Online & home tutors *Registered business name: itute ABN: 96 297 924 083*

Further Mathematics

2011

Trial Examination 2

Core – Data analysis
Module 2 – Geometry and trigonometry
Module 3 – Graphs and relations
**Module 4 – Business-related mathematics**

Instructions:

Answer all questions in the core and the three modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Core – Data analysis

Question 1 The following graph shows the median age of population at 30 June 1990 and 2010.

Source: ABS
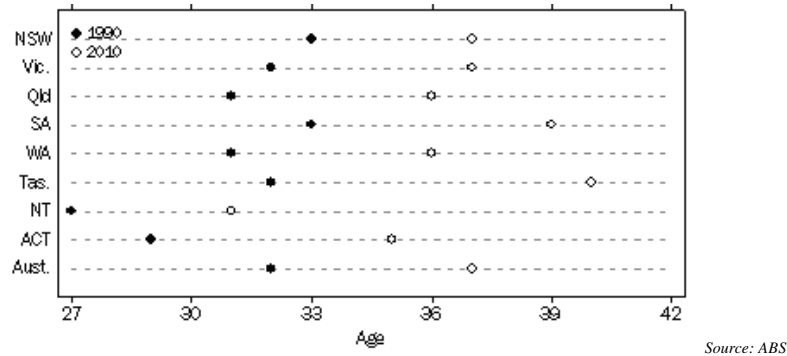
a. What percentage of Australians were older than 37 years of age at 30 June 2010?   1 mark

b. Do you expect a higher or lower percentage of Australians older than 37 years of age at 30 June 2012? Explain your answer.   1 mark

c. Which state/territory experienced the largest change in median age over the 20-year period?   1 mark

d. What was the change in median age over the 20-year period in Victoria?   1 mark

Question 2 There are many reasons people use the internet. They include work and education purposes, and personal reasons, such as emailing, banking, gaming, preparing job applications and online shopping.

PURPOSE OF INTERNET USE AT HOME (2008-09) *Source: ABS*
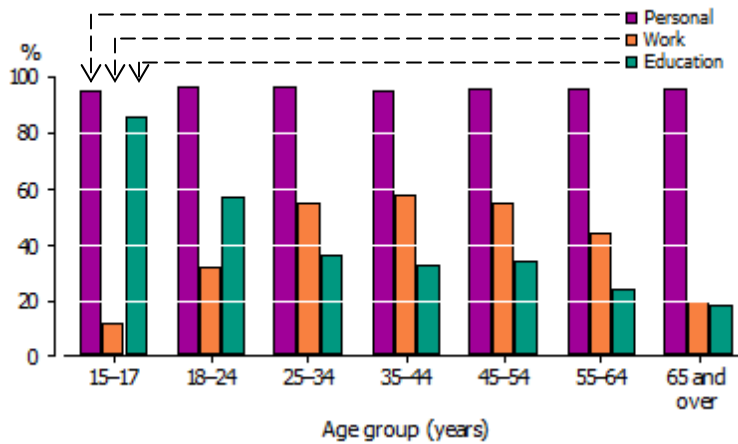
a. In 2008-09 the majority (96%) of people aged 15 years and over who accessed the internet at home did so for *personal reasons*, making it the most common purpose for internet use across all age groups.
Describe the trend in internet use at home for *work purposes* by the different age groups in 2008-09.

1 mark

b. Complete the following table showing the internet use at home for *education purposes* by the different age groups in 2008-09. 1 mark

Age group	Centre of age group	Percentage for education
15-17	16	86
18-24	21	58
25-34	30	
35-44	40	34
45-54	50	
55-64	60	24
65 and over	75 (estimated)	19

Note: The centre of each age group is rounded to the nearest whole number.

c. For the data set of *centre of age group* and *percentage for education*, an equation of the least squares regression line is: *percentage for education* $= a \times$ *centre of age group* $+ b$.

Find the values of a and b. 1 mark

d. Find a suitable transformation that could be used to linearised the data set of *centre of age group* and *percentage for education* with the value of Pearson's product moment correlation coefficient greater than 0.95. State the value of Pearson's product moment correlation coefficient.

