 marks

Modules

Module 1: Number patterns

Question 1

Alexander is 14 years old this year (2012). Alexander dreams of one day becoming a professional basketball player, but he is a little vertically challenged. He believes that in order to achieve his goal of becoming a professional basketball player he needs to have a height of 150 cm.

Below is a sequence of Alexander's height, measured each year, since he was 10.

1.0 , 1.30 m, 1.42m, 146.8m, 148.72m

a. Write out the sequence of Alexander's change in height each year in centimetres.

1 mark

b. From the above sequence, show it has a common ratio of $r = 0.4$.

1 mark

c. If this pattern of growth were to continue forever, assuming Alexander lives forever, what height, in meters, will Alexander eventually grow to?

3 marks

d. Will Alexander ever reach his dream of being 150cm in height? If so, when can he expect to reach such a height?

1 mark

Total: 6 marks

Question 2

Alexander's older brother, Oscar, is thinking about investing his money in a bank savings account. Oscar has $5,000 that he wants to put into a savings account. Naturally, he wants to have the largest amount of money in his bank account. Being a smart investor, Oscar has decided to shop around at different banks to find the best deal.

Below is a table summarising each banks savings account plans. All following questions refer to the information in this table.

Bank	Simple Interest	Compound Interest	Yearly Account keeping fees
A	5% per annum	-	$0
B	-	4.5% per annum	$0
C	-	6% per annum	$90

a. If the simple interest plan for Bank A follows the arithmetic sequence $t_n = 5{,}000 + (n-1)d$. Find the common difference d. Hence, what is the value of the 7th term in this arithmetic sequence?

2 marks

b. If the compound interest plan for Bank B follows a geometric sequence, $t_n = 5{,}000 \times R^{n-1}$, where $R = 1 + \frac{r}{100}$. Write down the value for R using correct decimal notation. Hence find the value of the 7th term in this geometric sequence.

2 marks

c. If the plan for Bank C can be expressed as a first order difference equation $t_{n+1} = R \times t_n + b$. What are the values for R and b?

2 marks

The sequence generated by the first order difference equation for Bank C is

\$5,000	\$5,210	\$5,432.60	\$5,668.56	\$5,918.67

d. What are the values of the 6th and 7th terms? Show full working and answer to 2 decimal places.

2 marks

e. Which bank plan should Oscar choose in order to maximise the amount of money in his savings account at the end of 6 years?

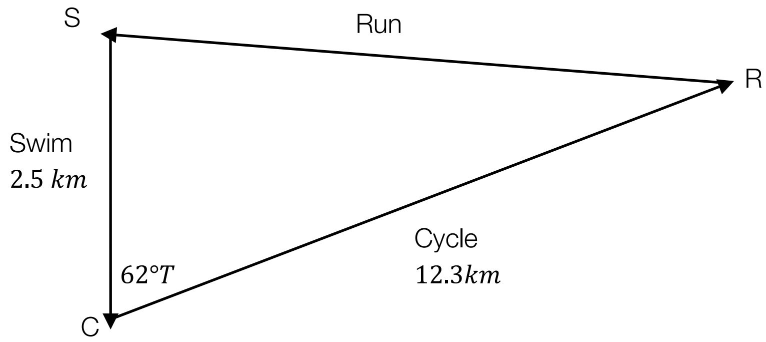
1 mark

Total: 9 marks

Module 2: Geometry and trigonometry

Question 1

Sunny Bay council is to hold a triathlon on the weekend. The triathlon involves 3 legs, swimming, cycling and finally running. The organisers are planning the course such that the finish line is the same as the starting point. The swim is planned to be $2.5km$ due south of the start, to the opposite side of the bay. After the swim, competitors are to cycle along a road, on a bearing of $62°T$ for $12.3km$. Competitors will then run all the way back to the finish (the starting point).



a. What is the distance that competitors will run during the triathlon? Give your answer to 2 decimal places.

1 mark

b. For spectators watching the race finish, what direction (given as a true bearing), should they face to see incoming competitors running to the finish line? Give your answer to 2 decimal places.

