2011

VCE
Further Mathematics
Trial Examination 2

Suggested Solutions

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Core

Question 1

a.
Using TI-84 calculator press stat edit and enter the data for Melbourne's maximum temperature in L_1

Press stat calc 1-Var stats enter L_1 enter. This gives $\bar{x} = 23.9$

$s = 4.1$

Using a ClassPad, select statistics, enter data into list 1, tap calc, then tap one-variable

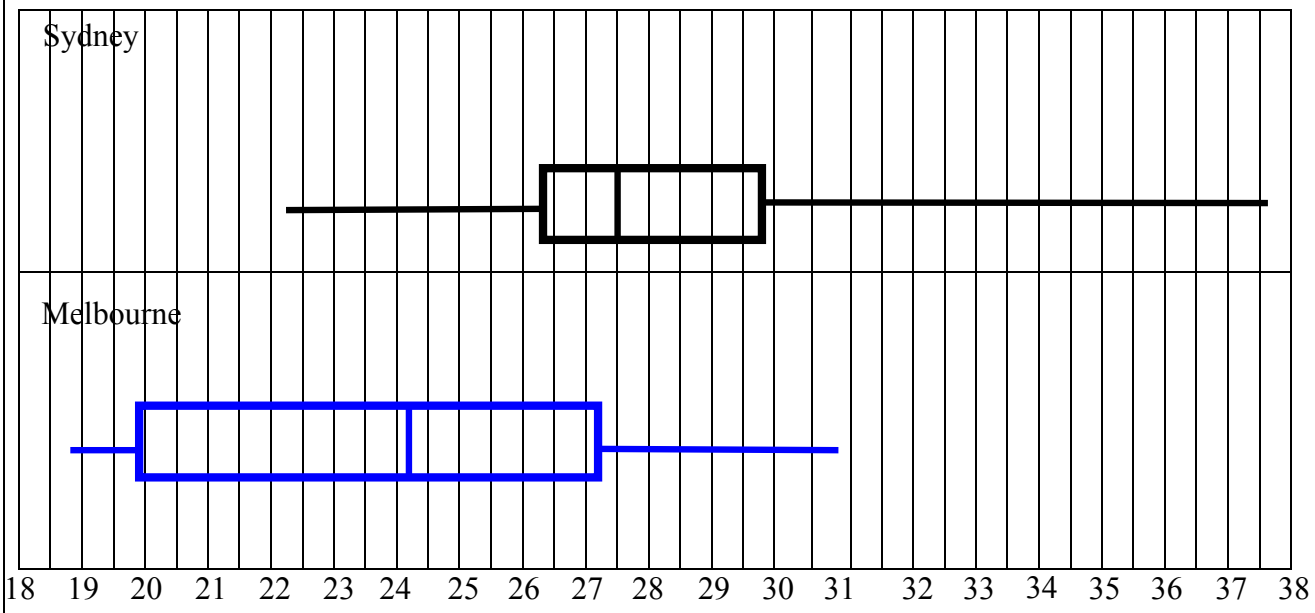
(1 mark)

b.
There are 7 temperatures above 23.9
7 out of 14 = 50%

(1 mark)

c. The median, lower and upper quartile, maximum and minimum values can be obtained from the same screen as **1 a**.

Minimum = 18.6, Maximum = 30.7, Lower Quartile = 19.8, Upper Quartile = 27.2,
Median = 24.2



1 mark for correct upper and lower quartile
1 mark for correct median and whiskers

(2 marks)

d.
$IQR = Q_3 - Q_1 = 27.2 - 19.8 = 7.4$

(1 mark)

Core

Question 1 (continued)

e.
On the whole, Sydney's temperatures are higher than Melbourne's temperatures. Sydney has the higher median, the higher lower quartile, the higher upper quartile and the higher maximum temperature. Melbourne has a higher interquartile range than Sydney, which shows that the middle 50% of maximum temperatures for Melbourne are more varied than the middle 50% maximum temperatures for Sydney.

(1 mark)

Question 2

a.

0	6
1	2 8
2	6
3	4 6
4	6 8
5	0
6	8
7	1 2
8	
9	2
1 0	8

(1 mark)

b.

