2010

VCE

Further Mathematics

Trial Examination 2

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PURPOSE OF THIS TRIAL EXAMINATION

This Further Mathematics Trial Examination is designed to assess

- understanding and communication of mathematical ideas
- interpretation, analysis and solution of routine problems
- interpretation, analysis and solution of non-routine problems

Assessment is by extended answer questions involving multi-stage solutions of increasing complexity.

IMPORTANT COPYRIGHT NOTICE

STUDENT NUMBER

										Letter
Figures										
Words										

VICTORIAN CERTIFICATE OF EDUCATION 2010

FURTHER MATHEMATICS

Trial Written Examination 2 (Analysis task)

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Core

Number of questions	Number of questions to be answered
3	3

Modules

Number of modules	Number of modules to be answered
6	3

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator.
Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 38 pages.
- Working space is provided throughout the book.
- There is a detachable sheet of miscellaneous formula supplied.

Instructions

- Detach the formula sheet from the book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \frac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index $= \frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a+d) + \ldots + (a+(n-1)d) = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\frac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\frac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\frac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{\Pr T}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

annuities: $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$, where $R = 1 + \frac{r}{100}$

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET

Specific Instructions

This task paper consists a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

Core: Data analysis

Question 1

Table 1

Batting Averages	
Male	**Female**
66	66
44	37
42	32
28	28
27	24
26	21
24	19
22	16
16	7
14	3
10	3
5	2

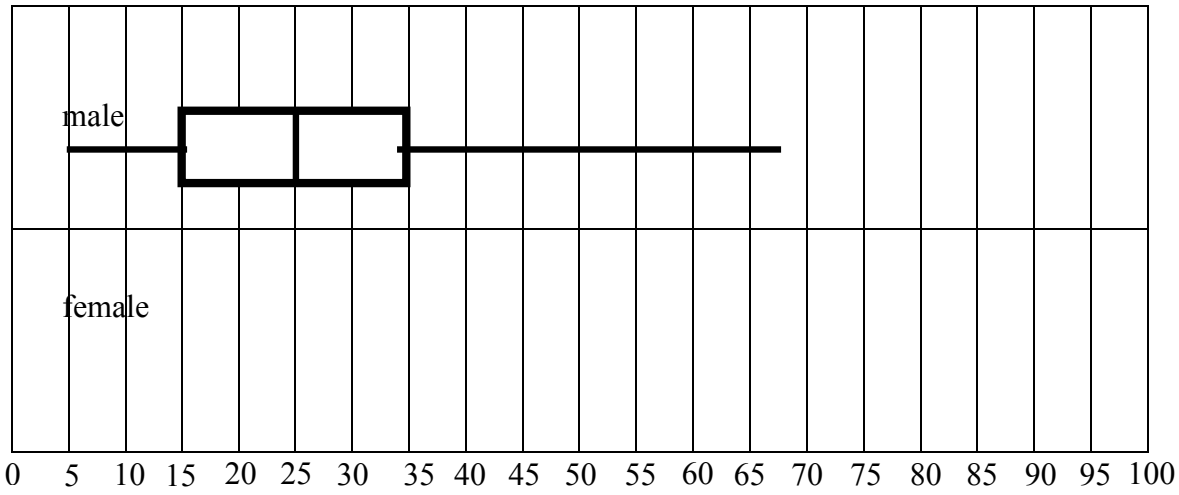
The above table shows the batting averages of a male and female cricket team.

a. What is the mean for the female team's batting averages?

_____

1 mark

b. In the space provided, draw the box plot for the batting averages for the female team.



2 marks

Core: Data analysis

Question 1 (continued)

c. The highest batting average for both male and female players is 66. Are these both outliers? Give a reason for your answer.