2 marks

Question 3 The following table shows the annual rainfall from 1999 to 2010.

Year	Time (year)	Annual rainfall (cm)	Three-point moving average (cm)
1999	0	800	*blank*
2000	1	900	850
2001	2	850	1000
2002	3	1250	1100
2003	4	1200	1200
2004	5	1150	1200
2005	6	1250	1250
2006	7	1350	1300
2007	8	1300	*a*
2008	9	1400	1400
2009	10	1500	*b*
2010	11	1450	*blank*

a. State the value of each of *a* and *b* in the table above.

1 mark

b. Comment on the trend shown by the 3-year moving average of annual rainfall.

1 mark

An equation of the least squares regression line for the data set of *time* and *three-point moving average of annual rainfall* is

$$three\text{-}point\ moving\ average\ of\ annual\ rainfall\ (cm) = 883.3 + 59.4 \times time\ (year)$$

c. Determine the residual for year 2000.

1 mark

d. What information can be inferred from the number 59.4 in the equation of the least squares regression line?
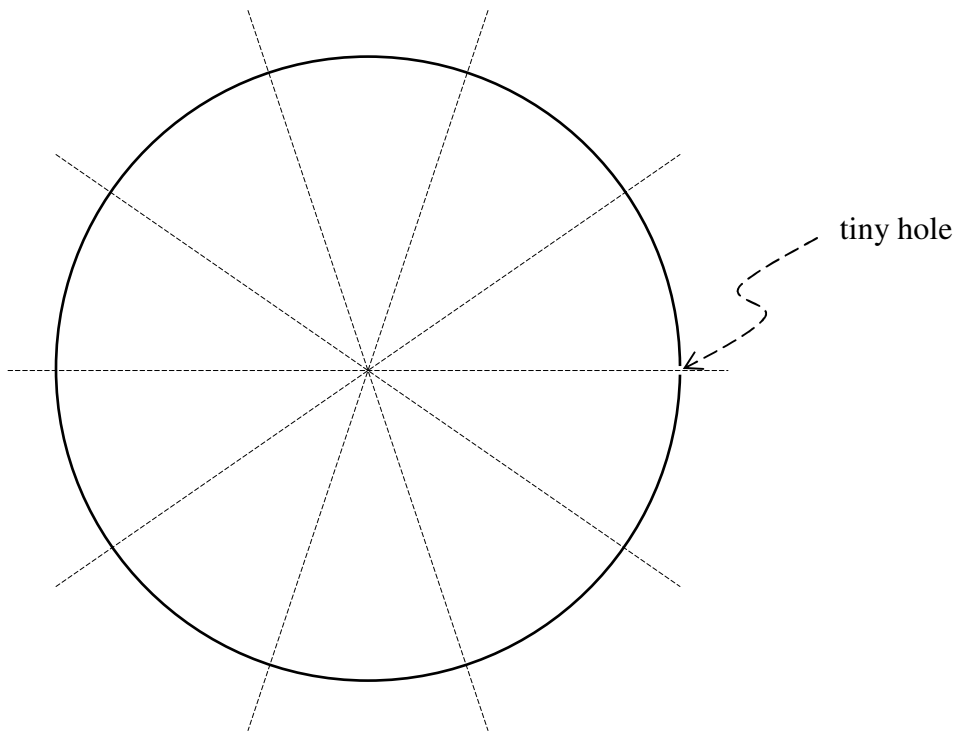
1 mark

Question 4 The least squares regression line $y = c + mx$ for the *average number of hours of study per week (x)*, and the *atar score (y)* for a group of year 12 students has a coefficient of determination of 0.5098. Also, $\bar{x} = 20.8966$, $s_x = 5.7155$, $\bar{y} = 71.4138$ and $s_y = 15.3960$.

a. Calculate the value (2 decimal places) of c in the least squares regression line $y = c + mx$.

