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FURTHER MATHEMATICS TRIAL EXAMINATION 2 SOLUTIONS 2011

SECTION A

Core - solutions

Question 1

a. 9C has 14 students with a concession card.
Four classes have more than this (8C, 9B, 9D and 10A).
$\left(\frac{4}{16} \times \frac{100}{1}\right)\% = 25\%$

(1 mark)

b. The circled value is 19 so 8C is represented by this value.

(1 mark)

c. Using the ordered stemplot, the 8^{th} value is 7 and the 9^{th} value is 9. The median is 8.

(1 mark)

d. The shape is positively skewed.

(1 mark)

e. The median is a better measure of the centre of the distribution because the shape of the distribution is skewed.

(1 mark)

Question 2

a. $r = -0.842042$
$r = -0.84$ (correct to 2 decimal places)

(1 mark)

b. $r^2 = (-0.842042...)^2$
$= 0.709035...$
$= 71\%$ (to the nearest percent)

So 71% of the variation in the number of times a student is late to school can be explained by the variation in the distance between their home and school.

(1 mark)

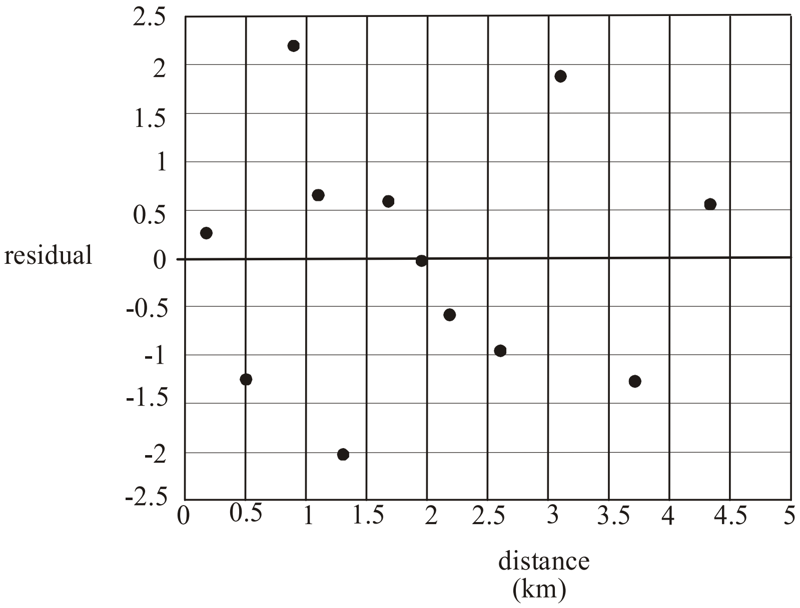
c. Use your calculator to find this

$\textit{number of times late} = -1.56 \times \textit{distance} + 13.04$

(1 mark) for – 1.56
(1 mark) for 13.04

d. Using part **c**, *number of times late* $= -1.56 \times 1.9 + 13.04 = 10.076$

$$\text{residual} = \text{actual value} - \text{predicted value}$$
$$= 10 - 10.076$$
$$= -0.076$$



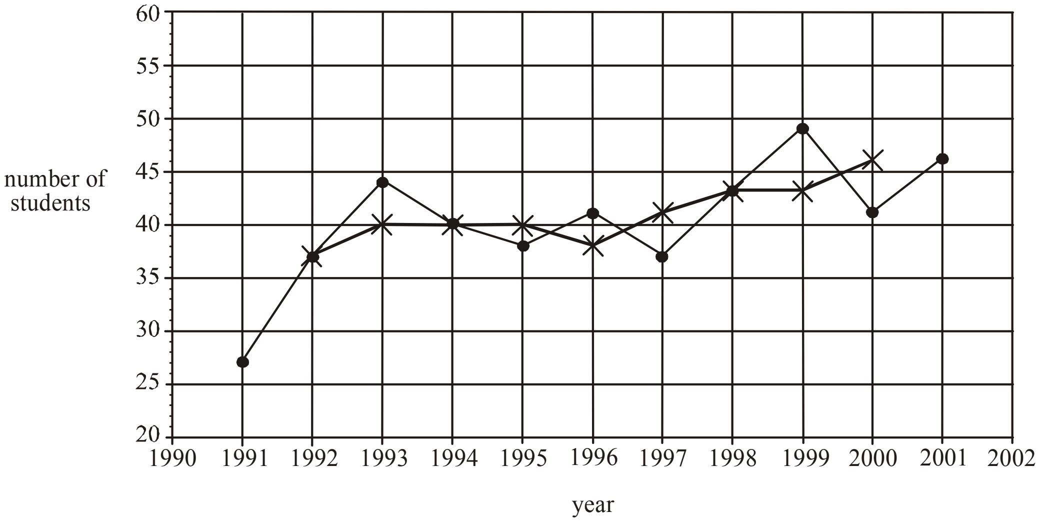
(1 mark)

e. The residual plot shows a collection of points scattered randomly around the horizontal line with a residual of zero; i.e. there is no clear pattern and hence the assumption of linearity is supported.

(1 mark)

Question 3

a.



(1 mark) 4 correctly placed crosses
(1 mark) correct smoothed time series plot

b. The *year* 2010 is referred to as *year number* 20.

