Further Mathematics

2011

Trial Examination 1

Core – Data analysis
Module 1 – Number patterns
Module 5 – Networks and decision mathematics
Module 6 – Matrices

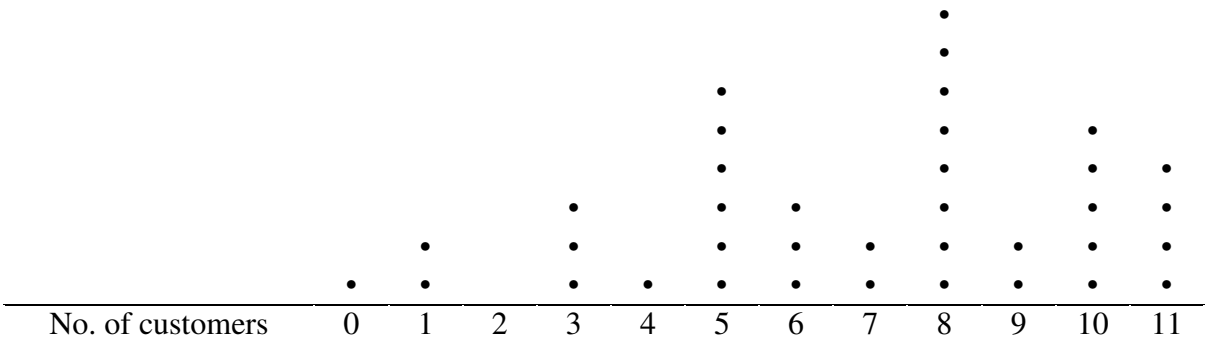
SECTION A Instructions

Answer **all** questions
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if **more than one** answer is completed for any question.

Core – Data analysis

The following information relates to Questions 1, 2 and 3

The results of a survey of the number of customers inside each shop in a shopping strip at 5.00 pm on a particular day are displayed in the following dot plot.



Question 1

The number of shops surveyed is

A. 11

B. 12

C. 36

D. 37

E. not determinable from the dot plot

Question 2

The shape of the distribution of the number of customers inside a shop is best described as

A. discrete

B. symmetric

C. negatively skewed

D. positively skewed

E. positively skewed with outliers

Question 3

The median of the distribution is

A. 7

B. 8

C. 17

D. 18

E. 19

The following information relates to Questions 4 and 5

The test results for a class of 26 students are displayed in the following back-to-back stem-and-leaf plot.

Boys					*Girls*				
			1	4	1	2	4		
			2	5	0	1	3	5	
	8	7	5	6	6	6	9	9	9
7	4	2	2	7	4				
		3	0	8	2	9			

Question 4

The interquartile range of the test results is

A. 19

B. 20

C. 21

D. 22

E. 23

Question 5

Let M_{girl}, M_{boy} and M_{all} be the medians for girls, boys and all the students in the class. Which one of the following statements is true?

A. $M_{girl} > M_{boy} > M_{all}$

B. $M_{girl} < M_{boy} < M_{all}$

C. $M_{boy} > M_{all} > M_{girl}$

D. $M_{boy} < M_{all} < M_{girl}$

E. $M_{boy} > M_{girl} > M_{all}$

Question 6

A survey of Year 9 students at a large school found a number of relationships with a high degree of correlation. Which one of the following relationships also demonstrates causality?

