

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 1



2009 Trial Examination

SOLUTIONS

SECTION A: Multiple-choice questions (1 mark each)

Question 1

Answer: B

Explanation:

$$y = a + bx \text{ where } b = \frac{rs_y}{s_x} \text{ and } a = \bar{y} - b\bar{x}$$

$$b = \frac{0.86 \times 2.15}{3.18} = 0.58$$

$$a = 22.1 - 0.58 \times 56.2 = -10.58$$

Question 2

Answer: D

Explanation:

The name of the course is categorical and the score is numerical data.

Question 3

Answer: A

Explanation:

Average St Andrews =

$$\frac{60 + 60 + 61 + 62 + 62 + 63 + 66 + 66 + 68 + 70 + 70 + 70 + 70 + 71 + 71 + 71 + 72 + 72 + 73 + 74 + 75 + 78 + 78 + 79 + 81}{25}$$

$$=69.72$$

Average Pebble Beach =

$$\frac{59 + 60 + 60 + 62 + 65 + 65 + 66 + 68 + 68 + 70 + 71 + 71 + 71 + 71 + 72 + 73 + 74 + 75 + 75 + 75 + 76 + 81 + 82 + 82 + 84}{25}$$

$$=71.04$$

Question 4

Answer: A

Explanation:

St Andrews range = $81 - 60 = 21$, median = 13th number $7|0 = 70$

Pebble Beach range = $84 - 59 = 25$, median = 13th number = $7|1 = 71$

Question 5

Answer: D

Explanation:

Actual score = mean + z-score × standard deviation

$$\text{Actual score} = 62 + 1.8 \times 12.2$$

$$\text{Actual score} = 83.92 \approx 84$$

Question 6

Answer: E

Explanation:

The standardised score of -1.8 is between mean – one standard deviation and mean – 2 standard deviations, 16% scores below the mean – 1 standard deviation, 2.5% scores below mean – 2 standard deviations.

Question 7

Answer: C

Explanation:

Height = $0.56 + 0.72 \times$ number of rings, where height is in metres, therefore for every ring the tree would have grown 0.72 metres or 72 centimetres.

Question 8

Answer: A

Explanation:

The correlation coefficient is 0.74, therefore it is positive and moderate

Question 9

Answer: B

Explanation:

$r = 0.74$ therefore $r^2 = 0.55$ therefore 55% of the variation in the dependant variable (height) can be explained by variation in the independent variable (number of rings)

Question 10

Answer: E

Explanation:

The 3 mean moving average for April takes the average of the data form March, April and May.

$$3 \text{ mean moving average} = \frac{36+101+85}{3} = 74$$

Question 11

Answer: A

Explanation:

$$\text{Seasonal index} = \frac{\text{monthly rain}}{\text{average rain}}$$

$$\text{Average rain} = \frac{12+25+36+101+85+62+121+191+102+56+42+80}{12} = 76.08mm$$

$$\text{Seasonal index} = \frac{56}{76.08} = 0.74$$

Question 12

Answer: C

Explanation:

Using the circle of transformation an x^2 , $\frac{1}{y}$ or $\log(y)$ transformation is suitable.

Question 13

Answer: B

Explanation:

Using a graphics calculator and making the appropriate transformations results in the following output $y = ax + b$, $a = 53.32$, $b = -602.03$. As it is an x^2 transformation the answer is $y = 53.32x^2 - 602.03$.