1 mark

b. Describe the goodness of fit of the least squares regression line to the set of bivariate data in terms of percent (nearest whole number) of explained variation.
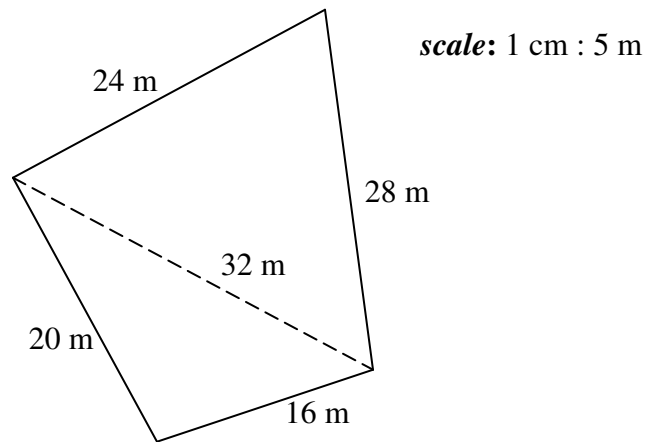
1 mark

Module 2: Geometry and trigonometry

Question 1   A hollow sphere has an internal reflective surface. A tiny hole of negligible size is made through its thin wall. A narrow laser beam is sent through the hole into the sphere and it escapes through the hole after *four* reflections inside the sphere.

a.   Draw a regular polygon (use ***dotted lines***) to show the path formed by the laser beam and its reflections inside the sphere shown above.   1 mark

b.   Calculate the exact angle between any pair of adjacent sides of the polygon.   2 marks

c.   On the same diagram above draw another possible path (use ***solid lines***) of the laser beam entering the tiny hole and escaping through the hole after *four* reflections inside the sphere.   1 mark

Question 2   The scale drawing of a block of land is shown below. The actual linear measurements of the block of land are shown in the drawing.


scale: 1 cm : 5 m

24 m

28 m

32 m

20 m

16 m

a.   Calculate the *actual* area of the block of land (nearest m^2).   2 marks

b.   Calculate the area of the *drawing of the block of land* (nearest cm^2).   1 mark

c.   There are two corners opposite to the dotted line. Select the corner which has a greater angle and calculate this angle (nearest degree).   2 marks

Question 3 A small cottage is marked as point *C* in the following contour map. A person marked as point *P* is 1.2 km horizontally and at a bearing of 210° from *C*.

300 m

N

C

200 m

100 m

P •

0 m
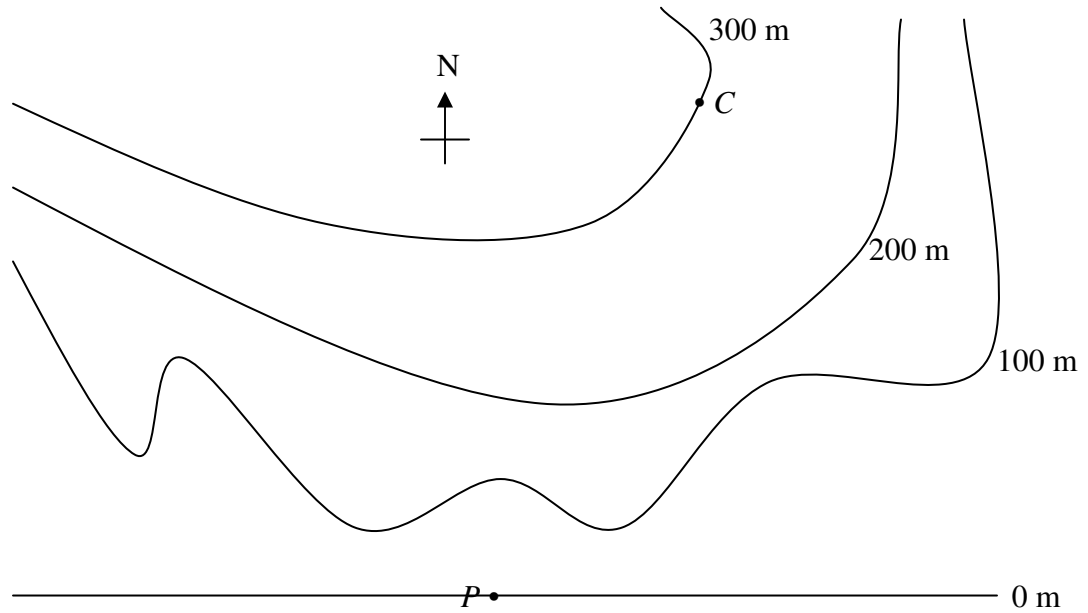
a. Determine the angle (nearest degree) of elevation of *C* from *P*. 1 mark