So, $\text{number of students} = 1.57 \times 20 + 31.62$
$$= 63.02$$

Predicted number of students is 63 (correct to nearest whole number). **(1 mark)**

c. Average, annual increase is 1.57 or 2 (correct to nearest whole number).

(1 mark)

Total 15 marks

SECTION B

Module 1: Number patterns

Question 1

a. 55, 100, 145, 190,…

$t_2 - t_1 = 100 - 55 = 45$

$t_3 - t_2 = 145 - 100 = 45$

$t_4 - t_3 = 190 - 145 = 45$

There is a common difference of 45 between successive terms so the sequence is arithmetic.

(1 mark)

b. Method 1 – using a calculator

Generate the sequence. The 15th term is 685.

(1 mark)

Method 2 – by hand

$t_n = a + (n-1)d$ (formula sheet)

$t_{15} = 55 + 14 \times 45$

$= 685$

(1 mark)

c. Method 1 – using a calculator

Generate the sequence. The 29th term is \$1315 and the 30th term is \$1360.

Keisha can purchase a maximum of 29 tickets.

(1 mark)

Method 2 – by hand

$t_n = a + (n-1)d$ (formula sheet)

$1350 = 55 + (n-1) \times 45$

$1295 = (n-1) \times 45$

$\frac{1295}{45} = n - 1$

$n - 1 = 28.77...$

$n = 29.77...$

Keisha can purchase a maximum of 29 tickets.

(1 mark)

d. The difference equation defines an arithmetic sequence so $a = 1$ and $b = 45$.

(1 mark) for 1
(1 mark) for 45

Question 2

a. Method 1 – using a calculator
Generate the sequence.
65, 58.5, 52.65, 47.39, 42.65…
The cost of the 5th ticket is \$42.65.

(1 mark)

Method 2 – by hand
The sequence is geometric so

$$t_n = ar^{n-1}$$

$$\begin{aligned} t_5 &= 65 \times 0.9^4 \\ &= \$42.65 \end{aligned}$$

(1 mark)

b. Method 1 – using a calculator
Generate the sequence.
The 12th term is 20.40.
The 13th term is 18.36.
So 13 tickets must be purchased before the cost of a ticket drops below \$20.

(1 mark)

Method 2 – using CAS

$$t_n = ar^{n-1}$$

$$20 = 65 \times 0.9^{n-1}$$

Solve for n. So $n = 12.1869$.
So 13 tickets must be purchased before the cost drops below \$20.

(1 mark)

c. The sequence is a decreasing geometric sequence (i.e. $r < 1$) so

$$\begin{aligned} S_\infty &= \frac{a}{1-r} \quad \text{(formula sheet)} \\ &= \frac{65}{1-0.9} \\ &= 650 \end{aligned}$$

The maximum amount that can be spent is \$650.

(1 mark)

d. From Ticketget, we need to find the 8th term of the arithmetic sequence because the nth term gives us the cost of purchasing n tickets.

$$\begin{aligned} t_8 &= 55 + 7 \times 45 \\ &= \$370 \end{aligned}$$

(1 mark)

From Dodgeybros, we need to find the sum of the first 8 terms since the nth term gives us the cost of the nth ticket.

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{(formula sheet)}$$

$$\begin{aligned} S_8 &= \frac{65(1-0.9^8)}{0.1} \\ &= \$370.20 \end{aligned}$$

So Keisha saves \$0.20 by purchasing from the Ticketget website.

(1 mark)

e. The cost of each ticket on the Ticketget website is \$45. On the Dodgeybros website the cost of the 9th ticket is $t_9 = 65(0.9)^8 = \$27.98$.
The cost of subsequent tickets is now cheaper with Dodgeybros so Keisha should purchase them from the Dodgeybros website. **(1 mark)**

Question 3

a. Method 1 – using a calculator
Generate the sequence.
4200, 4204, 4208.08, 4212.24
The number of tickets predicted to be sold in the fourth month is 4212.

(1 mark)

Method 2 – by hand
$S_1 = 4200$
$S_2 = 1.02 \times 4200 - 80 = 4204$
$S_3 = 1.02 \times 4204 - 80 = 4208.08$
$S_4 = 1.02 \times 4208.08 - 80 = 4212.24$
The number of tickets predicted to be sold in the fourth month is 4212.

(1 mark)

b. Using the results from either method in part **a.**,

$$\frac{S_2}{S_1} = \frac{4204}{4200} = 1.00095$$

$$\frac{S_3}{S_2} = \frac{4208.08}{4204} = 1.00097$$

Since $\frac{S_2}{S_1} \neq \frac{S_3}{S_2}$ the sequence does not have a common ratio and is therefore not geometric.

(1 mark)

c. Method 1 – using a calculator
Generate the sequence.
$S_{n+1} = 1.02S_n - 80,\ S_1 = 4000$
4000, 4000, 4000,…
The monthly sales remain on 4000 so there is no growth. **(1 mark)**
Generate the sequence.
$S_{n+1} = 1.02S_n - 80,\ \ S_1 = 3990$
(i.e. choose a value less than 4000)
3990, 3989.8, 3989.6, …
The monthly sales are decreasing so there is no growth.

