Further Mathematics

2009

Trial Examination 1

Core – Data analysis
Module 1 – Number patterns
Module 5 – Networks and decision mathematics
Module 6 – Matrices

SECTION A Instructions

Answer **all** questions
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if **more than one** answer is completed for any question.

Core – Data analysis

The following information relates to Questions 1, 2, 3 and 4

The heights (nearest cm) of 74 male and female students are grouped according to gender and shown in the following back-to-back ordered stemplot below.

4 3	14	0 3 3
8 8 6	14	5 5 7 8 9
4 4 2 2	15	1 1 1 3 4 4
8 8 7 5 5	15	6 6 6 7 8
4 3 3 2 2 0	16	2 3 4 4 4
9 8 7 7 7 5 5	16	5 8 9
4 4 4 3 2 1 1	17	3 3 4
9 7 6 6 5	17	6 8
2 0 0	18	

Question 1
The modal height interval for the 74 students is

A. 150 - 154

B. 155 - 158

C. 160 - 164

D. 165 - 169

E. 171 - 174

Question 2
The mean height of female students is closest to 164 cm, the mean height of male students is closest to

A. 157 cm

B. 158 cm

C. 160 cm

D. 161 cm

E. 163 cm

Question 3
The mean height of the 74 students is closest to

A. 157 cm

B. 158 cm

C. 160 cm

D. 161 cm

E. 163 cm

Question 4
Select the best statement about the back to back ordered stemplot.

A. It has 2 numerical variables.

B. It has a numerical variable and a categorical variable.

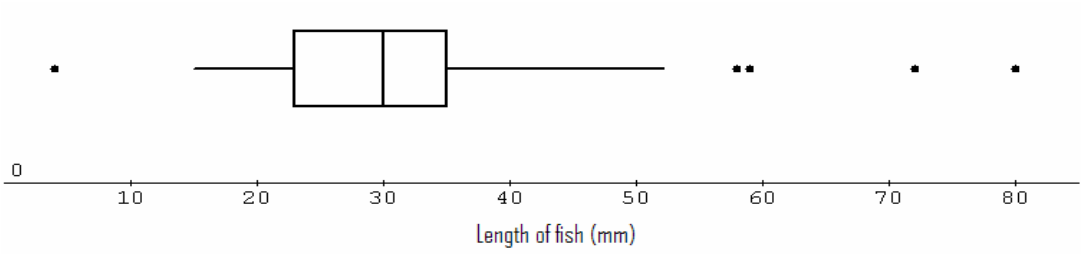
C. It has 2 numerical variables and a categorical variable.

D. It is a set of univariate data.

E. It shows the strength of the relationship between the 2 numerical variables.

The following information relates to Questions 5 and 6

The lengths of 323 fish in a pond were recorded and the resulting data is summarized in the boxplot below.



Question 5
Which one of the following statements is true?

A. The number of fish in the interval 15-23mm is greater than the number of fish in the interval 35-52mm.

B. The number of fish in the interval 15-23mm is less than the number of fish in the interval 35-52mm.

C. The number of fish in the interval 23-30mm is greater than the number of fish in the interval 30-35mm.

D. The number of fish in the interval 23-30mm is less than the number of fish in the interval 30-35mm.

E. The range of the distribution is 37mm.

Question 6
The percentage of fish greater than 35mm considered as outliers of the distribution is closest to

A. 1%

B. 2%

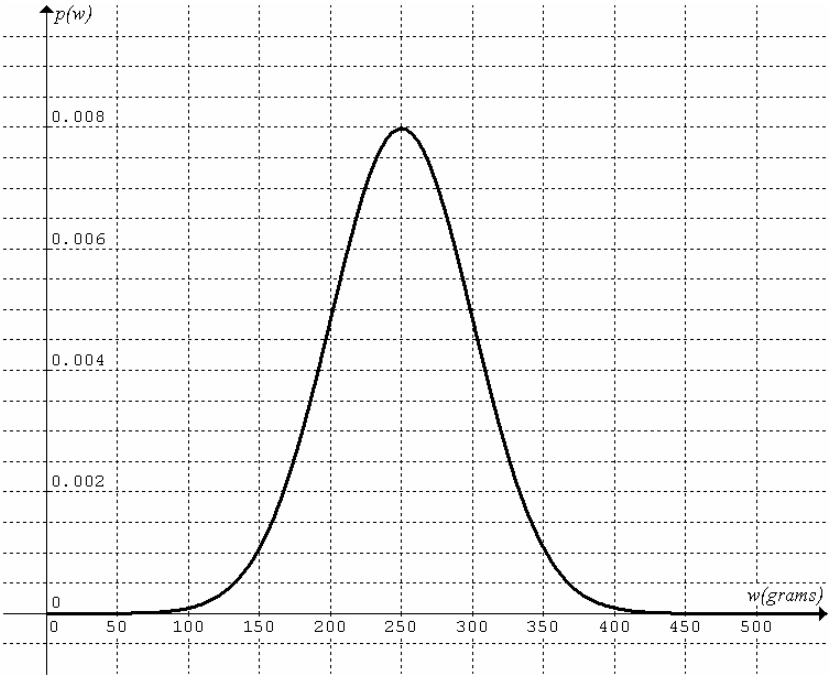
C. 3.5%

D. 5%

E. 6%

The following information relates to Questions 7 and 8

The weight *w* (grams) distribution of oranges from an orchard in Victoria is bell shape and can be modelled by the normal distribution shown below.