A. The size of students' left hands and the size of their right hands

B. Students weight and height

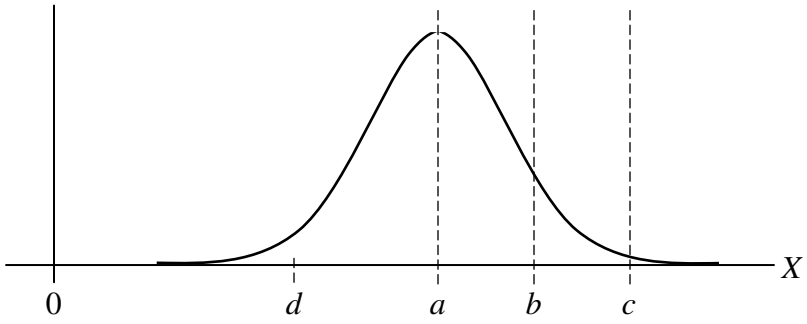
C. The size of students' hands and the size of their feet

D. Students' examination results and their test results in Science

E. The number of hours students spent studying for a test and their results in that test

Question 7

A set of data has a bell-shape distribution as shown below. 34% of the data are between values a and b, and 13.5% are between values b and c.



The z-score of value d is

A. $2 < z < 3$

B. $1 < z < 2$

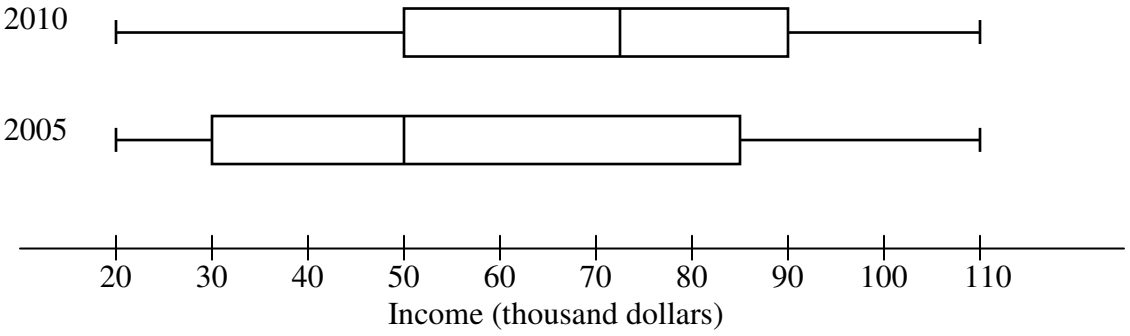
C. $0 < z < 1$

D. $-1 < z < 0$

E. $-2 < z < -1$

The following information relates to Questions 8 and 9

The graphs show the distribution of the incomes of households in a suburb in 2005 and 2011.



In 2005 there were 1952 households with income 20 to 110 thousand dollars.

The number of households with income 20 to 50 thousand dollars is the same in both 2005 and 2011.

Question 8

The number of households with income 30 to 110 thousand dollars in 2005 was

A. more than 1600

B. more than 1500

C. between 1400 and 1500

D. less than 1400

E. less than 1000

Question 9

Since 2005 the number of households in the suburb in 2011 has increased ***by***

A. 200%

B. 100%

C. 50%

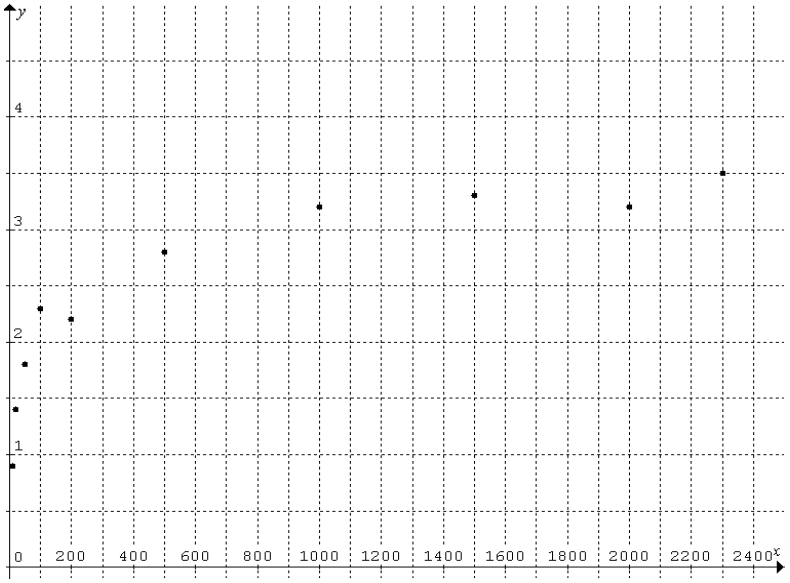
D. 25%

E. 10%

The following information to Questions 10 and 11

The values of variables x and y are recorded in the table and displayed in the graph below.