b. Calculate the shortest distance (m) travelled by the person to arrive at a position directly south of the small cottage *C*. 1 mark

c. Draw accurately on the above contour map a line and label it as *XY* which is definitely steepest in the actual terrain.

1 mark

Question 4   A container in the shape of an *irregular* pyramid has its base on a horizontal surface. It is filled with water to a depth of 12 cm. The height of the pyramid is 20 cm and its base has an area of 125 cm^2.

depth 12 cm

air

water

height 20 cm

Not drawn to scale
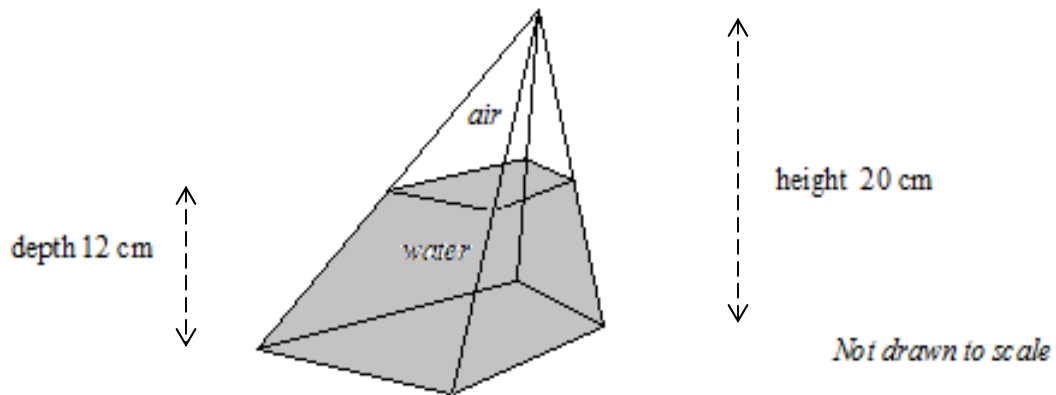
a.   Determine the fraction $\dfrac{volume\ of\ air}{volume\ of\ water}$ in simplest form.   2 marks

b.   Calculate the volume (cm^3) of water in the container.   1 mark

Module 3: Graphs and relations

**Question 1**   Individual income tax rates 2011/12 Source: ATO

The following rates for 2011-12 apply from 1 July 2011.

These rates apply to individuals who are Australian residents for tax purposes.

These rates **do not** include the Medicare levy of 1.5% . To help fund the scheme, most taxpayers pay a Medicare levy of 1.5% of their taxable income. These rates **do not** include the Flood levy.

Taxable income	Tax on this income
0 - $6,000	Nil
$6,001 - $37,000	15c for each $1 over $6,000
$37,001 - $80,000	$4,650 plus 30c for each $1 over $37,000
$80,001 - $180,000	$17,550 plus 37c for each $1 over $80,000
$180,001 and over	$54,550 plus 45c for each $1 over $180,000