_____

_____

_____

_____

_____

_____

2 marks

Question 2

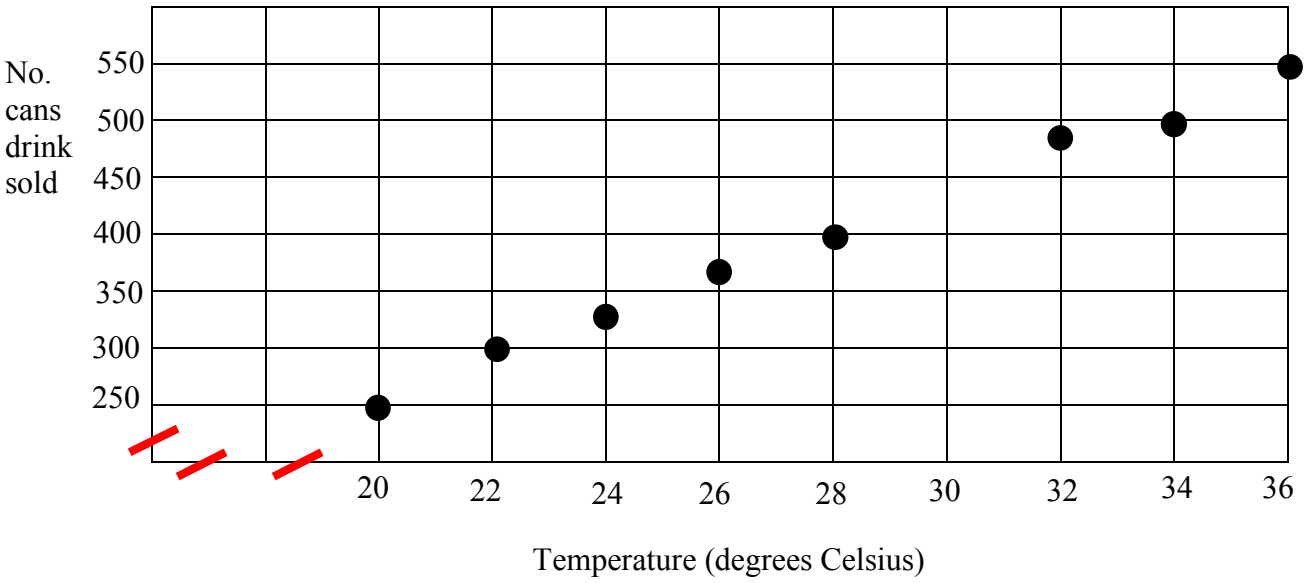
Table 2

Temperature (Celsius)	Number cans of drink sold
20	250
22	300
24	330
26	360
28	400
30	**350**
32	470
34	500
36	550

The above table gives the number of cans of drink sold at the cricket, compared with the temperature of the day, and the following is a scatter plot of this data.

Core: Data analysis

Question 2 (continued)



a. On the above scatter plot mark the point that has been omitted with an **X.**

1 mark

b. Using the data from table 2, complete the following sentence.
Give the percentage to one decimal place.

	%	of the variation in	

can be explained by the variation in

3 marks

Core: Data analysis

c. Complete the equation for the least squares regression line for this data.

Number of drinks sold = ☐ + ☐ × *temperature*

2 marks

d. From the equation for the regression line, what would you expect the temperature to be on a day when 440 drinks were sold? Give your answer to one decimal place.

_____

_____

_____

1 mark

Core: Data analysis

Question 3

Table 3

Temperature ^{0}C	20	22	24	26	28	30	32	34	36
No. Hot dogs sold	130	120	111	101	97	92	87	83	79

Table 3 gives the number of hot dogs sold at the cricket, compared with the temperature of that day. Since this data is not linear, it is decided to linearise the relationship by using the transformation $\frac{1}{\text{independent variable}}$

a. What would the transformed value be when the temperature is 26^0C?
Give your answer to 2 decimal places.

_____

1 mark

b. What is the equation for the least squares regression line for the transformed data?
Give all values to the nearest whole number.

_____

_____

1 mark

c. How many hot dogs would you expect to be sold if the temperature dropped to 15^0C?

_____

_____

_____

1 mark

Total = 15 marks

End of Core: Data analysis

Module 1 : Number patterns and applications

If you choose this module, all questions are to be answered.

Question 1

Bacteria *A* spreads through a town infecting 25 people the first day. After that, the number of people infected each day increases by 12.

a. i. How many people were infected by bacteria *A* on day 3?

_____

_____

_____

1 mark

a. ii. How many people were infected by bacteria *A* on day *n*?
Give your answer in terms of *n*.

_____

_____

_____

_____

1 mark

Module 1: Number patterns and applications

Question 1 (continued)

b. How many people will have been infected by the end of day 16?

_____

_____

_____

_____

_____

1 mark

Question 2

Bacteria *B* spreads through another town, infecting 5 people on the first day. After this, the number of people infected each day by bacteria *B* increases by 40 %.

a. i. How many people were infected by bacteria *B* on day 2?

_____

_____

_____

1 mark

a. ii. How many people were infected by bacteria *B* on the *n* th day?
Give your answer in terms of *n*.

_____

_____

_____

1 mark

Module 1: Number patterns and applications

Question 2 (continued)

b. How many people will have been infected by bacteria *B* by the end of the 10th day? Give your answer to the nearest whole number.

_____

_____

_____

1 mark

c. What is the least number of days required for the number of people infected by bacteria *B* on a particular day to be greater than the number of people infected by bacteria *A* on that particular day?

_____

_____

_____

1 mark

d. Bacteria *C* spreads through another town infecting 25 people the first day, 40 people the next day, and after that the number of people infected each day increases by 20% of the number infected the previous day. How many people will have been infected by bacteria *C* by the end of the 20^{th} day?