x	10	20	50	100	200	500	1000	1500	2000	2300
y	0.9	1.4	1.8	2.3	2.2	2.8	3.2	3.3	3.2	3.5



Question 10

The pearson's product-moment correlation coefficient is closest to

A. 0.90

B. 0.85

C. 0.80

D. 0.75

E. 0.70

Question 11

Which one of the following graphs is most likely to display linearity?

A. $\log_{10} y$ versus x

B. y versus $\log_{10} x$

C. $\log_{10} y$ versus $\log_{10} x$

D. y versus x^2

E. y versus $\frac{1}{x}$

The following information relates to Questions 12 and 13

The table below shows the average temperature ($°C$) in a country town by month in 2008, 2009 and 2010.

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	**Yearly av**
2008	40	32	30	28	23	19	15	15	26	26	29	32	26.25
2009	38	33	29	25	20	18	15	19	23	25	30	32	25.58
2010	39	35	29	26	19	19	14	18	24	25	31	31	25.83

Question 12

The seasonal index for August based on the 3-year figures shown above was closest to

A. 0.6

B. 0.7

C. 0.8

D. 0.9

E. 1.0

Question 13

Based on the 3-year figures the seasonally adjusted average temperature for May 2008 was $28.8°C$. The seasonally adjusted average temperature for May 2010 was closest to

A. $21.8°C$

B. $22.8°C$

C. $23.8°C$

D. $24.8°C$

E. $25.8°C$

SECTION B Instructions

Answer **all** questions
A correct answer scores 1, an incorrect answer scores 0.
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No marks will be given if **more than one** answer is completed for any question.

Module 1: Number patterns

Question 1

Oranges of the same size are stacked to form a *square-base* pyramid. The pyramid has 6 layers of oranges. The number of oranges required to form the pyramid is

A. 140

B. 91

C. 55

D. 28

E. 21

Question 2

The sum of the first seven terms of the sequence 2, 3, 5, 8, 13, ……. is

A. 141

B. 86

C. 85

D. 53

E. 52

Question 3

Consider the sequence 4, 5, 7, 9, 13, 15. The next 3 terms of the sequence are

A. 17, 19, 23

B. 19, 21, 25

C. 20, 23, 27

D. 21, 25, 31

E. 23, 25, 29

The following information relates to Questions 4, 5 and 6

Three athletes A, B and C plan their training programs for a swimming competition so that each will follow a sequence to increase their weekly training to 250 laps in 52 weeks.

Question 4

Athlete A is currently swimming 80 laps per week. She decides to increase her weekly distance by a fixed amount each week. The weekly increase in distance in laps is closest to

A. 3

B. $\frac{85}{26}$

C. $\frac{10}{3}$

D. 4

E. $\frac{125}{26}$

Question 5

Athlete B is currently swimming 25 laps per week. He decides to increase his weekly distance by a fixed percentage each week. The weekly percentage increase in distance is closest to

A. 4.26

B. 4.32

C. 4.40

D. 4.53

E. 4.62

Question 6

Athlete C is currently swimming 7.5 laps per week. Each week she decides to increase her weekly distance by a fixed percentage of the previous week's distance plus an additional 3 laps. The sequence used by athlete C as a first-order linear difference equation is in the form