(1 mark)

Method 2 – by hand

If $S_1 = 4000$

$S_{n+1} = 1.02S_n - 80$

becomes $S_2 = 1.02 \times 4000 - 80$

$= 4000$

So $S_3 = 4000$ and so on.

The monthly sales remain on 4000 so there is no growth. **(1 mark)**

If $S_1 = 3990$

$S_{n+1} = 1.02S_n - 80$

becomes $S_2 = 1.02 \times 3990 - 80$

$= 3989.8$

So $S_3 = 1.02 \times 3989.8 - 80$

$= 3989.6$ and so on

The monthly sales are deceasing so there is no growth. **(1 mark)**

Total 15 marks

Module 2: Geometry and trigonometry

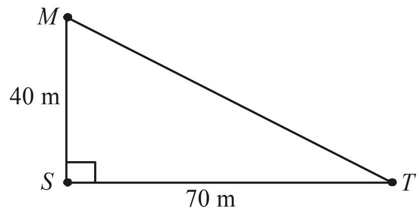
Question 1

a. In ΔMST,

$MT = \sqrt{70^2 + 40^2}$ (Pythagoras Theorem)

$= 80.622...$

The distance is 80.6m correct to 1 decimal place.



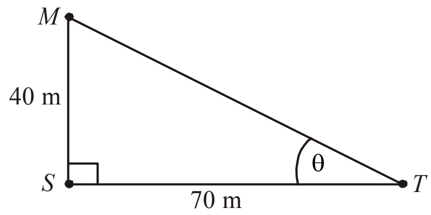
(1 mark)

b. In ΔMST,

$\tan\theta = \dfrac{40}{70}$

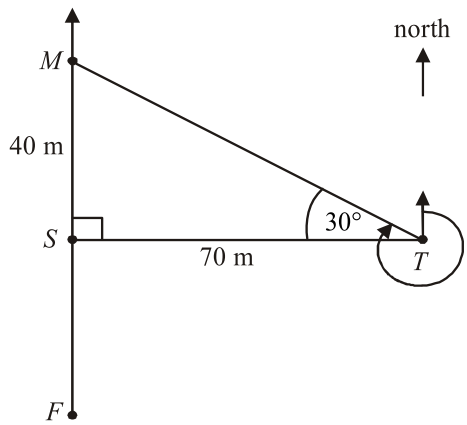
$\theta = 29.7449...^\circ$

So $\angle MTS = 30^\circ$ to the nearest whole degree.



(1 mark)

c. The bearing of *M* from *T* is $270^\circ + 30^\circ = 300^\circ$.



(1 mark)

d. Since the bearing of *F* from *T* is 230°, $\angle FTS = 40^\circ$. **(1 mark)**

So in ΔFST,

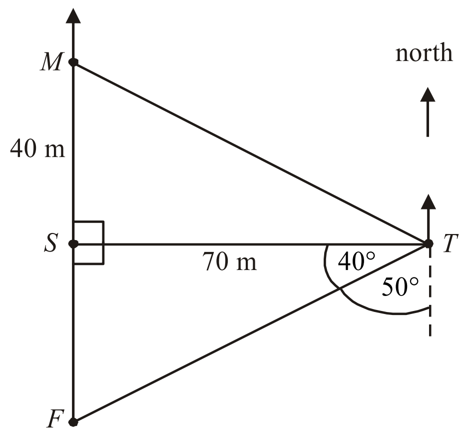
$\tan(40^\circ) = \dfrac{FS}{70}$

$70 \times \tan 40^\circ = FS$

$FS = 58.737...$

The distance from *F* to *S* is 58.7m correct to 1 decimal place.

(1 mark)



Question 2

a. $\dfrac{\sin(\angle ACB)}{80} = \dfrac{\sin 65^\circ}{110}$

$\sin(\angle ACB) = \dfrac{\sin 65^\circ}{110} \times 80$

$\angle ACB = 41.23^\circ$ (correct to 2 decimal places)

(1 mark)

b. Since $\angle ACB = 41.23^\circ$,

$\angle ABC = 180^\circ - 65^\circ - 41.23^\circ$

$= 73.77^\circ$

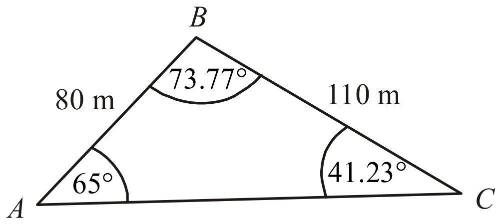
So $(AC)^2 = (AB)^2 + (BC)^2 - 2 \times (AB) \times (BC) \times \cos 73.77^\circ$ (cosine rule)

$= 13\,580.9...$

$AC = \sqrt{13\,580.9...}$

$= 116.537...$

So $AC = 116.5$ m (correct to 1 decimal place).