SECTION B: Multiple-choice questions

Module 1: Number Patterns

Question 1

Answer: E

Explanation:

There is a common difference of 3 in each of the terms

Question 2

Answer: A

Explanation:

There must be a common ratio between the successive terms.

$$\frac{x}{-7} = \frac{-63}{x} \text{ and } x^2 = 441. \text{ Therefore, } x = \pm 21.$$

Question 3

Answer: D

Explanation:

The Lucas sequence follows the rule $t_n = t_{n-1} + t_{n-2}$

$$t_{31} = t_{32} - t_{30}$$

Therefore $t_{31} = 4870847 - 1860498$

$$t_{31} = 3010349$$

Question 4

Answer: B

Explanation:

$$38 = 8a + 6$$

$$38 - 6 = 8a$$

$$32 = 8a$$

$$a = \frac{32}{8} = 4$$

Question 5

Answer: C

Explanation:

2009-2005 = 4 years

2009 value = 245000×1.12^4

2009 value = \$385112

Question 6

Answer: B

Explanation:

$$M_n = (1 + 0.12)M_{n-1}$$

$$M_n = 1.12M_{n-1}$$

Question 7

Answer: A

Explanation:

$$r = \frac{0.48}{0.60} = 0.8$$

$$S_\infty = \frac{a}{1-r} = \frac{0.6}{1-0.8} = 3m$$

Question 8

Answer: D

Explanation:

Difference between 8 and 12 = 4

Difference between 12 and 20 = 8

Difference between 20 and 36 = 16

\therefore not $t_{n+2} = t_{n+1} + t_n$ where $t_1 = 8$, $t_2 = 12$, $t_{n+1} = 1.5t_n$ or $t_{n+1} = t_n + 4$

$t_{n+1} = t_n + 4n$, would give 8, 16, 28 ...

Question 9

Answer: C

Explanation:

$$t_0 = 2, t_1 = 2, t_2 = 4, t_3 = 6, t_4 = 10, t_5 = 16, t_6 = 26, t_7 = 42, t_8 = 68, t_9 = 110, t_{10} = 178, t_{11} = 288, t_{12} = 466$$

Module 2: Geometry and Trigonometry

Question 1

Answer: D

Explanation:

$a = 55^\circ$ (vertically opposite)

The lines are parallel, therefore 55° and b° are supplementary $\therefore 55 + b = 180^\circ$

$\therefore b = 125^\circ$

Question 2

Answer: C

Explanation:

$$\sin \theta = \frac{O}{H}$$

$$\therefore \sin \theta = \frac{16}{20} = 0.8$$

Question 3

Answer: C

Explanation:

$$\tan \theta = \frac{O}{A}$$

$$\therefore \tan 42 = \frac{x}{120}$$

$$\therefore x = 120 \tan 42$$

$$\therefore x = 108.05m$$

The building is 20m tall therefore the balloon is 88.05m above the building.

Question 4

Answer: C

Explanation:

Heron's formula: $Area = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

$$s = \frac{480+360+520}{2} = 680$$

$$\therefore Area = \sqrt{680(680-480)(680-360)(680-520)}$$

Question 5

Answer: D

Explanation:

Scale factor = 1:1750. Scale factor for volume = 1:1750³

$$1:1750^3 = 1:5359375000$$

$$15 \times 5359375000 = 80390625000 \text{ L or } 80.4 \text{ gigalitres}$$

Question 6

Answer: E

Explanation:

Cos rule: $a^2 = b^2 + c^2 - 2bc \cos A$

$$\therefore a = \sqrt{120^2 + 20^2 - 2 \times 120 \times 20 \cos 62}$$

$$\therefore a = \sqrt{12546.5365}$$

$$\therefore a = 112.01\text{m}$$

Question 7

Answer: A

Explanation:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\therefore \frac{25}{\sin 11} = \frac{82}{\sin x}$$

$$\therefore 25 \sin x = 82 \sin 11$$

$$\sin x = \frac{82 \sin 11}{25}$$

$$x = \sin^{-1} \frac{82 \sin 11}{25} = 39.03^\circ \text{ or } 180^\circ - 39.05^\circ = 140.97^\circ$$

As the required angle is obtuse, $x = 140.97^\circ$

Question 8

Answer: A

Explanation:

length of bottom diagonal squared : $a^2 = 36^2 + 82^2 = 8020$

length of diagonal: $x = \sqrt{8020 + 68^2} = \sqrt{12644} = 112.446\text{cm}$

Question 9

Answer: D

Explanation:

The bearing of A from O is 53° , B from O is 83° $\therefore \angle AOB = 83 - 53 = 30^\circ$

AOB is an isosceles triangle, so $\angle BAO = \frac{180 - 30}{2} = 75^\circ$

Bearing of O from A = $180 + 53 = 233^\circ$

Bearing of B from A = $233 - 75 = 158^\circ$

Module 3: Graphs and Relations

Question 1

Answer: D

Explanation:

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 5}{2 - 0} = -1$$

Question 2

Answer: C

Explanation:

$$F = 100 + 60n$$

$$F = 100 + 60 \times 5$$

$$F = 100 + 300$$

$$F = 400$$

Question 3

Answer: B

Explanation:

$$4x = 50 + 1.2x$$

$$2.8x = 50$$

$$x = 17.85 = 18 \text{ puzzles}$$

Question 4

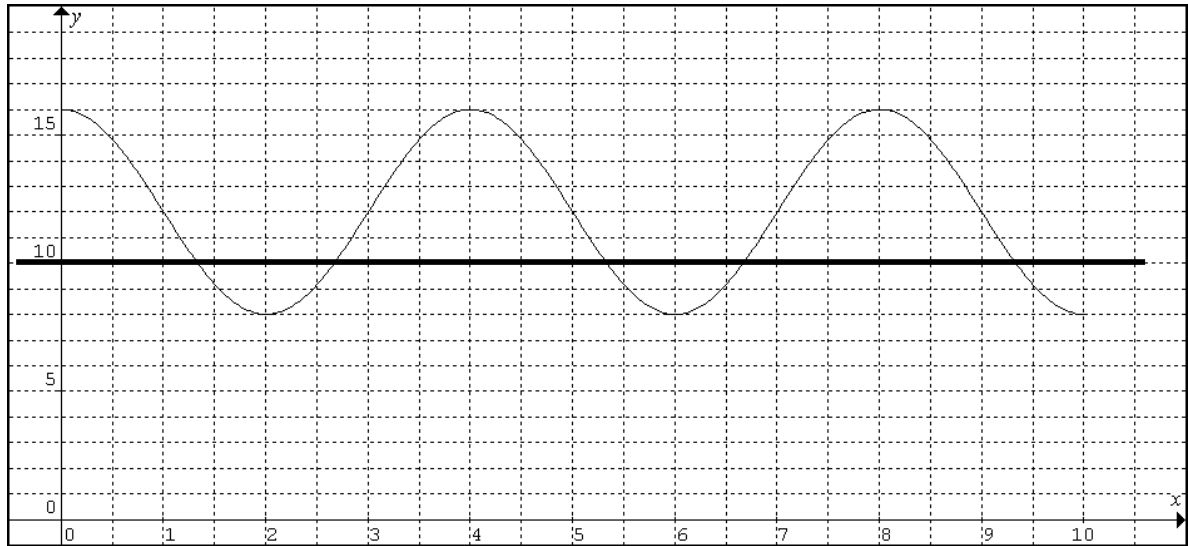
Answer: A

Explanation:

$$\$80 = \$30 (0 < x \leq 1 \text{ hour}) + \$50 (1 < x \leq 3 \text{ hours})$$

Question 5*Answer:* E*Explanation:*

The graph crosses the 10 m line 5 times in the period shown

**Question 6***Answer:* D*Explanation:*

For a set of simultaneous equations to have no solutions the lines must be parallel.

From $4x + 2y = 16$, $y = -2x + 8$

The line has the same gradient as $y = 9 - 2x$.