A. $t_{n+1} = a \times (t_n + 3)$, where $a > 1$

B. $t_{n+1} = a \times (t_n + 3)$, where $a < 1$

C. $t_{n+1} = a \times t_n + 3$, where $a > 1$

D. $t_{n+1} = a \times t_n + 3$, where $a < 1$

E. $t_{n+1} = a \times (t_n + 7.5)$, where $a > 1$

Question 7

The series $1+2+3+4+.....+n$ can be simplified to

A. $\frac{(n-1)n}{2}$

B. $\frac{(n-1)^2}{2}$

C. $\frac{(n+1)^2}{2}$

D. $\frac{n(n+1)}{2}$

E. $\frac{n^2}{2}$

Question 8

Starting at midday a clock slows down by 30 minutes in the first hour, 45 minutes in the second hour, 52.5 minutes in the third hour and so on. Eventually the clock will point at

A. 1:00 pm

B. 1:30 pm

C. 2:00 pm

D. 2:30 pm

E. 6:00 pm

Question 9

S_A is an arithmetic series of *n* terms with $a=1$ and $d=2$, and S_G is a geometric series of *n* terms with $a=1$ and $r=\frac{1}{2}$.

Which one of the following statements is true?

A. $S_A > S_G$ for $n \geq 1$

B. $S_G \geq S_A$ for $n \geq 1$

C. $S_A \geq S_G$ for $n \geq 1$

D. $S_A > S_G$ when $0 < n < 1$

E. $S_G > S_A$ when $0 < n < 1$

Module 5: Networks and decision mathematics

Question 1 The degree of each vertex in a particular graph is odd. The possible number of vertices in the graph is

A. 1

B. 5

C. 10

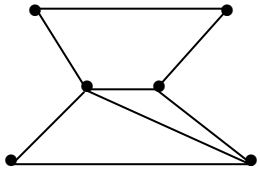
D. 15

E. 25

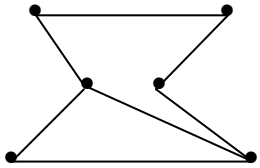
Question 2

Which one of the following graphs does NOT have an Euler **path**?

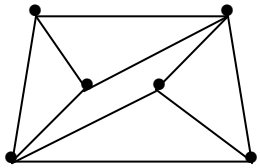
A.



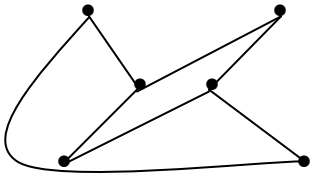
B.



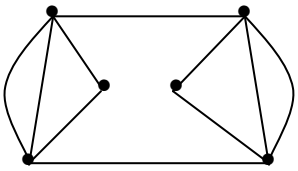
C.



D.



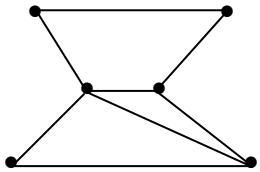
E.



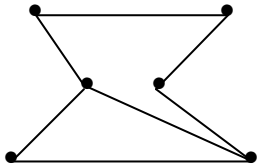
Question 3

Which one of the following graphs does NOT have a hamiltonian **circuit**?

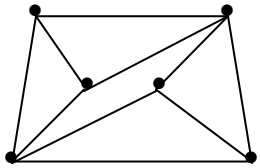
A.



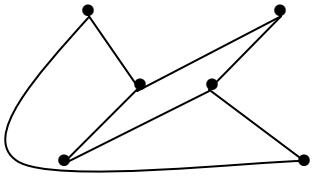
B.



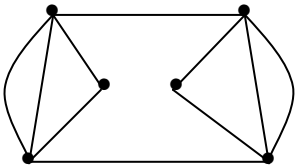
C.



D.



E.



Question 4 A particular complete graph has 10 vertices. The number of edges in this graph could be

A. 9

B. 10

C. 25

D. 26

E. 51

Question 5

Which one of the following graphs with an unknown number of edges in each case is **definitely** planar?