(1 mark)

c.

area of a triangle $= \dfrac{1}{2} bc \sin A$ (formula sheet)

$= \dfrac{1}{2} ac \sin B$ (in our case)

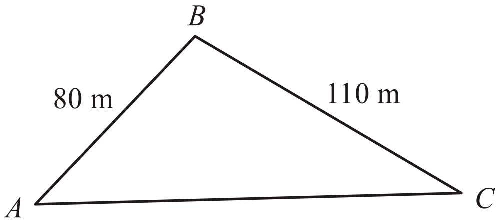
$= \dfrac{1}{2} \times 110 \times 80 \times \sin 73.77^\circ$

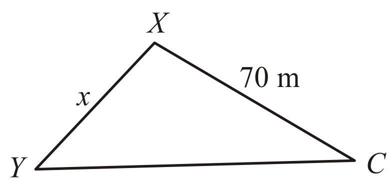
$= 4224.64888...$

$= 4224.6 \text{m}^2$ (correct to one decimal place)

(1 mark)

d. Use similar triangles.





$\dfrac{x}{80} = \dfrac{70}{110}$

$x = 50.9090...$

$XY = 50.9$ metres (correct to one decimal place)

(1 mark) an attempt using similar triangles
(1 mark) correct answer

Question 3

$$\begin{aligned}\text{Volume of prism} &= \text{area of cross-section} \times \text{length}\\ &= \left(\frac{1}{2}\times 1\times 1+3\times 1+\frac{1}{2}\times 2\times 1\right)\times 5\\ &= \left(\frac{1}{2}+3+1\right)\times 5\\ &= \frac{9}{2}\times 5\\ &= \frac{45}{2}\end{aligned}$$

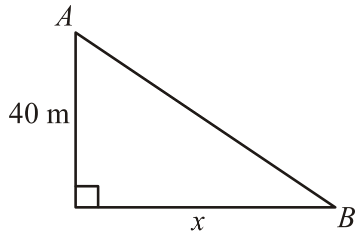
So 22.5 cubic metres of cement is required.

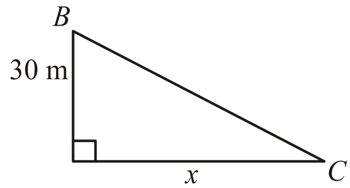
(1 mark) – attempt to find area of cross-section
(1 mark) – correct answer

Question 4

a. The horizontal distance between the towers (marked as x in the diagrams to the right) is equal. From the contour lines, point A is 40m vertically above point B and point B is 30m vertically above point C.

Since $\text{gradient} = \frac{\text{rise}}{\text{run}}$, the section of cable between A and B has the steeper gradient.

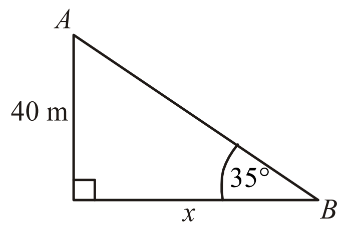




(1 mark)

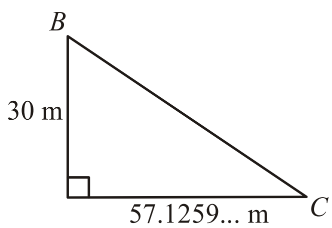
b.

$$\begin{aligned}\tan 35^\circ &= \frac{40}{x}\\ x\times\tan 35^\circ &= 40\\ x &= \frac{40}{\tan 35^\circ}\\ &= 57.1259...\end{aligned}$$



(1 mark)

$$\begin{aligned}(BC)^2 &= 30^2+(57.1259...)^2\\ &= 4163.37...\\ BC &= \sqrt{4163.37...}\\ &= 64.5242...\end{aligned}$$



So the length of cable is 64.5m (correct to one decimal place).

(1 mark)
Total 15 marks

Module 3: Graphs and relations

Question 1

a. 6.1m

(1 mark)

b. 6pm

(1 mark)

c. Between 12pm and 8pm the height of the river was increasing.

$\frac{8}{12}=\frac{2}{3}$

The height of the river was increasing for $\frac{2}{3}$ of the 12 hour period. **(1 mark)**

d.

$$\begin{aligned}\text{average decrease} &= \frac{6.6-6.55}{4}\\ &= \frac{0.05}{4}\\ &= 0.0125 \text{ metres/hour}\end{aligned}$$

(1 mark)

Question 2

a. $R = 95n$

(1 mark)

b. $C = 320 + 55n$

At break-even,

$$\begin{aligned}R &= C\\ 95n &= 320 + 55n\\ 40n &= 320\\ n &= 8\end{aligned}$$

(1 mark)

c. When 15 cabins are occupied,

$$\begin{aligned}R &= 95 \times 15 = 1425\\ C &= 320 + 55 \times 15 = 1145\end{aligned}$$

$$\begin{aligned}\text{Profit} &= \text{Revenue} - \text{Cost}\\ &= 1425 - 1145 = \$280\end{aligned}$$

(1 mark)

d.

$$\begin{aligned}\text{Profit} &= \text{Revenue} - \text{Cost}\\ -200 &= 95n - (320 + 55n)\\ -200 &= 95n - 320 - 55n\\ 120 &= 40n\\ n &= \frac{120}{40}\\ n &= 3\end{aligned}$$