14 terms
Median lies half way between the 7^{th} and 8^{th} terms
i.e. $(46 + 48) \div 2 = 47$ mm

(1 mark)

c.

$IQR = 7.1 - 2.6 = 4.5$

$1.5 \times 4.5 = 6.75$

$7.1 + 6.75 = 13.85$

$2.6 - 6.75 < 0$

Hence, there are no outliers.

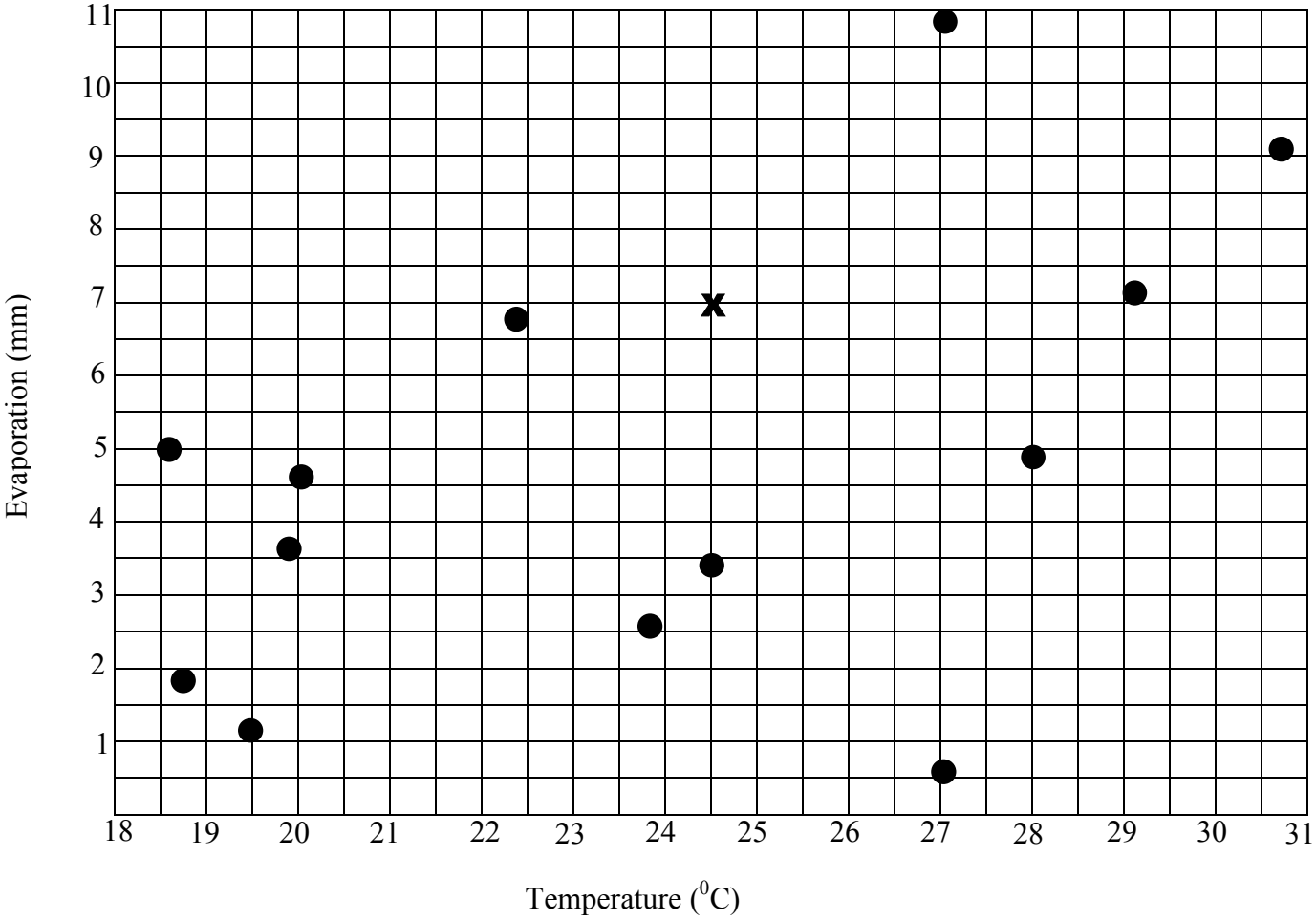
(1 mark)

Core

Question 3

a.