2 marks

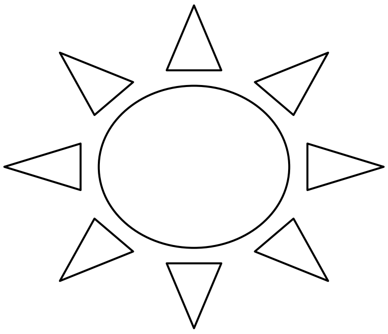
c. A television crew will be filming the triathlon. They have decided to set up a camera 22m due south of point C, to film the transition between the swim and the cycle legs. To get a good view, the television crew have also decided to position their camera on top of a lighthouse, which is 14m high. Calculate the angle that the camera will point down to have point C in view.

1 mark

Total: 4 marks

Question 2

Tiffany owns a jewellery shop, and she enjoys making her own jewellery. She is currently working on some new designs for gold earrings. Her sketch for a new pair of earrings is below:



The circle will be made from a thin sheet of gold, and each of the spikes will be made from a thin sheet of silver. Tiffany wants to make the earrings such that the amount of gold used is equal to the amount of silver.

Tiffany thinks that the spikes will look best if they are isosceles triangles with the two equal sides 15mm in length and a height of 12mm.

a. Calculate the base length of the spike.

1 mark

b. Calculate the total area of silver leaf that Tiffany will use for one earring. Hence, calculate the radius of the circle.

2 marks

c. Tiffany is contemplating only using 3 spikes instead of 8. She still wants for the amount of silver used to be equal to the amount of gold.
If she wants to maintain the base length what should be the new height of the spike?
if she wants to maintain the height what should be the new base length of the spike?

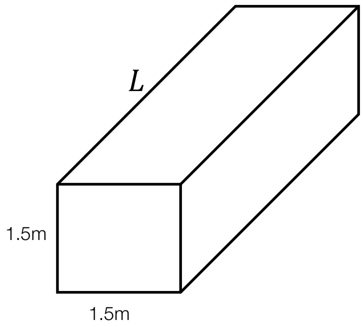
2 marks

Total: 5 marks

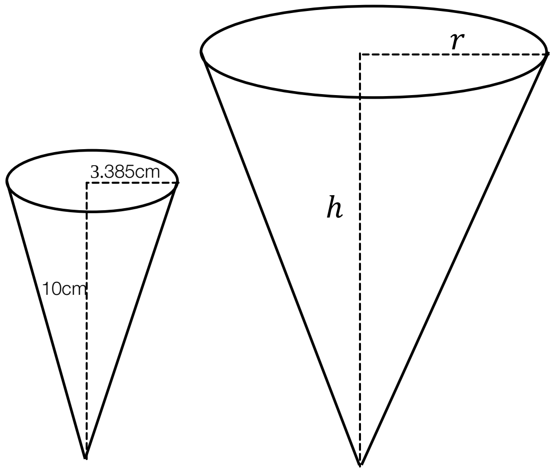
Question 3

A farmer owns a water trough for his horses. The trough is in shape of prism (see diagram below). The farmer wishes to fill the trough, unfortunately his hose is broken. Instead, the farmer has two cones which he can use to fill up the trough. The farmer only knows the dimensions of the smaller cone, but not the larger one. He does however know that these cones are similar in shape.

Note that diagrams are not to scale.



Water trough



Small cone Large cone

a. Given that the volume of the larger cone is 125 times greater than the smaller cone, calculate r and h respectively for the larger cone.

2 marks

b. Calculate the volumes of the two cones. Express your answer to the nearest cm^3.

2 marks

c. If the farmer must fill up the large cone 1,000 times in order to fill up the trough completely, calculate the length of the trough. Give your answer to the nearest centimetre.

2 marks

Total: 6 marks

Module 3: Graphs and relations

Question 1

During the holiday Bruce decides to go on a long drive up the highway, to no destination in particular. He gets in his car at his home, point O, and drives due north at a constant speed of 65km/h.