A. A graph with 4 vertices

B. A graph with 5 vertices

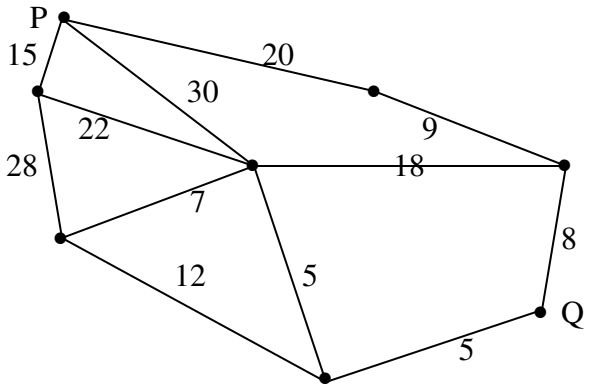
C. A graph with 6 vertices

D. A graph with 7 vertices

E. A graph with 8 vertices

Question 6

The distances in kilometres between towns connected by roads are shown in the following weighted graph.



The shortest distance (in kilometers) of a trip starting from P, finishing at Q and visiting all the towns once *or more* is

A. 94

B. 95

C. 104

D. 105

E. 114

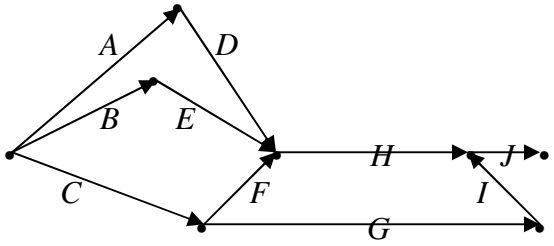
The following information relates to Questions 7, 8 and 9

A task consists of 10 activities *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I* and *J*. The time (hours) required completing an activity and the pre-requisite for it are shown in the following table.

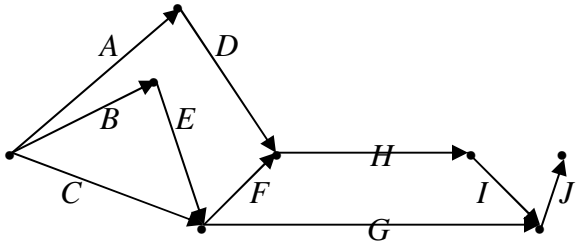
Activity	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Time required	2	5	6	9	11	15	16	17	20	8
Pre-requisite	-	-	-	*A*	*B*	*C, E*	*C*	*D, F*	*G*	*H, I*

Question 7 An activity graph for the task is

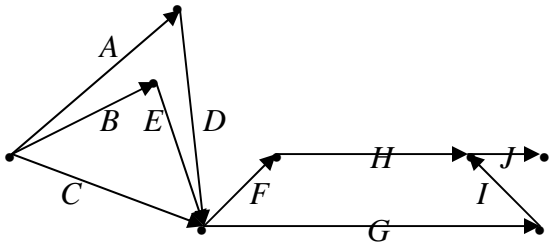
A.



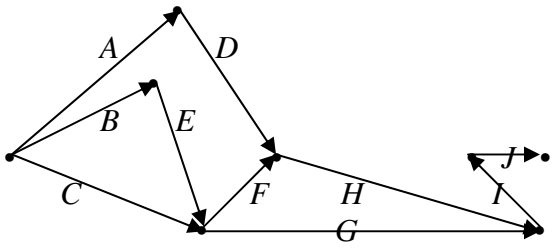
B.



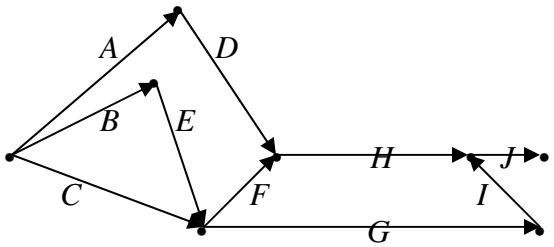
C.