Melbourne



(1 mark)

b.

On calculator press stat edit and enter temperature in L_1 and evaporation in L_2. Go to stat calc LinReg ($ax + b$) $L_1 L_2$. This gives

$Evaporation = 0.4 \times temperature - 4.0$

On the Casio ClassPad, enter temperatures in list 1 and evaporation in list 2.
From calc, select Linear Reg.

(1 mark)

c.

$r = 0.51$

Hence, there is a moderate positive relationship between the two variables.

(1 mark)

Core

Question 4

a.
Taking the first five points to the left, the median point is (25.5,4.8)
Taking the last five points to the right, the median point is (30,8.5)

(1 mark)

b.

$$\text{gradient} = \frac{8.5 - 4.8}{30 - 25.5} = 0.8$$

(1 mark)

c.
The gradient of the three median line tells us that for every 1^0C increase in temperature the evaporation increases by 0.8 mm. Hence, for an increase of 0.5^0C in temperature, the evaporation will increase by 0.4 mm.

(1 mark)

Module 1 Number patterns and applications

Question 1

a. i.

10.00 am is 20 minutes after 9.40 am.
Hence, we want
t_4 of the arithmetic sequence
The sequence is formed by adding 3 to the previous term.
Hence, $t_4 = 8 + 3 = 11$

(1 mark)

a. ii.

This is an arithmetic sequence with $a = 2$ and $d = 3$

$t_n = a + (n-1)d$

$t_7 = 2 + (7-1)3$

$t_7 = 2 + 18$

$t_7 = 20$

(1 mark)

a. iii.

10.00 am will be the fourth term and
11.00 am will be the seventh term

(1 mark)

a. iv.

$t_n = a + (n-1)d$

$41 = 2 + (n-1)3$

$39 = (n-1)3$

$13 = n - 1$

$n = 14$ (1 mark)

$t_7 = 11.00 \text{ am}$

$t_{10} = 12.00 \text{ pm}$

$t_{13} = 1.00 \text{ pm}$

$t_{14} = 1.20 \text{ pm}$ (1 mark)

a. v.

$S_n = \frac{n}{2}[2a + (n-1)d]$

1.20 pm is the 14th term, so 2.00 pm is the 16th term.

$S_{16} = \frac{16}{2}[4 + (16-1)3]$

$S_{16} = 8[4 + 15 \times 3] = 392$

(1 mark)

Module 1 Number patterns and applications

Question 2

a. i.

Each term in the geometric sequence is found by multiplying the previous term by 2.
10.20 am is the next time slot after 10.00 am. Hence, number of emails responded to
$= 4 \times 2 = 8$

(1 mark)

a. ii.

A table can be set up as follows.

Time	Total Emails in	Total Emails out	Emails left unanswered
9.00	2		2
9.20	2 +5 = 7	1	6
9.40	2 + 5 + 8 = 15	1 + 2 = 3	**12**

(1 mark)

a. **iii**

Total number emails answered 10.40 is t_5

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_5 = \frac{1(2^5 - 1)}{2 - 1} = 31$$

(1 mark)

a. **iv.**

Number emails received =

$$S_6 = \frac{6}{2}\left[2 \times 2 + (6-1)3\right]$$

$$S_6 = 3\left[4 + 5 \times 3\right] = 57$$

From a.iii, Number emails answered = 31
Number unanswered = 57 – 31 = 26

(1 mark)

a. v.

Make an S_n table of values on the calculator for incoming and outgoing emails and compare.

Use $U(n) = \left(\frac{n}{2}\right)(4 + (n-1) \times 3)$ and $V(n) = 2^n - 1$

The first time that the incoming is less than the outgoing is at 11.20 am. At 11.20 the total number of incoming emails is 100 and the total outgoing emails would be 127 if there were 127 emails to answer.

(1 mark)

Module 1 Number patterns and applications

Question 3

a. i.