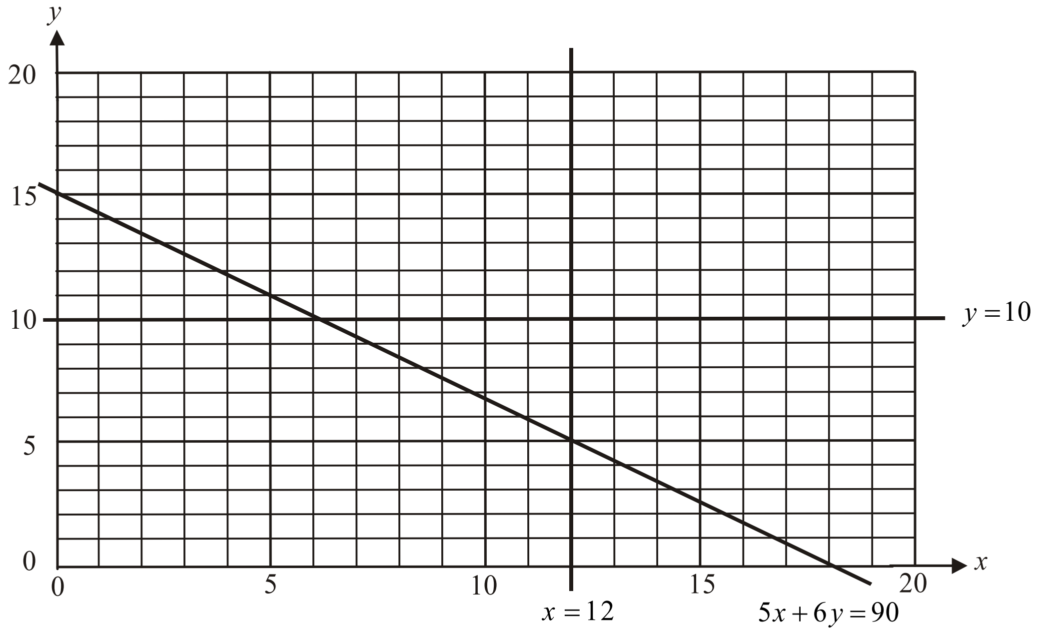
(1 mark)

Question 3

a. The inequality $5x+6y\le 90$ tells us that for every on-site-van that is rented overnight a maximum of 6 people can stay in it.

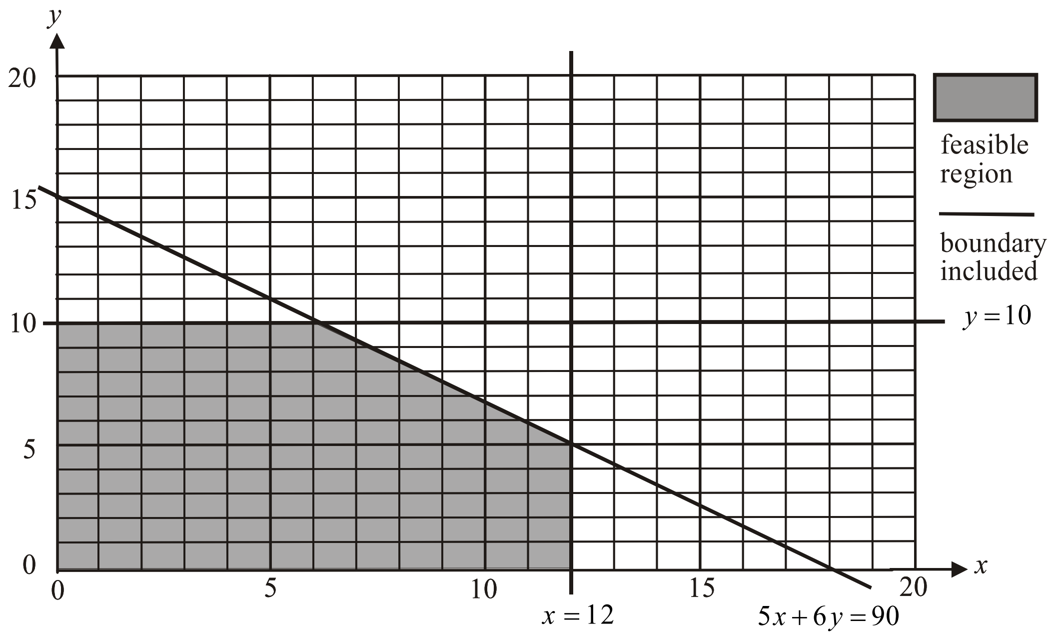
(1 mark)

b.



(1 mark)

c.



(1 mark)

d. $P = 40x + 50y$

(1 mark)