_____

_____

_____

_____

_____

_____

1 mark

Module 1: Number patterns and applications

Question 3

a. The number of bacteria being grown on an agar plate in a laboratory double every 20 minutes. If there are initially 4 bacteria on the plate, how many bacteria will be on the plate

i. in 20 minutes?

_____

1 mark

ii. in 1 hour?

_____

1 mark

b. How many times greater than the original 4 bacteria is the number of bacteria after one hour?

_____

1 mark

c. Complete the following difference equation that gives the number of bacteria after t hours?

$B_t =$ ☐ $B_0 = 4$

1 mark

d. How many bacteria would there be after 5 hours?

_____

_____

_____

1 mark

Module 1: Number patterns and applications

Question 3 (continued)

e. What is the minimum number of hours it would take for the number of bacteria to reach at least 16,000?

_____

_____

1 mark

f. After 4 hours of growth, the scientist sprays the bacteria with a new antibiotic and observes that 1200 bacteria are killed every 20 minutes. How many hours will it take for the bacteria to all be destroyed? Give your answer to one decimal place.

_____

_____

_____

1 mark

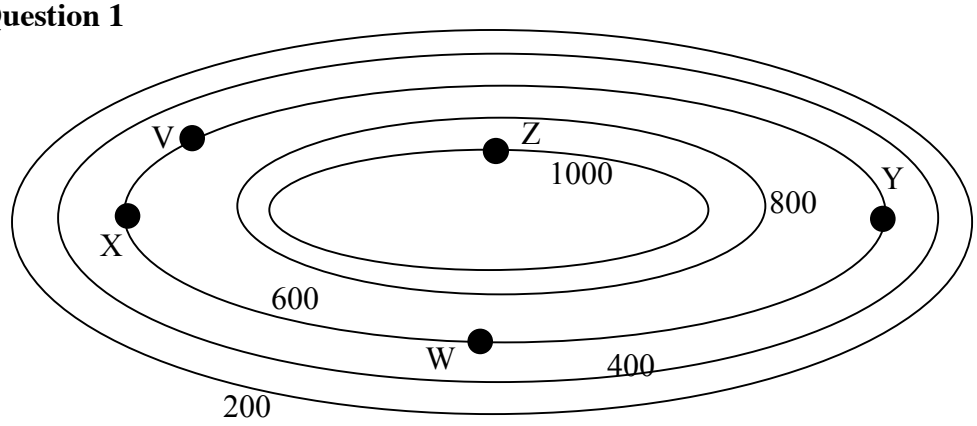
Total = 15 marks

End of Module 1: Number patterns and applications

Module 2: Geometry and trigonometry

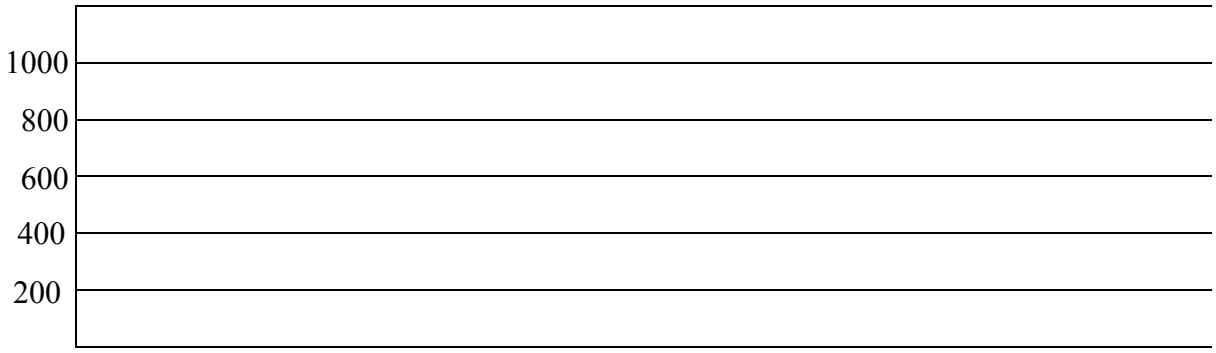
If you choose this module, all questions are to be answered.

Question 1



The above is a contour map showing five towns, V, W, X, Y, Z.
All measurements are given in metres.

a. On the axes below, draw a cross section from X to Y



1 mark

Module 2: Geometry and trigonometry

Question 1 (continued)

b. What is the gradient from W to Y?

_____

_____

1 mark

c. If Z is 1 kilometre from W, what is the angle of depression from a fire lookout at Z to a fire lookout of the same height at W? Give your answer to the nearest degree.

_____

_____

_____

_____

_____

_____

1 mark

d. What is the gradient from W to Z? Give your answer to 2 decimal places.