$E_n = 2E_{n-1} - a \qquad E_1 = 16$

For a constant value

$E_n = E_{n-1} = 16$

$16 = 2 \times 16 - a$

$16 = 32 - a$

$a = 16$

(1 mark)

b. i.

$E_2 = 2 \times 16 - 4 = 28$

(1 mark)

b. ii

Midday is E_5

12.20 pm is E_6

$E_6 = 2 \times E_5 - 4 = 388$

$2 \times E_5 = 392$

$E_5 = 196$

(1 mark)

c.

This is the sum to infinity of a geometric sequence.

$a = p$

$r = \frac{1}{10}$

$S_\infty = \frac{p}{1 - \frac{1}{10}} = \frac{p}{\frac{9}{10}} = \frac{10p}{9}$

(1 mark)

Module 2 Geometry and trigonometry

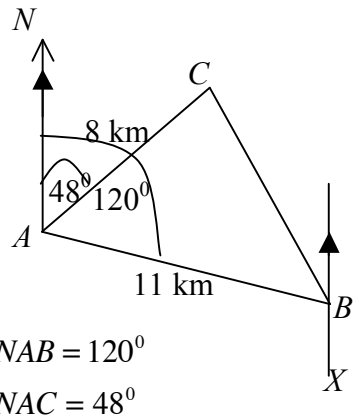
Question 1

a.

$\angle NAC = 48^0$ because Chris sails on a bearing which is 48^0 in a clockwise direction from north

(1 mark)

b.



$\angle NAB = 120^0$

$\angle NAC = 48^0$

$\angle CAB = \angle NAB - \angle NAC = 120^0 - 48^0 = 72^0$

(1 mark)

c.

Using the cosine rule

$(BC)^2 = 11^2 + 8^2 - 2 \times 11 \times 8 \cos 72^0$

$BC = \sqrt{11^2 + 8^2 - 2 \times 11 \times 8 \cos 72^0} = 11.43 \text{ km}$

(1 mark)

d.

AN is parallel to XB, so $\angle NAB = \angle ABX = 120^0$

(1 mark)

e.

$$\frac{8}{\sin \angle ABC} = \frac{11.43}{\sin 72^0}$$

$$\sin \angle ABC = \frac{8 \sin 72^0}{11.43}$$

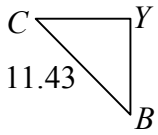
$$\angle ABC = \sin^{-1}\left(\frac{8 \sin 72^0}{11.43}\right) = 41.7^0$$

Bearing of C from B =

$180 + 120 + 41.7 = 342^0$ to the nearest degree

(1 mark)

f.



$\angle CBY = 360^0 - 341.73^0 = 18.27^0$

$$\cos 18.27^0 = \frac{BY}{11.43}$$

$BY = 11.43 \cos 18.27^0 = 11$ km to the nearest km.

(1 mark)

Module 2 Geometry and trigonometry

Question 2

a.

$AD = \sqrt{8^2 + 12^2} = 14.4 \text{ m}$

(1 mark)

b.

$$\frac{17.9}{\sin \angle ADC} = \frac{14.4}{\sin 51^0}$$

$$\sin \angle ADC = \frac{17.9 \times \sin 51^0}{14.4}$$

$$\angle ADC = \sin^{-1}\left(\frac{17.9 \times \sin 51^0}{14.4}\right) = 75^0$$

$$\angle DAC = 180 - (75 + 51) = 54^0$$

(1 mark)

c.

Area of tringle, $DAC = \frac{1}{2} \times 14.4 \times 17.9 \times \sin 54^0$

Area of tringle, $DAC = 104.3 \text{ m}^2 = 104 \text{ m}^2$ to the nearest whole number.

(1 mark)

Module 2 Geometry and trigonometry

Question 3

a. i.

$\angle AOB = 360 \div 6 = 60^0$

(1 mark)

a. ii.

AOB must be an equilateral triangle

$AO = BO$

(so base angles are equal and

third angle is 60^0)

Hence, all angles are 60^0

Area of triangle AOB

$= \frac{1}{2} \times 1.5 \times 1.5 \sin 60^0$

$= 0.9743 \text{ m}^2$

Area of 6 triangles $= 6 \times 0.9743 = 5.85 \text{ m}^2$

(1 mark)

b.