a.   Draw a line-segment graph showing the income tax payable versus taxable income, *not* including the Medicare levy and the Flood levy. Label the axes including units.   2 marks
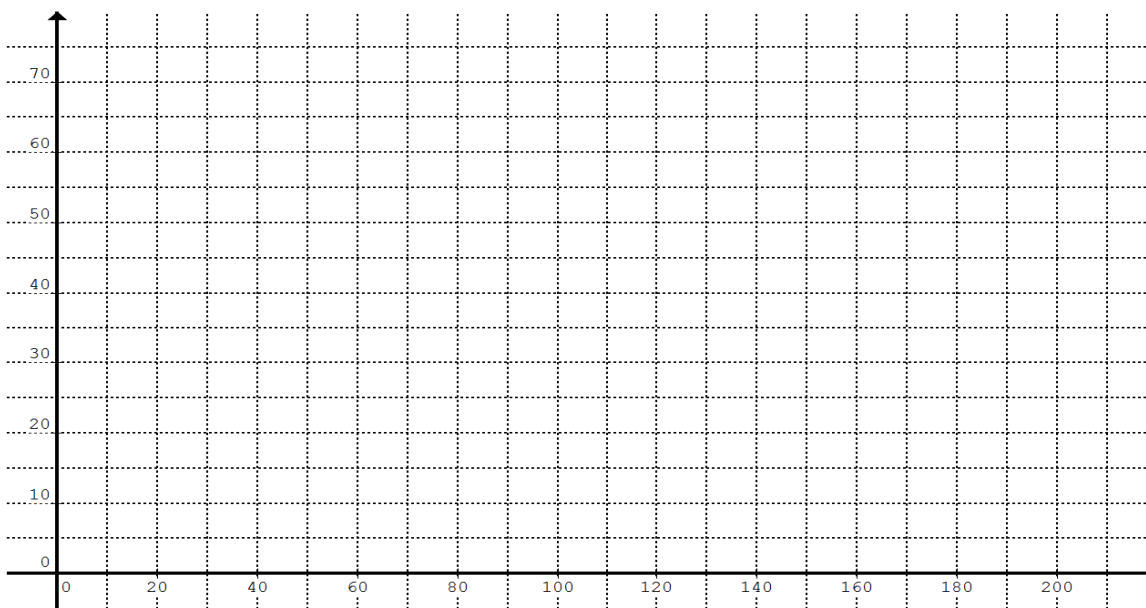
The flood levy only applies to taxable income you earn over $50,000.

Taxable income	Flood levy on this income
$0 to $50,000	Nil
$50,001 to $100,000	Half a cent for each $1 over $50,000
Over $100,000	$250 plus 1c for each $1 over $100,000

b.   Calculate the tax (*not* including the Medicare levy but *including* the Flood levy) payable for a taxable income of $120000.   2 marks

Question 2   The distance-time graph of a postal van on its round of picking up mails from post boxes marked as A, B and C is shown below. An Australian Post supervisor started his car from the same post office as the postal van five minutes after the van left the post office and travelled along the same route as the van. The supervisor's car passed the van the first time after 14 minutes of driving at constant speed.
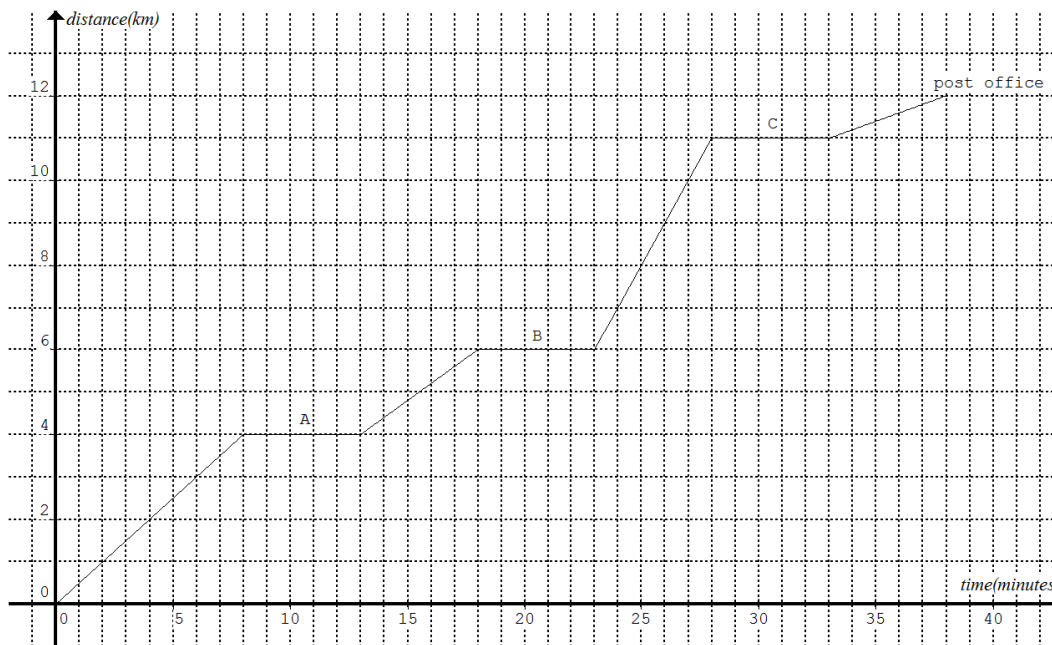
a.   Calculate the average speed (nearest kilometres per hour) of the postal van from the moment it left the post office to the moment it returned to the post office.   1 mark