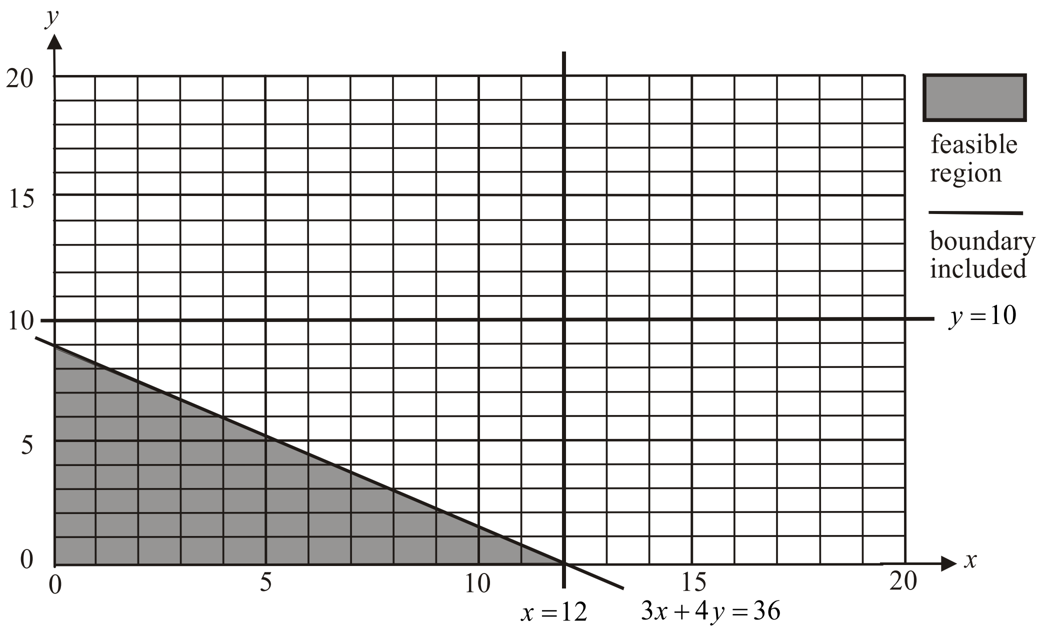
From the graph, the corner points of the feasible region are

$(0,0)$ where $P = 40 \times 0 + 50 \times 0 = 0$
$(12,0)$ where $P = 40 \times 12 + 50 \times 0 = 480$
$(12,5)$ where $P = 40 \times 12 + 50 \times 5 = 730$
$(6,10)$ where $P = 40 \times 6 + 50 \times 10 = 740$
$(0,10)$ where $P = 40 \times 0 + 50 \times 10 = 500$

The maximum profit is $740.

(1 mark)

e. The inequality $5x + 6y \le 90$ is replaced with $3x + 4y \le 36$. The revised feasible region is shown below.



From part **d.**, $P = 40x + 50y$.
From the graph above, the corner points of the feasible region are

$(0,0)$ where $P = 40 \times 0 + 50 \times 0 = 0$
$(12,0)$ where $P = 40 \times 12 + 50 \times 0 = 480$
$(0,9)$ where $P = 40 \times 0 + 50 \times 9 = 450$

The maximum profit now possible is $480.

(1 mark) – correct corner points
(1 mark) – correct answer
Total 15 marks

Module 4: Business-related mathematics

Question 1

a. $$\text{percentage discount} = \left(\frac{28\,000 - 24\,500}{28\,000} \times \frac{100}{1}\right)\%$$
$$= 12.5\%$$

(1 mark)

b. 10% of $\$24\,500 = \$2450$

(1 mark)

Question 2

a. 6% of $\$42\,000 = \$2\,520$

After 5 years its book value
$= \$42\,000 - 5 \times \$2\,520$
$= \$29\,400$

(1 mark)

b. $$\text{book value} = 42\,000 \times \left(1 - \frac{7.2}{100}\right)^5$$
$$= \$28\,906.10$$

(1 mark)

c. $\$42\,000 - \$10\,000 = \$32\,000$

$\$32\,000 \div 0.20 = 160\,000$

It will have been driven 160 000km.

(1 mark)

Question 3

a.
$$SI = \frac{PrT}{100} \quad \text{(formula sheet)}$$
$$= \frac{12\,000 \times 4 \times 3}{100}$$
$$= 1\,440$$

The investment is worth $\$12\,000 + \$1\,440 = \$13\,440$ after 3 years.

(1 mark)

b.
$$A = PR^n \qquad \text{where} \quad R = 1 + \frac{r}{100} \text{ (formula sheet)}$$
$$= 12\,000 \times 1.04^3 \qquad = 1 + \frac{4}{100}$$
$$= 13\,498.37 \qquad = 1.04$$

The investment is worth \$13 498.37 after 3 years. **(1 mark)**

c. The interest needs to be calculated over a shorter period of time; for example, quarterly or monthly or weekly in order to add to the value of the investment.

(1 mark)

For example, if interest is calculated quarterly,
$$A = PR^n \qquad \text{where} \quad R = 1 + \frac{r}{100}$$
$$= 12\,000 \times 1.01^{12} \qquad = 1 + \frac{1}{100}$$
$$= 13\,521.90 \qquad = 1.01$$

This amount is greater than 13 498.37 which was found in part **b.** **(1 mark)**

Question 4

a. Use *TVM*

N ?

$I(\%)$ 6

PV $-64\,000$

PMT 2000

FV 0

P/Y 12

C/Y 12

$N = 34.9577...$

The annuity lasts for 35 months (to the nearest whole month). **(1 mark)**

b. Use *TVM* – where 5 years is 60 months

N 60

$I(\%)$ 6

PV $-64\,000$

PMT ?