Question 7*Answer:* B*Explanation:*

Substituting (3, 4) into the equations we get:

$$y = 4x - 8 \quad 4 = 4 \times 3 - 8 = 4$$

$$2x + 5y = 16 \quad 2 \times 3 + 5 \times 4 \neq 16$$

$$2x + 3 = 3y - 3 \quad 2 \times 3 + 3 = 9 = 3 \times 4 - 3$$

$$3x - 2y = 1 \quad 3 \times 3 - 2 \times 4 = 1$$

$$2x - 3y = -6 \quad 2 \times 3 - 3 \times 4 = -6$$

Question 8

Answer: C

Explanation:

Checking each of the options

200 frames at \$2 each $R = 200 \times 2 = 400$ $C = 200 + 200 \times 2 = 600$

500 frames at \$3 each $R = 500 \times 3 = 1500$ $C = 200 + 500 \times 2 = 1200$

50 frames at \$6 each $R = 50 \times 6 = 300$ $C = 200 + 50 \times 2 = 300$

100 frames at \$5 each $R = 100 \times 5 = 500$ $C = 200 + 100 \times 2 = 400$

300 frames at \$2.50 each $R = 300 \times 2.5 = 750$ $C = 200 + 300 \times 2 = 800$

Question 9

Answer: D

Explanation:

Not A: (15, 10) $2x + 3y < 60$ because $2 \times 15 + 3 \times 10 = 60$, on the line

Not B: (-5, 20) $x \geq 0$ because $-5 < 0$, outside the region

Not C: (4, 4) $3x + 2y > 20$ because $3 \times 4 + 2 \times 4 = 12 + 8 = 20$, on the line

Not E: (5, 2) $3x + 2y > 20$ because $3 \times 5 + 2 \times 2 = 15 + 4 = 19$, on the line

Module 4: Business-Related Mathematics

Question 1

Answer: B

Explanation:

$$\frac{\$48000}{\$160000} \times 100\% = 30\%$$

Question 2

Answer: D

Explanation:

$$\frac{\$1699}{110} \times 10 = \$154.45$$

Question 3

Answer: A

Explanation:

$$\begin{aligned} 6000 &= 22000 - 0.06n \\ 6000 - 22000 &= -0.06n \\ -16000 &= -0.06n \\ n &= \frac{-16000}{-0.06} \\ n &= 266667 \end{aligned}$$

Question 4

Answer: A

Explanation:

$$\begin{aligned} I &= PRT \\ I &= \$5000 \times 0.06 \times 1.5 \\ I &= \$450 \\ \text{Investment} &= I + P \\ &= \$450 + \$5000 \\ &= \$5450 \end{aligned}$$

Question 5

Answer: E

Explanation:

\$1.95 end 2006

$\$1.95 \times 1.032 = \2.0124 end 2007

$\$2.0124 \times 1.024 = \2.06 end 2008

Question 6

Answer: D

Explanation:

Using TVM solver,

$n = 30 \times 26 = 780, I\% = 6.49, PV = 220000,$

$FV = 0, P/Y \ \& \ C/Y = 26$

$PMT = \text{End.}$ Solve for PMT .

Question 7

Answer: A

Explanation:

Using TVM solver,

$I = 4.5, PV = -50000, PMT = 500,$

$FV = 0, P/Y \ \& \ C/Y = 12$

$PMT = \text{End.}$ Solve for n .

Question 8

Answer: B

Explanation:

$10000 \times (1 - 0.048)^{10} = \6114.62

Question 9

Answer: D

Explanation:

$$r_e = \frac{100I}{Pt} \times \frac{2n}{(n+1)} \quad I = 80 \times 24 - 1499 = 421, P = 1499, t = 2, n = 24$$

$$r_e = \frac{100 \times 421}{1499 \times 2} \times \frac{2 \times 24}{(24+1)} = 26.96\%$$

Module 5: Network and Decision Mathematics

Question 1

Answer: D

Explanation:

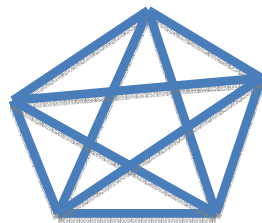
1 vertex has four edges, 2 vertices have two edges
= 3 vertices have an even degree

Question 2

Answer: B

Explanation:

10 edges



Question 3

Answer: C

Explanation:

$$v - e + f = 2$$

$$v - e + 5 = 2$$

$$v - e = -3$$

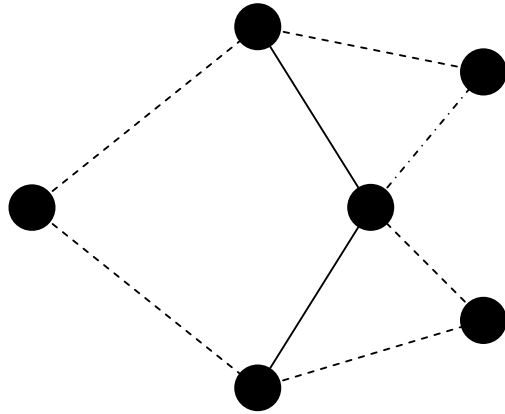
$$3 - 6 = -3$$

Question 4

Answer: A

Explanation:

The graph has a Hamilton circuit running around the outside (dotted lines), but there is no Euler circuit as there are 2 even degree vertices.

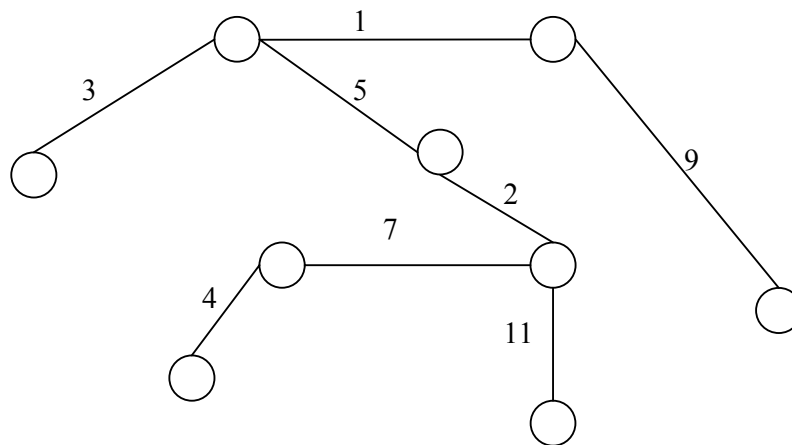


Question 5

Answer: C

Explanation:

Minimum spanning tree shown below, 6 is not present



Question 6

Answer: E

Explanation:

The flow is going from left to right therefore the flow of 10 going right to left is not counted.

Question 7

Answer: D

Explanation:

A minimum spanning tree best denotes the minimum length needed to cover a network. Therefore, the correct option is the minimum length of cable to connect computers.

Question 8

Answer: E

Explanation:

A and B can both start at the same time, the critical path is A, C, D, F, H which takes 17 hours not 18 hours

Question 9

Answer: B

Explanation:

Reducing D by 2 hours will reduce A, C, D, F, H by 2 hours to 15 hours; however, the path B, E, G, H will take 16 hours and become the new critical path.

Module 6: Matrices**Question 1***Answer:* C*Explanation:*

$$\begin{bmatrix} 2 & -2 & 0 \\ 3 & -1 & 1 \\ 2 & 2 & -3 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 2 \\ -2 & 3 & 1 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 2+(-1) & -2+2 & 0+2 \\ 3+(-2) & -1+3 & 1+1 \\ 2+0 & 2+(-1) & -3+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 2 \\ 2 & 1 & 0 \end{bmatrix}$$

Question 2*Answer:* C*Explanation:*

The number of columns in matrix A does not equal the number of rows in matrix C .