_____

_____

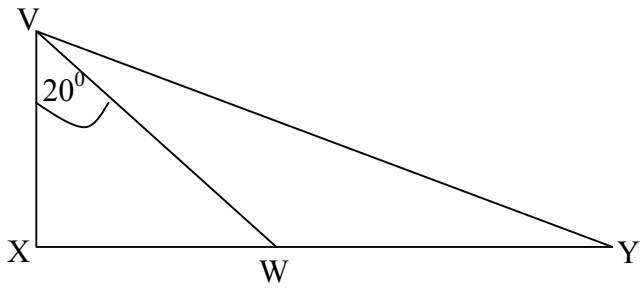
_____

1 mark

Module 2: Geometry and trigonometry

Question 2

The towns, V, X, W and Y can be represented in the following diagram where V is due north from X and Y is due east from X. W is equidistant from V and Y.



a. If $\angle XVW = 20^0$, find $\angle VWY$.

_____

_____

_____

1 mark

b. Show that $\angle WVY = 35^0$.

_____

_____

_____

_____

1 mark

Module 2: Geometry and trigonometry

Question 2 (continued)

c. What is the true bearing of V from W?

_____

_____

_____

_____

1 mark

d. What is the true bearing of Y from V?

_____

_____

_____

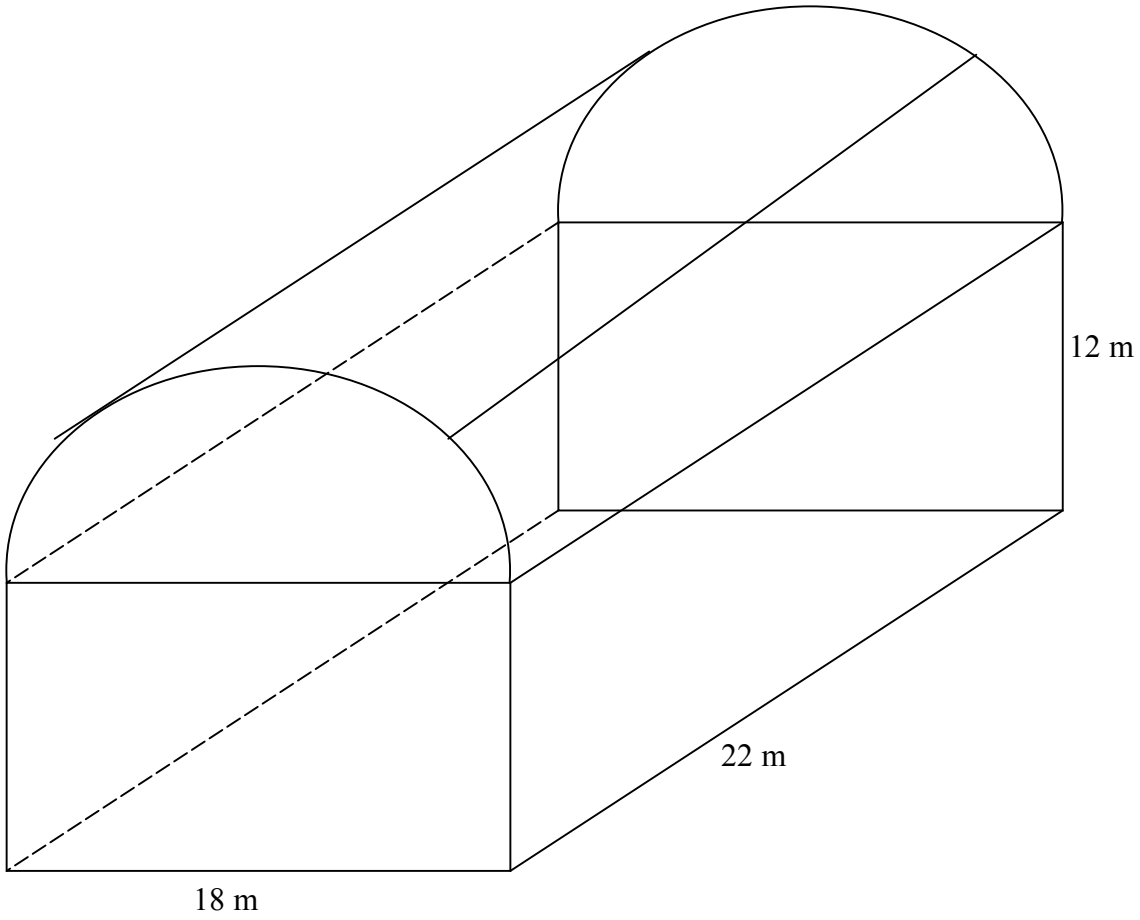
_____

1 mark

Module 2: Geometry and trigonometry

Question 3

In town Z there is a cabin that has a roof in the shape of half a cylinder. The width of the cabin is 18 m, the length is 22 m and the height is 12 m.



a. What is the radius of the base of the half cylinder roof?