Surface area

= Area of 2 hexagons + Area of 6 rectangles

$= 2 \times 5.8457 + 6 \times 1.5 \times 2 = 29.69 \text{ m}^2$

(1 mark)

c. i.

Area remaining = area of hexagon − area of circle

Area remaining $= 5.8457 - \pi \times 0.5^2$

Area remaining $= 5.06 \text{ m}^2$

(1 mark)

c. ii.

Area of curved surface $= 2 \times \pi \times r \times h$

Area of curved surface $= 2 \times \pi \times 0.5 \times 2 = 6.3 \text{ m}^2$

(1 mark)

c. iii.

Total surface area

= (Area of top hexagon − Area of circle) + (Area of bottom hexagon − Area of circle) + Area of 6 rectangles + Area of curved surface of cylinder

$= 5.0603 \times 2 + 6 \times 1.5 \times 2 + 6.2832$

$= 34 \text{ m}^2$

(1 mark)

Module 3 Graphs and relations

Question 1

a.
Shampoo costs more than conditioner, so shampoo costs $28

(1 mark)

b. i.
$20 is the fixed cost of producing both the shampoo and the conditioner.

(1 mark)

b. ii.

$C = an + b$
$b = 20$
$C = an + 20$
When $n = 4, C = 28$
$28 = 4a + 20$
$a = 2$
$C = 2n + 20$
Cost for x bottles is
$C = 2x + 20$

(1 mark)

b. iii.

Cost of shampoo $= 2x + 20$
Cost of conditioner $= ax + 20$
When $x = 4, C = 22$
$22 = 4a + 20$
$a = 0.5$
Cost of conditioner $= 0.5x + 20$
Cost of 100 conditioners $= 0.5 \times 100 + 20 = 70$
Cost of 100 shampoos $= 2 \times 100 + 20 = 220$
Total cost $= 220 + 70 = \$290$

(1 mark)

b. iv.
$73 = 20 + 0.5x$
$x = 106$ (1 mark)
$517 - 73 = 444$
$444 = 20 + 2x$
$x = 212$
Total number of bottles of shampoo and conditioner
$= 212 + 106 = 318$

(1 mark)

Module 3 Graphs and relations

Question 2

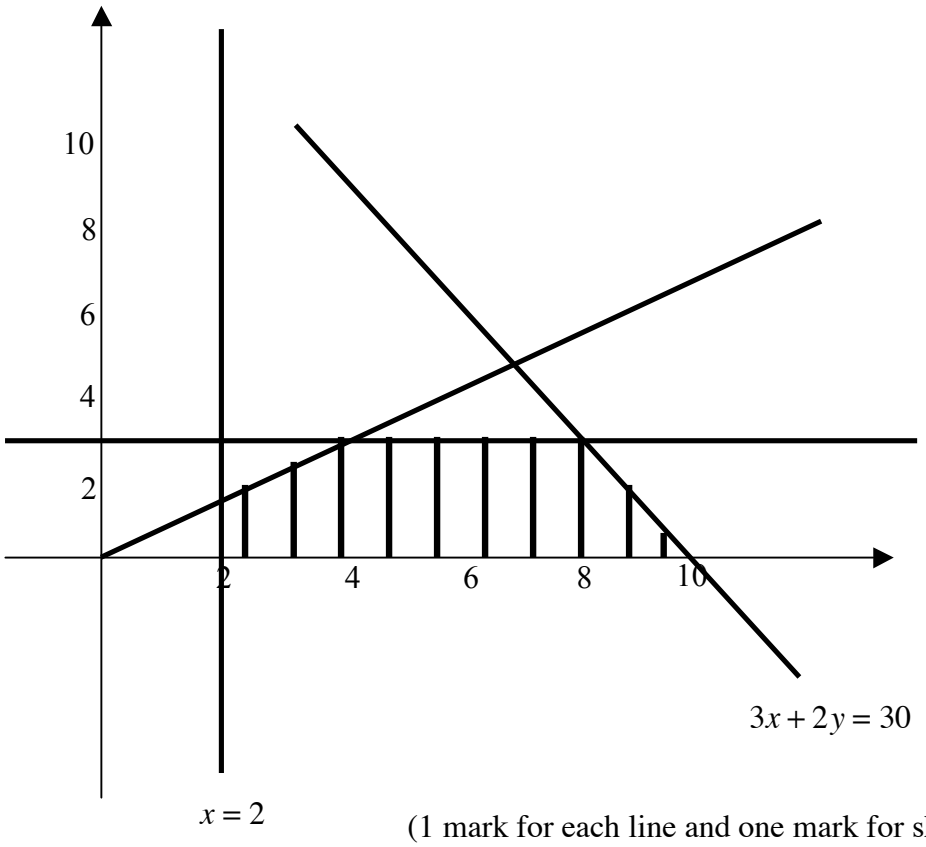
a. $\$20x$ (1 mark)	**b.** $20 + 2 \times 10000 = \$20020$ (1 mark)
c. Break even point $20x = 20020$ $x = 1001$ (1 mark)	