On the same day, Bruce's friend Sally also decides to go on a long drive on the same stretch of highway. Sally's house, point S, is 385kms directly south of Bruce's home. Sally drives north at a constant speed of 100km/hr.

Both Bruce and Sally leave on their drives at exactly the same time.

a. Write the linear equation describing the distance that Bruce has travelled north, with respect to his home point O.

1 mark

b. Write the linear equation describing the distance Sally travels north, with respect to Bruce's home.

2 marks

c. How long (in minutes) will it take for Sally to pass Bruce's home?

1 mark

d. How many hours will it take before Sally and Bruce meet on the highway?

1 mark

e. How far **in total**, will Sally have travelled when they meet on the highway?

1 mark

Total: 6 marks

Question 2

Darren and William are workers at a beverage factory. The factory makes beverages, Xtreme Cola, and dYet Cola. Darren and William are both able to operate the beverage making machines for each type of Cola.

If Darren operates the Xtreme Cola machine it takes him 20 minutes to produce 1000L of Xtreme Cola. While it only takes William only half the amount of time to produce the same amount of Xtreme Cola.

If Darren operates the dYet Cola machine, it takes him 10 minutes to produce 1000L of dYet Cola. However William takes twice the amount of time to produce the same amount of dYet Cola.

In a given day, Darren does not want to work for more than 7 and half hours, while William must work less than 5 hours. The factory must always be producing cola each day.

a. State the decision variables

____

____

1 mark

b. Hence, what are the constraints imposed by the maximum amount of time Darren and William will work, respectively. Answer in minutes

____

____

____

____

2 marks

c. Graph the set of inequations from part b in the space provided. Shade the feasible region, and include the coordinates for the vertices of the feasible region.

4 marks

d. Given that the factory can make a profit of $150 for every 1000L of Xtreme Cola made, and $250 profit for every 1000l of made. State the objective function and hence determine the maximum amount of profit the factory can make in a day.

2 marks

Total: 9 marks

Module 4: Business-related mathematics

Question 1

The cash price of a plasma televison is $2500. Frankie buys the TV under a hire-purchase agreement. She does not pay a deposit and will pay $50 per month for five years.

a. Calculate the total amount, in dollars, the customer will pay.

1 mark

b. Find the total interest the customer will pay over five years.

1 mark

c. Determine the annual flat interest rate that is applied to this hire-purchase agreement. Give your answer as a percentage.

1 mark

Next year the cash price of the plasma will rise by 2.5%. The following year it will rise by a further 2.0%.

d. Calculate the cash price of the plasma after these two price rises.

1 mark

Total: 4 marks

Question 2

Sam decided to invest some of his money. He deposited $5000 in an account paying 7.5% per annum, compounding monthly.

a. Write down an expression involving the compound interest formula that can be used to find the value of Sam's $5000 investment at the end of two years. Find this value correct to the nearest cent.

2 marks

b. How much interest will the $5000 investment earn over a four-year period? Write your answer correct to the nearest cent.

1 mark

Total: 3 marks

Question 3

Gracie paid $1445 for a photocopier. This price includes 10% GST (goods and services tax).

a. Determine the price of the photocopier before GST was added. Write your answer correct to the nearest cent.

1 mark

Gracie will depreciate her $1445 photocopier for taxation purposes. She considers two methods of depreciation.

b. Flat rate depreciation: Under flat rate depreciation the fax machine will be valued at $700 after five years. Calculate the annual depreciation in dollars.

1 mark

c. Unit cost depreciation: Suppose Gracie photocopies 350 documents a year. The $1445 photocopier is depreciated by 36 cents for each copy it makes. Determine the value of the photocopier after five years.

1 mark

Total: 3 marks

Question 4

Tim's Honda D60 motorcycle is valued $10 000.

a. Calculate the value of his motorcycle after five years if it depreciates by 12% per annum using the reducing balance method. Write your answer correct to the nearest dollar.