_____

_____

1 mark

Module 2: Geometry and trigonometry

Question 3 (continued)

b. What is the surface area of the roof? Give your answer to one decimal place.

_____

_____

_____

_____

_____

_____

1 mark

c. Heating costs are dependant on the volume of air contained in the structure. What is the volume of air in this cabin? Give your answer to two decimal places.

_____

_____

_____

_____

_____

_____

1 mark

d. If heating costs are 0.02 cents per cubic metre, per hour, how much will it cost to heat the cabin for a day?

_____

_____

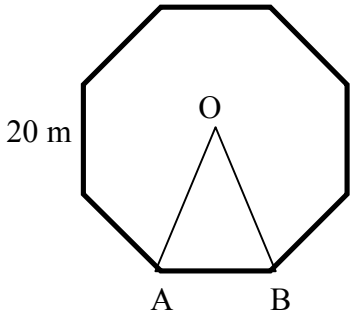
_____

1 mark

Module 2: Geometry and trigonometry

Question 4

Town Z has a piazza in the shape of a regular octagon, with side 20 m.
O is the central point of the piazza.



a. Show that $\angle OAB = 67.5^0$

_____

_____

_____

_____

1 mark

b. What is the length of AO? Give your answer to two decimal places.

_____

_____

_____

_____

1 mark

c. What is the area of the piazza? Give your answer to two decimal places.

_____

_____

_____

_____

1 mark

Total 15 marks

End of Module 2: Geometry and trigonometry

Module 3: Graphs and relations

If you choose this module, all questions are to be answered.

Question 1

Highblooms is a highly automated firm that produces glassware which it sells for $65 per unit.

a. i. When the firm sells 10 units of glassware, what is the total selling price?

_____

1 mark

ii. When the firm sells x units of glassware, what is the total selling price?

_____

1 mark

b. The variable production cost for this firm is $20 per unit and the annual fixed costs are $37,890. Write an equation for the cost, $C, of producing x units of glassware per year.

_____

1 mark

c. How many units of glassware must be sold each year for *Highblooms* to break even?

_____

_____

_____

1 mark

Module 3: Graphs and relations

Question 1 (continued)

d. *Boomgarten* is a small firm that does many operations by hand. It produces essentially the same type of glassware as *Highblooms* and also sells its product for $65 a unit. The cost of production when *Boomgarten* produces x units of glassware a year can be given by the equation

$$C = 40x + 3789$$

How much less are the annual fixed costs for *Boomgarten* than for *Highblooms*?

_____

_____

1 mark

e. In one particular year, *Boomgarten* sold 30,000 units of glassware. Did they make a profit or loss in this year and how much was this profit or loss?

_____

_____

_____

_____

_____

_____

2 marks

Module 3: Graphs and relations

Question 2

Boomgarten decides to expand and produce plastic materials as well as glassware. It finds that each day it produces p units of glassware and q units of plastic materials, where $p \geq 0$ and $q \geq 0$. The total daily number of units of both glassware and plastic materials cannot exceed 100 units.

a. Write an inequality to show this constraint.

_____

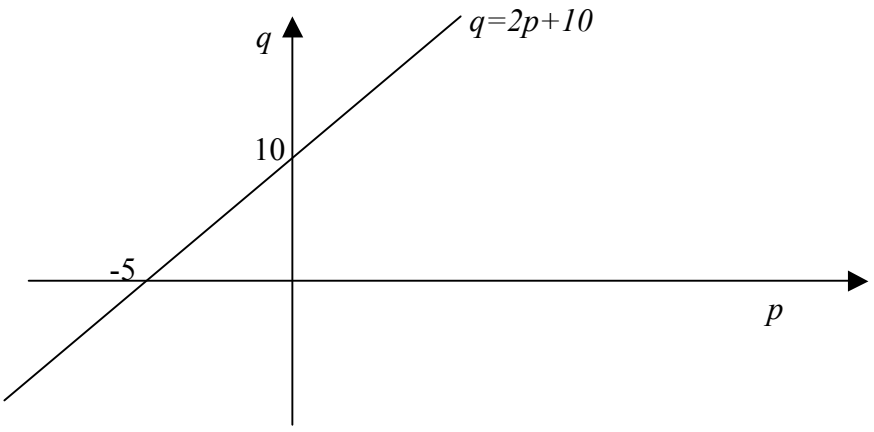
1 mark

b. Another daily constraint is that the number of units of glassware produced must not exceed three times the number of units of plastic produced. Write an inequality to show this constraint.