Question 3*Answer:* E*Explanation:*

$$\begin{aligned} A &= \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} \\ 2A - B &= \begin{bmatrix} 2 \times 2 & 2 \times 3 \\ 2 \times 2 & 2 \times 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 6 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 4-3 & 6-(-2) \\ 4-1 & 2-4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 8 \\ 3 & -2 \end{bmatrix} \end{aligned}$$

Question 4

Answer: B

Explanation:

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 0 & 2 & 3 \\ 2 & 1 & 2 \end{bmatrix} \quad XA = \begin{bmatrix} 3 & 2 & 6 \\ 2 & 2 & 1 \end{bmatrix} \quad X = ?$$

Number of rows of X = number of rows of $XA = 2$
 Number of columns of X = number of rows of $A = 3$

Question 5

Answer: A

Explanation:

$3x + 2y = 8$ and $1x + 3y = 7.8$ give a matrix equation as follows:

$$\begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 7.8 \end{bmatrix}$$

Question 6

Answer: D

Explanation:

Numbers expressed as decimals because they are percentages.

- 80% campers (C) will camp (C) next holiday
- 20% campers (C) will hotel (H) next holiday
- 5% hotel patrons (H) will camp (C) next holiday
- 95% hotel patrons (H) will book hotel (H) next holiday

$$\begin{array}{l} \text{Starting with Campers} \end{array} \quad \begin{array}{c} C \quad H \\ C \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} \\ H \end{array}$$

$$\begin{array}{l} \text{Then with Hotel patrons} \end{array} \quad \begin{array}{c} C \quad H \\ C \begin{bmatrix} 0.05 \\ 0.95 \end{bmatrix} \\ H \end{array} \quad \therefore \quad \begin{bmatrix} 0.8 & 0.05 \\ 0.2 & 0.95 \end{bmatrix}$$

Question 7*Answer:* A*Explanation:*

$$\begin{aligned}
 MP &= \begin{bmatrix} 1.15 & 0 \\ 0 & 1.2 \end{bmatrix} \begin{bmatrix} 20 & 35 & 60 \\ 15 & 30 & 50 \end{bmatrix} \\
 &= \begin{bmatrix} 1.15 \times 20 + 0 \times 15 & 1.15 \times 35 + 0 \times 30 & 1.15 \times 60 + 0 \times 50 \\ 0 \times 20 + 1.2 \times 15 & 0 \times 35 + 1.2 \times 30 & 0 \times 60 + 1.2 \times 50 \end{bmatrix} = \begin{bmatrix} 23 & 40.25 & 69 \\ 18 & 36 & 60 \end{bmatrix}
 \end{aligned}$$

Question 8*Answer:* D*Explanation:*

$$\begin{aligned}
 &\begin{bmatrix} 0.4 \times 2 + 0.2 \times 3 & 0.4 \times 4 + 0.2 \times 6 & 0.4 \times 1 + 0.2 \times 2 & 0.4 \times 3 + 0.2 \times 1 \\ 0.4 \times 1 + 0.2 \times 2 & 0.4 \times 3 + 0.2 \times 3 & 0.4 \times 8 + 0.2 \times 1 & 0.4 \times 1 + 0.2 \times 3 \\ 0.4 \times 9 + 0.2 \times 1 & 0.4 \times 2 + 0.2 \times 2 & 0.4 \times 3 + 0.2 \times 3 & 0.4 \times 4 + 0.2 \times 1 \\ 0.4 \times 1 + 0.2 \times 3 & 0.4 \times 2 + 0.2 \times 1 & 0.4 \times 1 + 0.2 \times 8 & 0.4 \times 1 + 0.2 \times 6 \end{bmatrix} = \\
 &\begin{bmatrix} 1.4 & 2.8 & 0.8 & 1.4 \\ 0.8 & 1.8 & 3.4 & 1 \\ 3.8 & 1.2 & 1.8 & 1.8 \\ 1 & 1 & 2 & 1.6 \end{bmatrix}
 \end{aligned}$$

Question 9*Answer:* C*Explanation:*

$$P_3 = T^3 P_0 = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}^3 \begin{bmatrix} 1500 \\ 800 \end{bmatrix} = \begin{bmatrix} 993 \\ 1307 \end{bmatrix}$$