D.



E.



Question 8 The critical path for the task is

A. *A-D-H-J*

B. *C-F-H-J*

C. *C-G-I-J*

D. *B-E-G-I-J*

E. *B-E-F-H-J*

Question 9 Which one of the following periods in hours is the float time for activity *D*?

A. 7

B. 14

C. 17

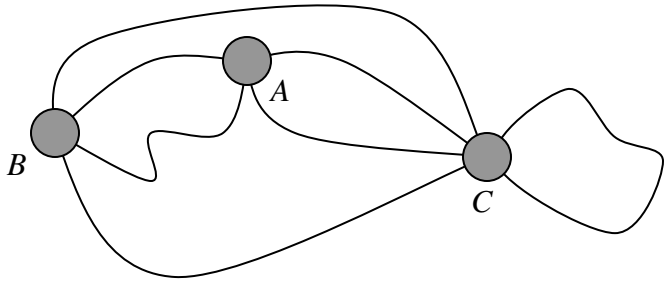
D. 24

E. 27

Module 6: Matrices

Question 1

Three towns are connected by roads as shown in the following diagram.



A matrix showing the number of possible routes from one town to another town is

A. $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 1 \end{bmatrix} \end{array}$

B. $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$

C. $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 2 & 2 & 1 \end{bmatrix} \end{array}$

D. $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \end{array}$

E. $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix} \end{array}$

The following information relates to Questions 2 and 3

A car dealer, who is not an expert working with matrices, sells 3 models of cars, *A*, *B* and *C*. He lists the current prices of the three models in dollars in matrix

$$P = \begin{matrix} A & B & C \end{matrix} \begin{bmatrix} 15000 & 22000 & 28000 \end{bmatrix}$$

Question 2

The car dealer intends to increase the price of each model by \$450, \$ 600 and \$750 respectively. The matrix giving the new prices is given by

A. $P + \begin{bmatrix} 450 \\ 600 \\ 750 \end{bmatrix}$

B. $P \times \begin{bmatrix} 450 \\ 600 \\ 750 \end{bmatrix}$

C. $\begin{bmatrix} 450 \\ 600 \\ 750 \end{bmatrix} \times P$

D. $\begin{bmatrix} 15000 \\ 22000 \\ 28000 \end{bmatrix} \begin{bmatrix} 450 & 600 & 750 \end{bmatrix}$

E. $P + \begin{bmatrix} 450 & 600 & 750 \end{bmatrix}$

Question 3

The car dealer decides to increase the price of each model by 3%, 2.5% and 2% respectively. The matrix giving the new prices is given by

A. $P + \begin{bmatrix} 0.03 & 0.025 & 0.02 \end{bmatrix}$

B. $P \times \begin{bmatrix} 0.03 & 0.025 & 0.02 \end{bmatrix}$

C. $\begin{bmatrix} 0.03 \\ 0.025 \\ 0.02 \end{bmatrix} \times P$

D. $P \times \begin{bmatrix} 1.03 \\ 1.025 \\ 1.02 \end{bmatrix}$

E. $P \times \begin{bmatrix} 1.03 & 0 & 0 \\ 0 & 1.025 & 0 \\ 0 & 0 & 1.02 \end{bmatrix}$

The following information relates to Questions 4 and 5

Consider the system of simultaneous equations

$$z - y + 2 = 0$$
$$w + x - 2y - 3 = 0$$
$$y - w - z = 0$$
$$x + 2z - w + 1 = 0$$