Question 3

a. $15x + 10y \leq 150$ $3x + 2y \leq 30$ (1 mark)	**b.** $x \geq 2$ (1 mark)

Module 3 Graphs and relations

c. and d.



(1 mark for each line and one mark for shading)

e.

Corner points of graph that have integer values are.
(2,0) Total number of bottles = 2 + 0 = 2
(10,0) Total number of bottles = 10+ 0 = 10
(4,3) Total number of bottles = 4 + 3 = 7
(8,3) Total number of bottles = 8 + 3 = 11
Maximum number of bottles = 8 shampoo and 3 conditioner.

(1 mark)

Module 4 Business-related mathematics

Question 1

a.
Balance on 1 June = 2825.07
Balance on 11 June = 2825.07 – 204 = $2621.07
Balance on 15 June = 2621.07 + 6000
= $8621.07
Balance on 23 June = $8441.07
Balance on 28 June = 8441.07 + 8000 = $16441.07
Balance on 29 June = 16441.07 – 13000 = $3441.07
Balance on 30 June = 3441.07 + Interest
> $3441.07
Hence, minimum monthly balance = $2621.07

(1 mark)

b.
On 15th June there was $8621.07 in the account.
On 23rd June there was $8441.07 in the account.
Amount withdrawn = 8621.07 – 8441.07
=$180

(1 mark)

c.

$$\text{Monthly interest} = \frac{4}{12} = 0.33\%$$

(1 mark)

d.

$$\text{Interest} = \frac{2621.07 \times 0.3333}{100} = \$8.74$$

(1 mark)

e.
Opening balance for 1st July = 3441.07 + 8.74 = $3449.81

(1 mark)

Question 2

a. i.

$$\text{Total paid} = 10000 + 652.50 \times 60 = \$49150$$

(1 mark)

a. ii.

$$\text{Interest} = 49150 - 39000 = \$10150$$

(1 mark)

a. iii.

$$I = \frac{PRT}{100}$$

$$R = \frac{100I}{PT} = \frac{100 \times 10150}{29000 \times 5} = 7\%$$ (1 mark)

Module 4 Business-related mathematics

Question 2 (continued)

b. i.

$A = 5000(1.005)^{60} = \$6744.25$

(1 mark)

b. ii.

On TI-84

Press Apps Finance Enter TVM Solver Enter

$N = 60$

$I = 6$

$PV = 5000$

$PMT = -100$

$FV =$

$P/Y = 12$

$C/Y = 12$

End

Put cursor on FV and press alpha solve.

$FV = \$232.75$

OR On a ClassPad, enter Financial on menu screen, then enter the same as for the TI-84

(1 mark)

b. iii.

On TI-84

Press Apps Finance Enter TVM Solver Enter

$N = ?$

$I = 6$

$PV = 5000$

$PMT = -652.50$

$FV = 0$

$P/Y = 12$

$C/Y = 12$

End

Put cursor on FV and press alpha solve.

$N = 7.8$

It would take 8 months to repay the loan

(1 mark)

Module 4 Business-related mathematics

Question 3

a.