1 mark

Tim believes his motorcycle should be valued at $4000 after five years.

b. Determine the annual reducing balance depreciation rate that will produce this value. Write your answer as a percentage correct to one decimal place.

2 marks

Total: 3 marks

Question 5

Shannon is looking to borrow \$500 000 to build her dream home. Bank A offers Shannon a fixed annual interest rate of 6.01% compounding monthly with monthly instalments of \$3585.04 over 20 years. Bank B offers Shannon a fixed rate of 5% compounding monthly for 10 years with monthly payments of \$3585.04 but at the 10 year mark will switch her to an annual rate of 7% compounding weekly.

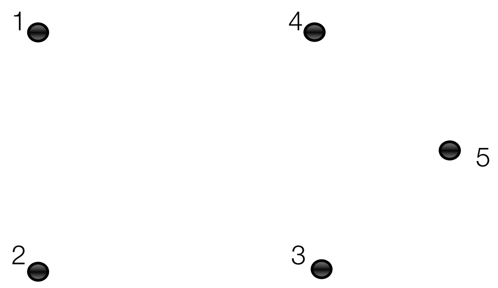
Which bank should Shannon lend with? (or, in other words, what loan will incur the least interest and time?)

2 marks

Module 5: Networks and decision mathematics

Question 1

A new retirement village is being developed. There are five apartments under construction in one location. These apartments are numbered as points 1 to 5 below.



The builders require the five houses to be connected by electrical cables to enable the workers to have a supply of power on each site.

a. What is the minimum number of edges needed to connect the five apartments?

____

____

1 mark

b. On the diagram above, draw a connected graph with minimum edges.

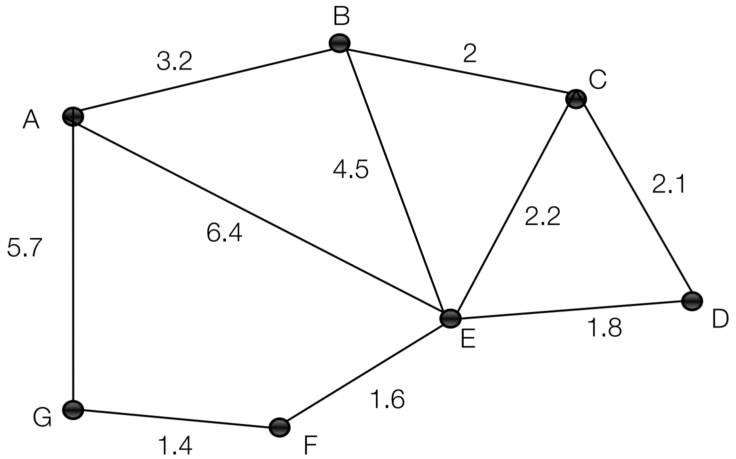
1 mark

Total: 2 marks

Question 2

A, B, C, D, E, F and G represent seven towns.

The network shows the road connections between these towns and the distances between each (in km).



a. What is the shortest distance between town A and D?

1 mark

b. Write a Hamilton path for the journey from town A to D.

1 mark

A road is to be added to the network in order to make it possible to travel along each road once without traversing the same road twice.

c. What is this the mathematical name given to this occurrence?

1 mark

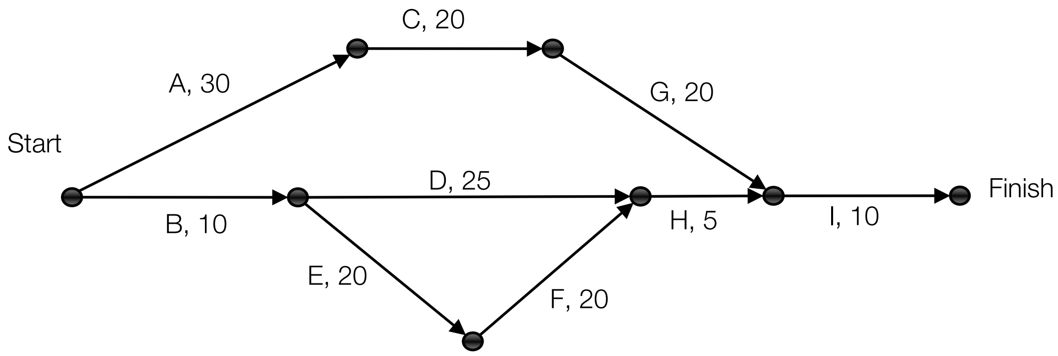
d. Between which towns should the road be added?