FV 0

P/Y 12

C/Y 12

$PMT = 1237.299...$

The monthly payment would be \$1237.30. **(1 mark)**

Question 5

a. Use *TVM*

$N: 20$

$I(\%): 5.2$

$PV: 180\,000$

$PMT: -5799.49$

$FV: ?$

$P/Y: 4$

$C/Y: 4$

After 5 years the principal still owed is \$101 560.35.

(1 mark)

b. Use *TVM*.

$N: 60$

$I(\%): 5.2$

$PV: 180\,000$

$PMT: ?$

$FV: 0$

$P/Y: 4$

$C/Y: 4$

The quarterly payment will be \$4339.09.

(1 mark)

c. Over 10 years, the amount paid back is $40 \times \$5799.49 = \$231\,979.60$

$\text{Interest} = \$231\,979.60 - \$180000$

$= \$51\,979.60$

(1 mark)

Over 15 years, the amount paid back is $60 \times \$4339.09 = \$260345.40$

$\text{Interest} = \$260\,345.40 - \$180000$

$= \$80345.40$

The couple will save $\$80\,345.40 - \$51\,979.60 = \$28\,365.80$ in interest.

(1 mark)

Total 15 marks

Module 5: Networks and decision mathematics

Question 1

a. Shortest route is 1050 metres.

(1 mark)

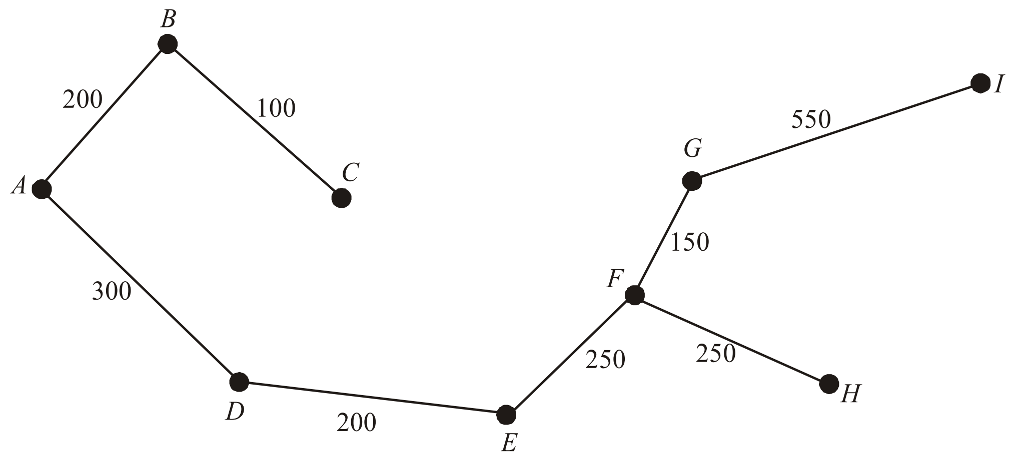
b. *A B C D E H F G I*
or *A B C G F D E H I*
or *A D C B G F E H I*
There are others. A Hamiltonian path starts at one vertex, finishes at another and passes through all the other vertices just once.

(1 mark)

c. The vertices that have an odd degree are *A* and *I* (which are the start and end points of the path) and *E* and *H*. The path between *E* and *H* needs to be removed.

(1 mark)

d. We are looking for a minimal spanning tree.



(1 mark)

Question 2

a. There are no paths that leave a performing area and return to the same performing area, that is, there are no loops.

(1 mark)

b. $m = 1, \quad n = 0$

(1 mark)

Question 3

a.

	Performing area			
	B	*C*	*D*	*E*
Kev	15	3	15	0
Pete	10	0	13	0
Lucy	20	0	10	2
Kiran	22	4	14	0

Step 1 -Subtract from each element in a row, the minimum value for that row.

(1 mark)

b. Step 2 – Repeat this process to columns *B* and *D* which have no zeros.

	Performing area			
	B	*C*	*D*	*E*
Kev	5	3	5	0
Pete	0	0	3	0
Lucy	10	0	0	2
Kiran	12	4	4	0

Step 3 – Cover the zeros with as few lines as possible, in this case 3 (ie cover column E and Pete and Lucy's rows).

Step 4 – The minimum uncovered entry is 3.
Add this to the covered rows and columns.

	Performing area			
	B	*C*	*D*	*E*
Kev	5	3	5	3
Pete	3	3	6	6
Lucy	13	3	3	8
Kiran	12	4	4	3

Step 5 – Subtract 3 from all entries.
Cover all zeros with as few lines as possible, in this case 4.

	Performing area			
	B	*C*	*D*	*E*
Kev	2	0	2	0
Pete	0	0	3	3
Lucy	10	0	0	5
Kiran	9	1	1	0

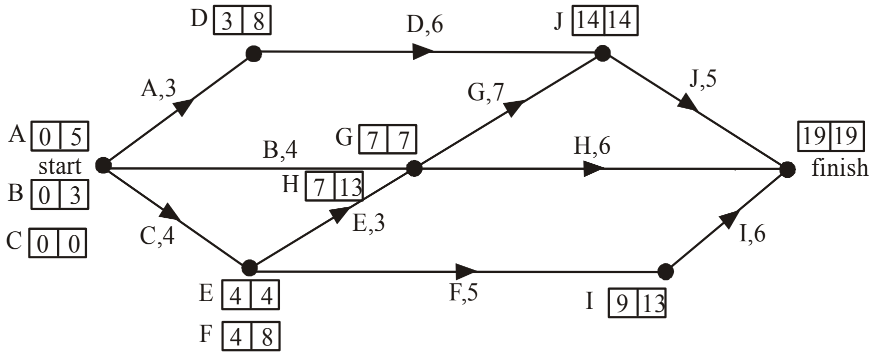
(1 mark)

Because we needed 4 lines to cover all the zeros, we can now allocate.
Kiran can only be allocated to performing area *E*.