b.   On the same axes above draw the distance-time graph of the supervisor's car travelling from the post office *back to the post office*. Assume the speed was constant throughout the round trip.   1 mark

c.   When and where (nearest whole number of minutes and kilometres) did the supervisor pass the van the second time?   1 mark

d.   Determine the range of speed (in whole number of kilometres per hour) of the supervisor's car not exceeding 60 kilometres per hour that would allow it to pass the van once *only* during the round trip.

                                                                                                                    2 marks

Question 3   A set of data is shown in the following table. v is the speed (kilometres per hour) of a truck and d is the braking distance (metres) of the truck.

v	40	50	60	70	80
d	12.3	19.3	27.8	37.8	49.4

a.   Plot the graph of d against v^2. Label and scale the axes.
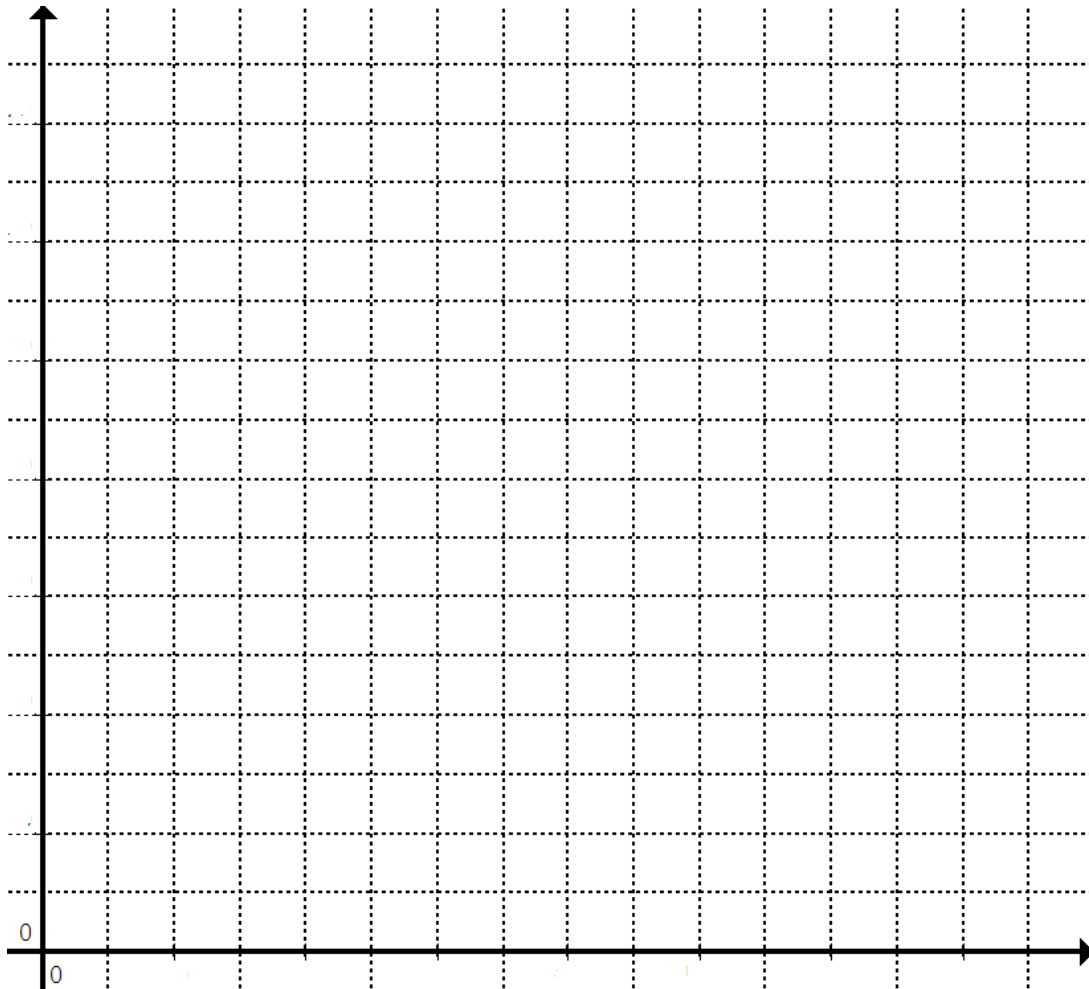
2 marks