Question 4

The system of simultaneous equations can be solved by using matrix equation

A. $\begin{bmatrix} 0 & 1 & -1 & 2 \\ 1 & 1 & -2 & -3 \\ 1 & -1 & -1 & 0 \\ 1 & 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & -1 & 1 \\ 1 & 1 & -2 \\ -1 & 1 & -1 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 0 \\ -1 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 1 & -2 & 0 \\ -1 & 0 & 1 & -1 \\ -1 & 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 0 \\ -1 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 1 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ 1 & -2 & -1 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 0 \\ -1 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 1 & -2 & 0 \\ -1 & 0 & 1 & -1 \\ -1 & 1 & 0 & 2 \\ -2 & 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Question 5

Matrix $\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} =$

A. $\begin{bmatrix} 2 \\ 3 \\ 1 \\ 1 \end{bmatrix}$ **B.** $\begin{bmatrix} -2 \\ 3 \\ -1 \\ 1 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$ **D.** $\begin{bmatrix} -1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$ **E.** $\begin{bmatrix} 2 \\ 3 \\ 1 \\ -1 \end{bmatrix}$

The following information relates to Questions 6, 7, 8 and 9

A country town has a constant population of 5000 since 2010 and there are 3200 mobile-phone users in 2011. Of the 3200 mobile-phone users 45% are with Extratel, 31% with Ophone and 24% with Vocotel.

In 2011 surveyed consumers in the country town have indicated their preferences for 2012 as:

	Extratel	Ophone	Vocotel	Nonuser
Extratel	48%	32%	6%	18%
Ophone	24%	55%	33%	9%
Vocotel	13%	9%	47%	7%
Nonuser	15%	4%	14%	66%

That is, 48% of consumers who are with Extratel in 2011 will stay with Extratel in 2012. 24% will switch to Ophone, 13% will switch to Vocotel and 15% will not use a mobile-phone.

Question 6

An appropriate matrix to show the number of users with each company and the number of nonusers in 2011 is $N = \begin{bmatrix} 1440 \\ 992 \\ 768 \\ n \end{bmatrix}$.

The value of n is

A. 1300

B. 1800

C. 1900

D. 1950

E. 2000

Question 7

The number of mobile-phone users with Vocotel in 2012 will be closest to

A. 600

B. 640

C. 680

D. 720

E. 760

Question 8

The matrix giving the number of users with each company and the number of nonusers in **2012** is given by *TN*, where transition matrix *T* is

A. $\begin{bmatrix} 0.48 & 0.24 & 0.13 & 0.15 \\ 0.32 & 0.55 & 0.09 & 0.04 \\ 0.06 & 0.33 & 0.47 & 0.14 \\ 0.18 & 0.09 & 0.07 & 0.66 \end{bmatrix}$

B. $\begin{bmatrix} 0.48 & 0.32 & 0.06 & 0.18 \\ 0.24 & 0.55 & 0.33 & 0.09 \\ 0.13 & 0.09 & 0.47 & 0.07 \\ 0.15 & 0.04 & 0.14 & 0.66 \end{bmatrix}$

C. $\begin{bmatrix} 0.66 & 0.07 & 0.09 & 0.18 \\ 0.14 & 0.47 & 0.33 & 0.06 \\ 0.04 & 0.09 & 0.55 & 0.32 \\ 0.15 & 0.13 & 0.24 & 0.48 \end{bmatrix}$

D. $\begin{bmatrix} 0.66 & 0.14 & 0.04 & 0.15 \\ 0.07 & 0.47 & 0.09 & 0.13 \\ 0.09 & 0.33 & 0.55 & 0.24 \\ 0.18 & 0.06 & 0.32 & 0.48 \end{bmatrix}$

E. $\begin{bmatrix} 48 & 24 & 13 & 15 \\ 32 & 55 & 9 & 4 \\ 6 & 33 & 47 & 14 \\ 18 & 9 & 7 & 66 \end{bmatrix}$

Question 9

Assume the trend of consumer preferences for 2011 surveyed in 2010 was the same as that for 2012 surveyed in 2011, the total number of mobile-phone users in the country town in 2010 was closest to

A. 2700

B. 2800

C. 2900

D. 3000

E. 3100

End of Exam 1