1 mark

Total: 4 marks

Question 3

Blue purchased a flat pack table and chairs set from a furniture store. Nine activities have been identified for this building project. The directed network below shows the activities and their completion times in minutes.



a. Determine the minimum time, in minutes, to complete this project.

1 mark

b. Determine the slack time, in minutes, for activity C.

2 marks

c. Blue is able to complete activity C in 10 minutes, what is the minimum completion time now?

1 mark

Blue has accidentally omitted an activity from his network. The activity takes 45 minutes, and it must be completed before activity I and has activity E as predecessor.

d. Draw in this activity on the above network. Label the activity as J.

1 mark

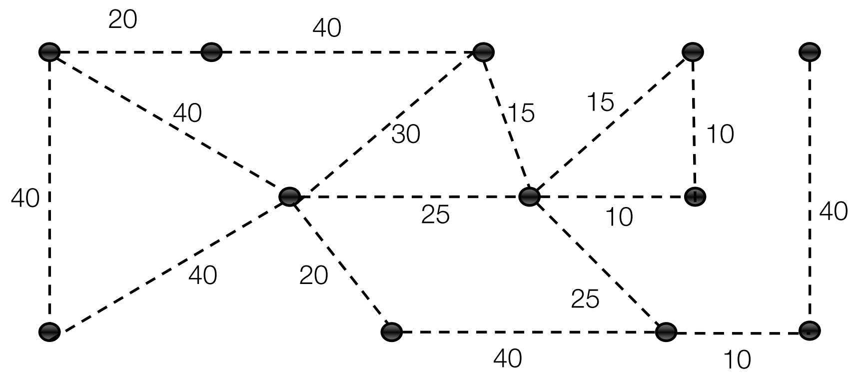
e. Including activity J, what is the minimum completion time for the project now?

1 mark

Total: 6 marks

Question 4

At Saint Elizabeth's Girls Preparatory School, twelve classrooms require access to water. These classrooms are represented by the vertices on the network diagram shown below. The dashed lines on the network diagram represent possible water pipe connections between adjacent classrooms. The numbers on the dashed lines show the minimum length of pipe required to connect these classrooms in metres.



All classrooms are to be connected using the smallest total length of water pipe possible.

a. On the diagram, show where these water pipes will be placed.

1 mark

b. Calculate the total length, in metres, of water pipe that is required

1 mark

c. If water piping costs $150 per metre, what is the total cost to the school to buy the piping?

1 mark

Total: 3 marks

Module 6: Matrices

Question 1

The table below displays the numbers sold and price of 4 clothing items; dress, top, jeans and shoes, in month in a boutique in inner city Melbourne.

Item of clothing	Number sold	Price ($)
Dress	60	150
Top	45	70
Jeans	18	200
Shoes	25	80

a. Write down a 2 × 2 matrix that displays the number sold and price of dresses and tops.

1 mark

A and B are two matrices defined as follows.

$$A = \begin{bmatrix} 2 & 2 & 3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 150 \\ 70 \\ 200 \\ 80 \end{bmatrix}$$

b. Evaluate the matrix product AB.

1 mark

c. Determine the order of matrix product BA.

1 mark

Matrix A displays the number of purchases by a customer of the four clothing items: dress, top, jeans and shoes. Matrix B displays the price of each clothing item.

d. Explain the information that the matrix product AB provides.

1 mark

e. Using the information provided, construct a matrices expression to find the total amount earned by the boutique in a month and then solve.

2 marks

Total: 6 marks

Question 2

The Dragons (D) and the Slayers (S) are two baseball teams that play in different leagues in the same city.