(1 mark)

Question 4

The network described by the table is given below.



Note that *A*, *B* and *C* have no immediate predecessors so they appear at the start and *H*, *I* and *J* are not immediate predecessors to any activities so they appear at the finish.
The EST and LST for each activity is shown for each activity.

a. The EST for activity *G* is 7 days.

(1 mark)

b. The LST for activity *H* is 13 days.

(1 mark)

c. The float time for activity *F* is 4 days.

(1 mark)

d. These activities lie on the critical path and are *C, E, G, J*.

(1 mark)

e. The shortest time possible is given by the length of the critical path which is $4+3+7+5=19$ days.

(1 mark)

f. Activity *B* had a float time of 3 days. It now becomes a critical activity and extends the shortest time possible to set up the infrastructure by 1 day. It will now take 20 days.

(1 mark)
Total 15 marks

Module 6: Matrices

Question 1

a. The order of M is 1×4.

(1 mark)

b.

$$C = \begin{bmatrix} 200 \\ 120 \\ 80 \\ 100 \end{bmatrix}$$

(1 mark)

c.

$$MC = \begin{bmatrix} 140 & 38 & 93 & 42 \end{bmatrix} \begin{bmatrix} 200 \\ 120 \\ 80 \\ 100 \end{bmatrix}$$

$$= \begin{bmatrix} 140 \times 200 + 38 \times 120 + 93 \times 80 + 42 \times 100 \end{bmatrix}$$

$$= \begin{bmatrix} 44\,200 \end{bmatrix}$$

(1 mark)

d. The matrix product MC represents the revenue for the gym made in a month from membership fees.

(1 mark)

Question 2

a. 3 guest passes were sold last month for mainstream zumba classes.

(1 mark)

b.

$$\begin{bmatrix} 7 & 8 & 10 \\ 4 & 5 & 3 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 440 \\ 209 \\ 103 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 & 8 & 10 \\ 4 & 5 & 3 \\ 2 & 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 440 \\ 209 \\ 103 \end{bmatrix}$$

(1 mark)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 18 \end{bmatrix}$$

The cost of a guest pass to a cardio class is \$20, to a jump class is \$15 and to a zumba class is \$18.

(1 mark)

Question 3

a. 30% of people change each week from a lunchtime class to an evening class.

(1 mark)

b.

$$S_2 = TS_1$$
$$= \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.1 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.8 \end{bmatrix} \begin{bmatrix} 30 \\ 20 \\ 40 \end{bmatrix}$$
$$= \begin{bmatrix} 26 \\ 17 \\ 47 \end{bmatrix}$$

There will be 47 people in the evening yoga class in week 2.

(1 mark)

c.

$$S_4 = T \times T \times T \times S_1$$
$$= \begin{bmatrix} 22.43 \\ 15.32 \\ 52.25 \end{bmatrix}$$

There will be 15 people (to the nearest whole number) in the lunchtime class in week 4.

(1 mark)

d. We are looking at the long term so we see if we can find a steady state.

Now, $S_{10} = T^9 \times S_1$

$$= \begin{bmatrix} 21.026 \\ 15.0013 \\ 53.9727 \end{bmatrix}$$

$$S_{15} = T^{14} \times S_1$$
$$= \begin{bmatrix} 21.0008 \\ 15 \\ 53.9991 \end{bmatrix}$$

Correct to 1 decimal place a steady state has been reached.
Over the long term, the minimum class size is 15 so no classes will be discontinued.

(1 mark) finding a steady state
(1 mark) – answering the question

Question 4

a.

$$
\begin{aligned}
N_2 &= AN_1 + B \\
&= \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 32 \\ 24 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} 0.5 \times 32 + 0 \times 24 \\ 0 \times 32 + 1 \times 24 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} 16 \\ 24 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} 20 \\ 26 \end{bmatrix}
\end{aligned}
$$

(1 mark)

b.

$$N_{n+1} = AN_n + B$$

So,

$$
\begin{aligned}
N_3 &= AN_2 + B \\
&= \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 26 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} 0.5 \times 20 + 0 \times 26 \\ 0 \times 20 + 1 \times 26 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} 10 \\ 26 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} 14 \\ 28 \end{bmatrix}
\end{aligned}
$$

(1 mark)

$$
\begin{aligned}
N_4 &= AN_3 + B \\
&= \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 14 \\ 28 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} 0.5 \times 14 + 0 \times 28 \\ 0 \times 14 + 1 \times 28 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} 7 \\ 28 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} 11 \\ 30 \end{bmatrix}
\end{aligned}
$$

The difference in the size of the two classes in week 4 is 19.

(1 mark)
Total 15 marks