$39000(0.8)^5 = \$12780$

(1 mark)

b.

Number of 100 km = $\frac{30000}{100} = 300$

Depreciation for 1 year $= 300 \times 4.40 = 1320$

Depreciation for 5 years $= 1320 \times 5 = 6600$

Book Value at end of 5 years

$= 15000 - 6600 = \$8400$

(1 mark)

c.

$39000R^5 = 8400$

$R^5 = 0.21538$

$R = (0.21538)^{\frac{1}{5}} = \sqrt[5]{0.21538} = 0.7356$

$1 - \frac{r}{100} = 0.7356$

$\frac{r}{100} = 0.2644$

$r = 26.4\%$

(1 mark)

d.

Book Value at end of 5 years = 12780

Depreciation = $15000 - 12780 = 2220$

$300 \times x \times 5 = 2220$

$x = 1.48$

Decrease by $(4.40 - 1.48) = \$2.92$

(1 mark)

Module 5 Networks and decision mathematics

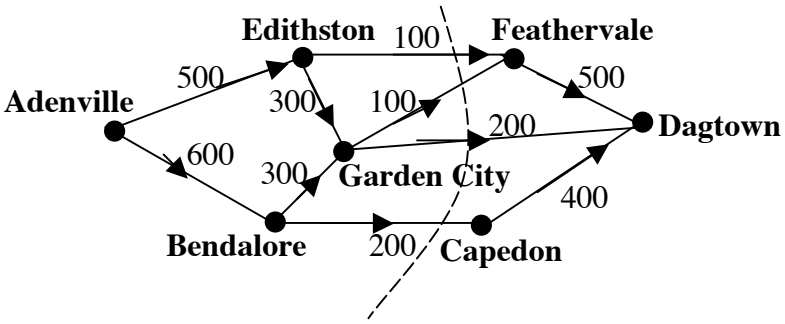
Question 1

a.
200
(1 mark)

b.
200 from A to D via B and C
100 from A to D via E and F
=300
(1 mark)

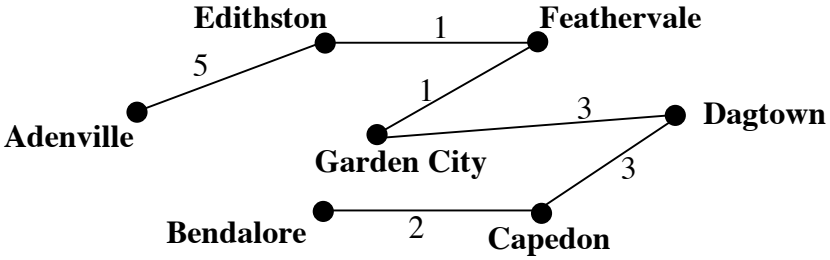
c. i.
100 + 100 + 200 + 200 = 600
(1 mark)

c. ii.



(1 mark)

d.



Minimum length of wire to connect all towns is 5 + 1 + 1 + 3 + 3 + 2 = 15 km.
(1 mark)

Module 5 Networks and decision mathematics

Question 2

<table>
<tr>
<td>a.
Activity A

(1 mark)</td>
<td>b.
Activity E

(1 mark)</td>
</tr>
<tr>
<td>c.
The critical path is the longest path.
A – C – D – F

(1 mark)</td>
<td>d.
10 + 13 + 7 + 4 = 34 days

(1 mark)</td>
</tr>
<tr>
<td>e.
10 + 13 + 7 = 30 days
(1 mark)</td>
<td>f.
Earliest start time = 10 + 13 = 23
Latest start time = 34 – 4 = 30 days
30 – 23= 7 days
(1 mark)</td>
</tr>
<tr>
<td>g. i.
E now takes 14 hours.
There is now a new critical path,
A – C – E so the time taken to complete the project is now 10 + 13 + 14 = 37 days. Hence, the project will take 3 more days to complete.
(1 mark)</td>
<td>g. ii.
37 – 4 = 33 days