b.   Determine the equation which shows the relationship between d and v.   1 mark

Question 4 There are two variables, x and y, in a linear programming problem.

The feasible region of variables x and y is the shaded area in the graph below.

a. Write down a set of four simultaneous linear inequalities which define the feasible region. 2 marks

b. The objective function of the problem is $P = 11x + 5y$. Find the maximum value of P. 1 mark

Module 4: Business-related mathematics

Question 1 A notebook computer was sold for $799.00 including GST. The advertised price was $999.00 including GST.

a. Calculate the % discount (nearest %) of the notebook computer.   1 mark

b. Calculate the amount of GST paid (to the nearest cent).   2 marks

Question 2 A $420 000 house was purchased with a housing loan of $300 000 from a bank at 7.25% p.a. interest compounded monthly. The loan is to be paid off over 25 years by monthly instalments.

a. Calculate the monthly repayment required.   1 mark

A year later the house owner had a pay rise and increased his monthly repayment to $2400.00.

b. How many months (nearest whole number) earlier will he repay the loan in full?   2 marks

c. How much interest (nearest $100) will he save?   2 marks

Question 3   The following table shows the Australian quarterly CPI from 2007 March quarter to 2011 June quarter.

Quarter	CPI
Mar-2007	155.6
Jun-2007	157.5
Sep-2007	158.6
Dec-2007	160.1
Mar-2008	162.2
Jun-2008	164.6
Sep-2008	166.5
Dec-2008	166.0
Mar-2009	166.2
Jun-2009	167.0
Sep-2009	168.6
Dec-2009	169.5
Mar-2010	171.0
Jun-2010	172.1
Sep-2010	173.3
Dec-2010	174.0
Mar-2011	176.7
Jun-2011	178.3

Source: ABS

a.   Calculate the average annual percentage rise (3 decimal places) in the cost of living in the three years 2008 – 2010.   2 marks

b.   In terms of the value of money, $100 at the end of the June quarter in 2011 is equivalent to how much (nearest cent) a year ago?

1 mark

c.   Would you be ahead/behind in real terms financially if you received 3.5% p.a. simple interest for your money in the period mentioned in part b?   1 mark

Question 4   Interest on a bank account is paid yearly on 30 June and is calculated on the minimum monthly balance. The interest rate is 5.25% p.a.

For the year 2010-11, the ***complete*** statement for an account, before adding interest, is shown below:

Date	Credit	Debit	Balance
30 June 2010	Interest $57.00		$6257.00
18 March 2011		$2130.00	$4127.00

Assume no other deposits or further withdrawals were made after 18 March 2011 in the following questions.

a.   What was the minimum balance in March 2011?   1 mark

b.   Calculate the total interest (nearest dollar) credited to this account on 30 June 2011.   2 marks

End of Exam 2