The matrix A_1 is the attendance matrix for the first game. This matrix shows the number of people who attended the first Dragons game and the number of people who attended the first Slayers game.

$$A_1 = \begin{bmatrix} 2500 \\ 1500 \end{bmatrix}$$

The number of people expected to attend the second game for each team can be determined using the matrix equation

$$A_2 = GA_1$$

Where G is the matrix

$$G = \begin{array}{c} \textit{this game} \\ \begin{array}{cc} D & S \end{array} \\ \begin{bmatrix} 1.2 & -0.3 \\ 0.2 & 0.7 \end{bmatrix} \end{array} \begin{array}{c} D \\ S \end{array} \quad \textit{next game}$$

a. Determine A_2, the attendance matrix for the second game.

1 mark

b. Every person who attends either the second Dragon game or the second Slayer game will be given a free cap. How many caps, in total, are expected to be given away?

1 mark

Assume that the attendance matrices for successive games can be determined as follows:

$$A_3 = GA_2$$

and so on such that $A_n = GA_{n-1}$.

c. Determine the attendance matrix (with the elements written correct to the nearest whole number) for game 10.

1 mark

d. Describe the way in which the number of people attending the Dinosaurs' games is expected to change over the next 80 or so games.

1 mark

The attendance at the first Dragon game was 2500 people and the attendance at the first Slayer game was 1500 people. Suppose, instead, that 2000 people attend the first Dragon game, and 1800 people attend the first Slayer game.

e. Describe the way in which the number of people attending the Dragon's games is expected to change over the next 80 or so games.

1 mark

Total: 5 marks

Question 3

A series of extra music lessons is being offered by Mozart's Music School. Each week participants could choose extra guitar lessons or extra drums lessons.

A matrix equation used to determine the number of students expected to attend these extra lessons is given by:

$$L_{n+1} = \begin{bmatrix} 0.80 & 0.35 \\ 0.20 & 0.65 \end{bmatrix} \times L_n - \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

Where L_n is the column matrix that lists the number of students attending in week n.

The attendance matrix for the first week of extra lessons is given by

$$L_1 = \begin{bmatrix} 105 \\ 107 \end{bmatrix} \begin{matrix} \textit{Guitar} \\ \textit{Drums} \end{matrix}$$

a. Calculate the number of students who are expected to attend the extra guitar lessons in week 3.

1 mark

b. Of the students who attended extra lessons in week 3, how many are not expected to return for any extra lessons in week 4?

2 marks

c. Explain what happens to the numbers of students in expected to attend extra lessons over time.

1 mark

Total: 4 marks

Formula Sheet

Core: Data analysis

Standardised score: $z = \frac{x - \bar{x}}{s_x}$

Least squares line: $y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

Residual value: $\text{residual value} = \text{actual value} - \text{predicted value}$

Seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

Arithmetic series: $a + (a + d) + \cdots + (a + (n-1)d) = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$

Geometric series: $a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

Infinite geometric series: $a + ar + ar^2 + ar^3 + \cdots = \frac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

Area of a triangle: $\frac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a + b + c)$

Circumference of a circle: $2\pi r$

Area of a circle: πr^2

Volume of a sphere: $\frac{4}{3}\pi r^3$

Surface area of a sphere: $4\pi r^2$

Volume of a cone: $\frac{1}{3}\pi r^2 h$

Volume of a cylinder: $\pi r^2 h$

Volume of a prism: $\text{area of base} \times \text{height}$

Volume of a pyramid: $\frac{1}{3}\text{area of base} \times \text{height}$

Pythagoras' theorem: $c^2 = a^2 + b^2$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

Gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation: $y = mx + c$

Module 4: Business-related mathematics

Simple interest: $I = \frac{PrT}{100}$

Compound interest: $A = PR^n$, where $R = 1 + \frac{r}{100}$

Hire purchase: $\text{effective rate of interest} \approx \frac{2n}{n+1} \times \text{flat rate}$

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

Determinant of a 2 x 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Inverse of a 2 x 2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$