(1 mark)</td>
</tr>
<tr>
<td colspan="2">h.
Time for completion of activity A = 5 hours.
Minimum time to complete project now
= 5 + 13 + 7 + 4 = 29 days.
Cost for extra person = 350 × 5 = $1750
Cost for project now = 1750 + 29 × 300 = $10450
(1 mark)

Cost without extra person = 300 × 34 = $10200
Extra cost = 10450 – 10200 = $250
(1 mark)</td>
</tr>
</table>

Module 6 Matrices

Question 1

a.

$$\begin{bmatrix} 50 & 1440 & 70 & 80 \\ 80 & 1000 & 90 & 85 \end{bmatrix}$$

(1 mark)

b.

$$S = \begin{bmatrix} 120 \\ 4 \\ 25 \\ 30 \end{bmatrix}$$

(1 mark)

c.

B would be a column matrix containing two elements. The element in the first row would be the total selling price for week1 and the element in the second row would be the total selling price for week 2.

(1 mark)

Question 2

a.

Reading across the rows, 1 means that the team won and 0 means that the team lost or that the team did not play itself. In the third row, the Condors won against the Anchors, Bull Dogs and Dragons so there is a 1 in theses positions.. They did not play themselves, and must have lost to the Emus as there is a 0 in these positions.

(1 mark)

b.

The Dragons defeated the Anchors so $y = 1$

(1 mark)

c.

In the last column we can see that the Emus were beaten by the Anchors and the Bull Dogs.

(1 mark)

Module 6 Matrices

Question 3

a. 80%

(1 mark)

b. 9 %

(1 mark)

c.

$$\begin{array}{c} \\ To \end{array} \begin{array}{c} \\ P \\ Q \\ R \end{array} \begin{array}{c} From \\ \begin{array}{ccc} P & Q & R \end{array} \\ \begin{bmatrix} 0.8 & 0.05 & 0.09 \\ 0.08 & 0.85 & 0.06 \\ 0.12 & 0.1 & 0.85 \end{bmatrix} \end{array}$$

(1 mark)

d.

50% of 800 = 400

25% of 800 = 200

$$N = \begin{array}{c} \\ P \\ Q \\ R \end{array} \begin{bmatrix} 400 \\ 200 \\ 200 \end{bmatrix}$$

(1 mark)

e.

$$\begin{bmatrix} 0.8 & 0.05 & 0.09 \\ 0.08 & 0.85 & 0.06 \\ 0.12 & 0.1 & 0.85 \end{bmatrix} \begin{bmatrix} 400 \\ 200 \\ 200 \end{bmatrix} = \begin{bmatrix} 348 \\ 214 \\ 238 \end{bmatrix}$$

Would expect 348 to belong to the Physical tennis centre.

(1 mark)

f.

1st. January 2011 = 238 (from **e**)

1st. January 2010 = 200

So Rigour has 38 more.

(1 mark)

g.

$$\begin{bmatrix} 0.8 & 0.05 & 0.09 \\ 0.08 & 0.85 & 0.06 \\ 0.12 & 0.1 & 0.85 \end{bmatrix}^5 \begin{bmatrix} 400 \\ 200 \\ 200 \end{bmatrix} = \begin{bmatrix} 250 \\ 240 \\ 310 \end{bmatrix}$$

Quality will have 240 members

(1 mark)

h.

$$\begin{bmatrix} 0.8 & 0.05 & 0.09 \\ 0.08 & 0.85 & 0.06 \\ 0.12 & 0.1 & 0.85 \end{bmatrix}^{30} \begin{bmatrix} 400 \\ 200 \\ 200 \end{bmatrix} = \begin{bmatrix} 214 \\ 249 \\ 337 \end{bmatrix}$$

$$\begin{bmatrix} 0.8 & 0.05 & 0.09 \\ 0.08 & 0.85 & 0.06 \\ 0.12 & 0.1 & 0.85 \end{bmatrix}^{50} \begin{bmatrix} 400 \\ 200 \\ 200 \end{bmatrix} = \begin{bmatrix} 214 \\ 249 \\ 337 \end{bmatrix}$$

Physical will have the least number

(1 mark)

i.

From h rigour will have 337 members in the long term.

(1 mark)

End of Suggested Solutions 2011 Further Mathematics VCE Trial Examination 2

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