_____

1 mark

c. Another constraint gives the inequality, $q \leq 2p + 10$. On the graph below shade the region that satisfies this particular constraint, and the constraints, $p \geq 0$ and $q \geq 0$

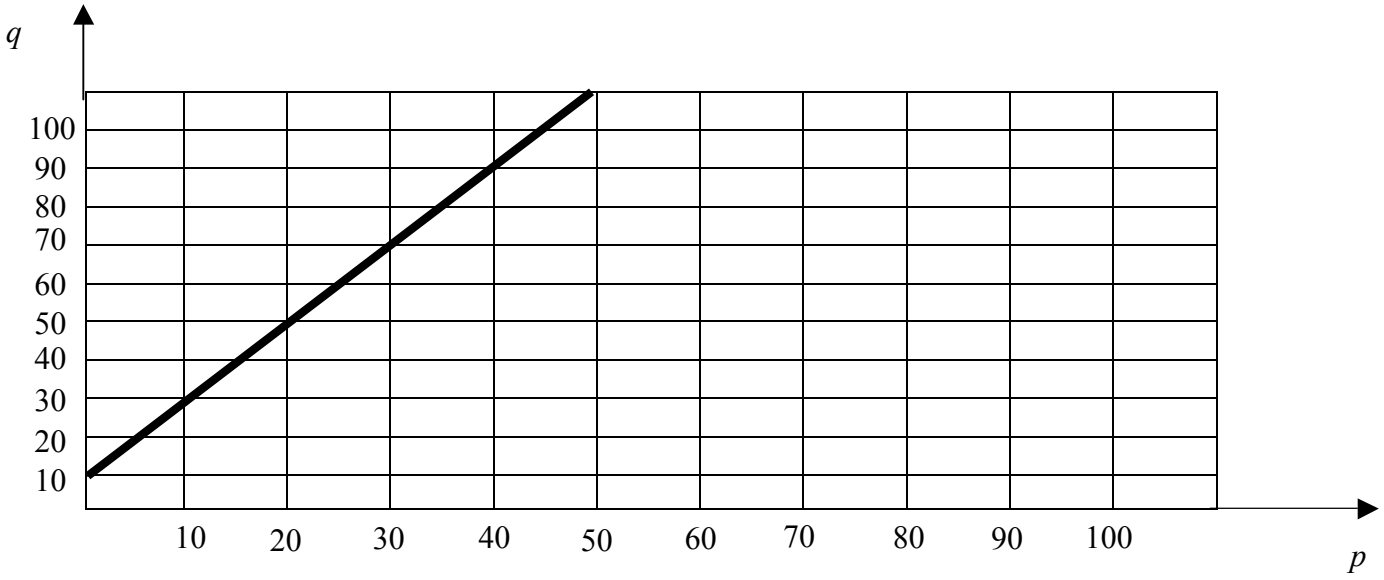


1 mark

Module 3: Graphs and relations

Question 2 (continued)

d. On the axes below, draw the two lines from your answers to **b.** and **c.** and shade the region that satisfies all the constraints for *Boomgarten*.



3 marks

e. If the profit on glassware is \$30 a unit and the profit on plastic is \$21 a unit, complete the following equation.

Profit =

1 mark

f. How many of each unit should be made each day to maximize this profit?

_____

_____

_____

1 mark

Total = 15 marks

End of Module 3: Graphs and relations

Module 4: Business-related mathematics

If you choose this module, all questions are to be answered.

Question 1

For a 25 km trip to the airport, a taxi company charges $48 from 5am to midnight and $57 from midnight to 5am.

a. What is the charge per kilometre for a 25 km trip taken at 7am?

_____

1 mark

b. What is the percentage increase for a 25 km trip between midnight and 5am?

_____

_____

1 mark

Question 2

a. Olivia is a taxi driver who earns $280 per week plus 12% on all fares. How much does she earn in a week when her fares come to $1300?

_____

1 mark

b. Tom works for a different taxi company. He earns $150 per week plus a percentage of his fares. If he earns $464.50 in a week when his fares came to $1850, what percentage of all fares does the company give him?

_____

_____

_____

1 mark

Module 4: Business-related mathematics

Question 3

Car Value	Stamp Duty
≤ $1000	1%
$1001 to $2000	$10 + 2% on the amount over $1000
$2001 to $3000	$30 + 3% on the amount over $2000
$3001 or more	$60 + 4% on the amount over $3000

a. If a new taxi costs $45,000, how much stamp duty will have to be paid?

_____

_____

1 mark

b. The taxi company owner has to pay $50,000 on road costs for the new taxi. She buys the taxi on hire purchase where she pays $5000 deposit and then $950 a month for 5 years.

i. What is the total amount she pays in monthly instalments?

_____

1 mark

ii. How much does she actually pay for the taxi?

_____

1 mark

iii. How much interest does she pay?

_____

1 mark

Module 4: Business-related mathematics

Question 3 (continued)

iv. What is the annual interest rate? Give your answer to one decimal place.

_____

_____

_____

1 mark

Question 4

An alternative way to buy the taxi that cost $50,000 including on-road costs, is to take out a bank loan for the full cost of the taxi at 6% per annum, compounded monthly.

a. What is the monthly interest rate?

_____

1 mark

b. How much will the taxi company have to pay back each month if it takes out an interest only loan?

_____

1 mark

c. How much will the taxi owner owe on her loan at the end of 5 years, if she does not repay any money until the end of the 5 years? Give your answer to the nearest cent.

_____

1 mark

d. The owner of the taxi company decides to repay the debt in equal monthly instalments so that the debt will be completely repaid by the end of 5 years. How much will each instalment need to be? Give your answer to the nearest cent.

_____

1 mark

Module 4: Business-related mathematics

Question 5

a. The taxi company found that its $45000 car was only worth $18,800 at the end of the first year. What was the unit depreciation cost per kilometre if it travelled 42,000 kilometres in that year? Give your answer to the nearest cent.

_____

1 mark

b. What would be the value of the car at the end of the second year if the depreciation cost per kilometre remained unchanged, but the number of kilometers travelled in the second year was 14,000?

_____

1 mark

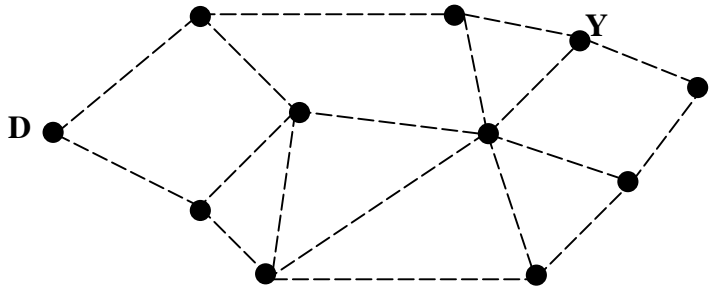
Total = 15 marks

End of Module 4: Business-related mathematics

Module 5: Networks and decision mathematics

If you choose this module, all questions are to be answered.

Question 1



The graph above shows each road in a network that connects a depot, D, to a series of supermarkets that are shown as nodes. A truck has to make deliveries to all of the ten supermarkets, visiting each supermarket only once.

a. i. On the above graph, clearly draw a complete route that the truck driver could take, beginning at D and finishing at the supermarket labeled Y.

1 mark

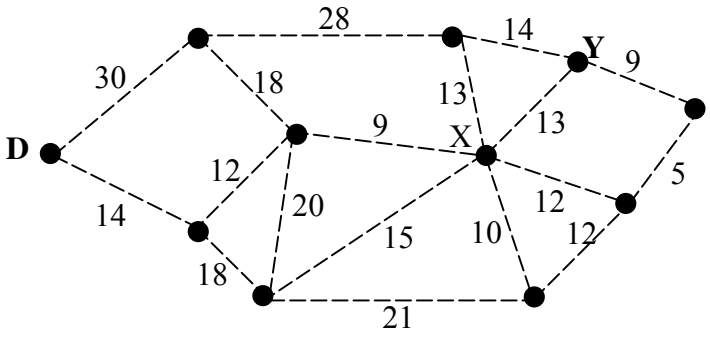
a. ii. What is the name of this type of path?

_____

1 mark

Module 5: Networks and decision mathematics

Question 1 (continued)



b. The above graph shows the distances between the supermarkets.

i. On the above graph, draw the shortest path the truck could take from the depot, **D**, if it only had a delivery for supermarket Y.

1 mark

ii. What is the length of this shortest route from D to Y?

_____

1 mark

c. The supermarket wishes to install tanks at X and then pipe water to each of the other 8 supermarkets (not the depot) so that they can water their trees. What is the shortest length of pipe required?

_____

_____

_____

1 mark

Module 5: Networks and decision mathematics

Question 2

Five yachts, *Alpha Star*, *Battering Ram*, *Challenger*, *Defending Champ* and *Earnest Endeavour* have all sailed against each other in a series of races for two yachts. The results were as follows.

- *Alpha Star* defeated *Challenger* and *Defending Champ*
- *Battering Ram* defeated *Alpha Star*, *Challenger* and *Earnest Endeavour*
- *Challenger* defeated *Earnest Endeavour*
- *Defending Champ* defeated *Battering Ram*, *Challenger* and *Earnest Endeavour*
- *Earnest Endeavour* defeated *Alpha Star*

a. Draw a dominance graph to show these results.

1 mark

b. Construct a first step dominance matrix.

1 mark

Module 5: Networks and decision mathematics

Question 2 (continued)

c. Complete the following one - step dominance table.

Yacht	One – Step Dominance
Alpha Star	2
Battering Ram	
Challenger	
Defending Champ	
Earnest Endeavour	1

1 mark

d. Which yacht/(s) is/are one - step dominant?

_____

1 mark

e. Construct a two – step dominance matrix for the results of the yacht race.

1 mark

Module 5: Networks and decision mathematics

Question 2 (continued)

f. Complete the following two – step dominance table.

Yacht	Two – Step Dominance
Alpha Star	
Battering Ram	
Challenger	
Defending Champ	
Earnest Endeavour	

2 marks

g. Rank the yachts from first to fifth place.

Place	Yacht
1	
2	
3	
4	
5	

2 marks

Module 5: Networks and decision mathematics

Question 2 (continued)

h. The one – step dominance table for a new set of races between the five yachts is given below.

Yacht	One – Step Dominance
Alpha Star	3
Battering Ram	1
Challenger	1
Defending Champ	2
Earnest Endeavour	3

If *Battering Ram* defeated *Alpha Star*, then whom did *Challenger* beat?

_____

1 mark

Total = 15 marks

End of Module 5: Networks and decision mathematics

Module 6: Matrices

If you choose this module, all questions are to be answered.

Question 1

In a football match it is possible to score goals, G, and behinds, B. The table below shows the results for the four quarters in a match between *Hawood* and *Essthorn*.

	1st quarter		2nd quarter		3rd quarter		4th quarter	
	G	B	G	B	G	B	G	B
Hawood	8	2	4	3	5	1	3	2
Essthorn	3	12	4	5	4	5	2	3

a. Write down the 2×2 matrix, A, that shows the number of goals and behinds kicked in the first quarter.

$$A = \begin{bmatrix} & \\ & \end{bmatrix}$$

1 mark

b. i. Explain how you would use matrices to find the overall goals and behinds kicked by each team.

_____

_____

_____

1 mark

b. ii. Give the answer in matrix form.

1 mark

Module 6: Matrices

Question 1 (continued)

c. What does the element in the first row, second column of your answer to **b. ii.** represent?

_____

_____

_____

1 mark

d. If there are six points for a goal and one point for a behind, write down the column matrix X that represents this.

1 mark

e. Write down the matrices and the process you would use to find the overall scores for the two teams.

_____

_____

_____

2 marks

Module 6: Matrices

Question 1 (continued)

f. i. Write down the matrix that shows the number of points scored by each side.

1 mark

f. ii. What is the order of this matrix?

_____

1 mark

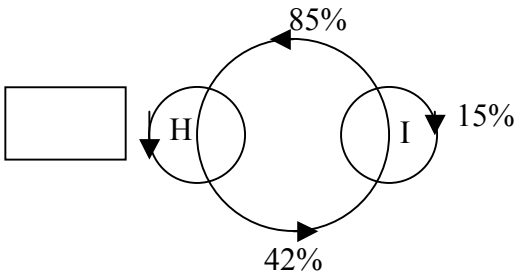
Module 6: Matrices

Question 2

Each week, footballers are either healthy or injured. Statistics show that

85% of footballers who are injured today will be healthy next week.
42% of footballers who are healthy today will be injured next week.

a. i. Use this information to complete the transition diagram below.



1 mark

a. ii. Use this information to complete the transition matrix below.

This Week

$$\begin{matrix} H & I \end{matrix}$$
$$\begin{bmatrix} & \\ & \end{bmatrix} \begin{matrix} H \\ I \end{matrix}$$

Next Week

1 mark

b. In a given week, out of 34 players, ten are injured. Write this as an initial state matrix, S_0

1 mark

Module 6: Matrices

Question 2 (continued)

c. In two weeks time, how many players do we expect to be injured?

_____

_____

_____

1 mark

d. In the long term, how many players would we expect to be healthy?

_____

_____

_____

1 mark

Module 6: Matrices

Question 3

At the football, a vendor sells four types of pie: beef, chicken, vegetable and curry. The transition matrix for the sale of these pies is given below.

$$T = \begin{array}{c} \\ \text{Beef} \\ \text{Chicken} \\ \text{Vegetable} \\ \text{Curry} \end{array} \begin{array}{c} \begin{array}{cccc} \text{Beef} & \text{Chicken} & \text{Vegetable} & \text{Curry} \end{array} \\ \begin{bmatrix} 0.1 & 0.6 & 0.3 & 0.5 \\ 0.4 & 0.1 & 0.4 & 0.3 \\ 0.3 & 0.1 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.1 & 0.1 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ \\ \\ \text{This Week} \end{array}$$

Last Week

If 12,000 people bought a chicken pie this week, and if last week there were three times as many chicken pies sold as each of the other types of pie, then what was the total number of pies sold last week?

_____

_____

_____

1 mark

Total = 15 marks

End of Module 6: Matrices

END OF QUESTION AND ANSWER BOOK
2010 Further Mathematics Trial Examination 2

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