Question 7
The percentage of oranges between 200 grams and 400 grams in weight is closest to

A. 72%

B. 76%

C. 80%

D. 84%

E. 89%

Question 8

The standard z score of $w = 105$ grams is closest to

A. -1

B. -2

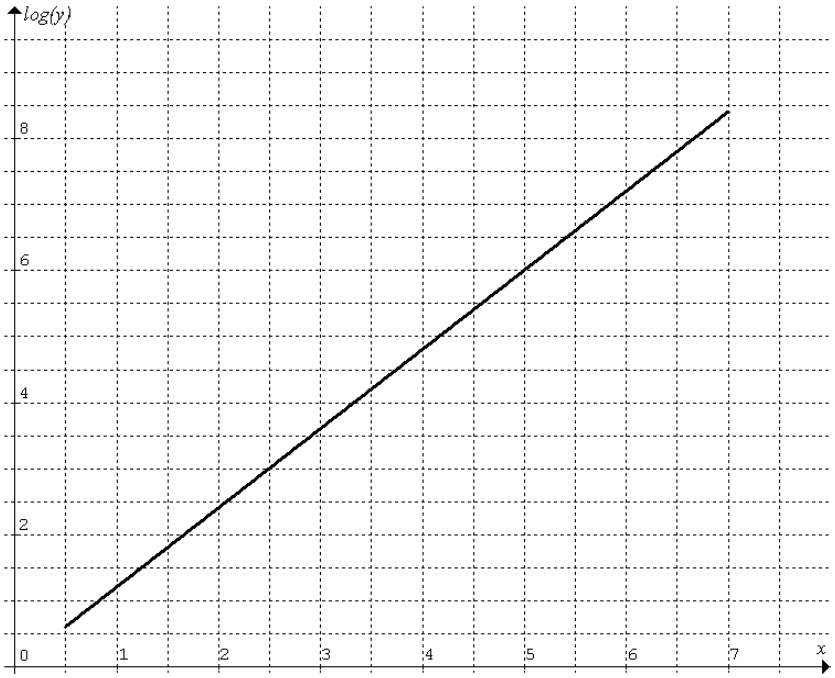
C. -3

D. 1

E. 2

The following scatterplot relates to Questions 9 and 10

The least squares regression line with $r = 0.80$ for a scatterplot of $\log y$ against x is shown below.



Question 9

The value of $\frac{s_{(\log y)}}{s_x}$ is closest to

A. 1.5

B. 1.7

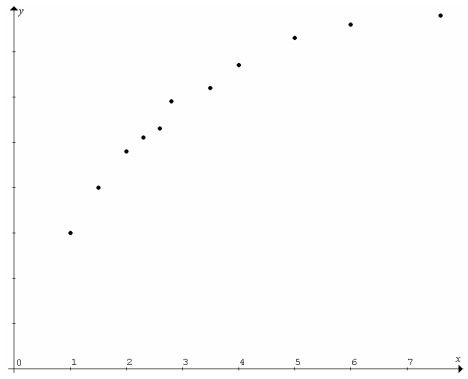
C. 1.8

D. 1.9

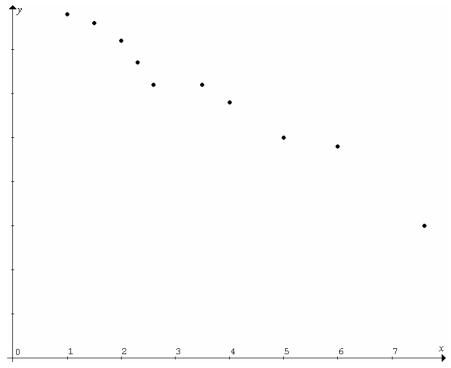
E. 2.1

Question 10
The most likely scatterplot of y versus x is

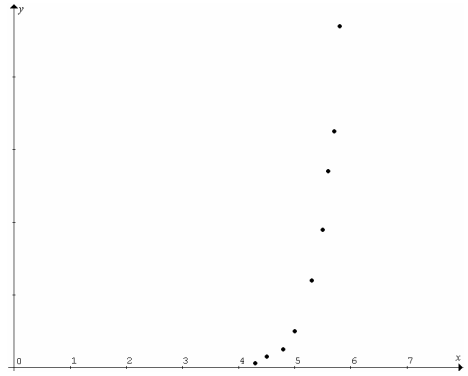
A.



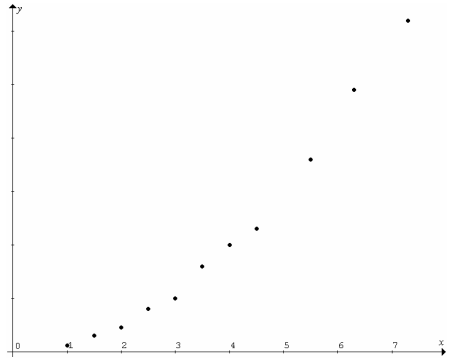
B.



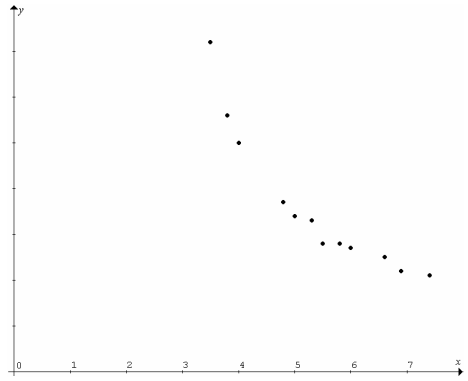
C.



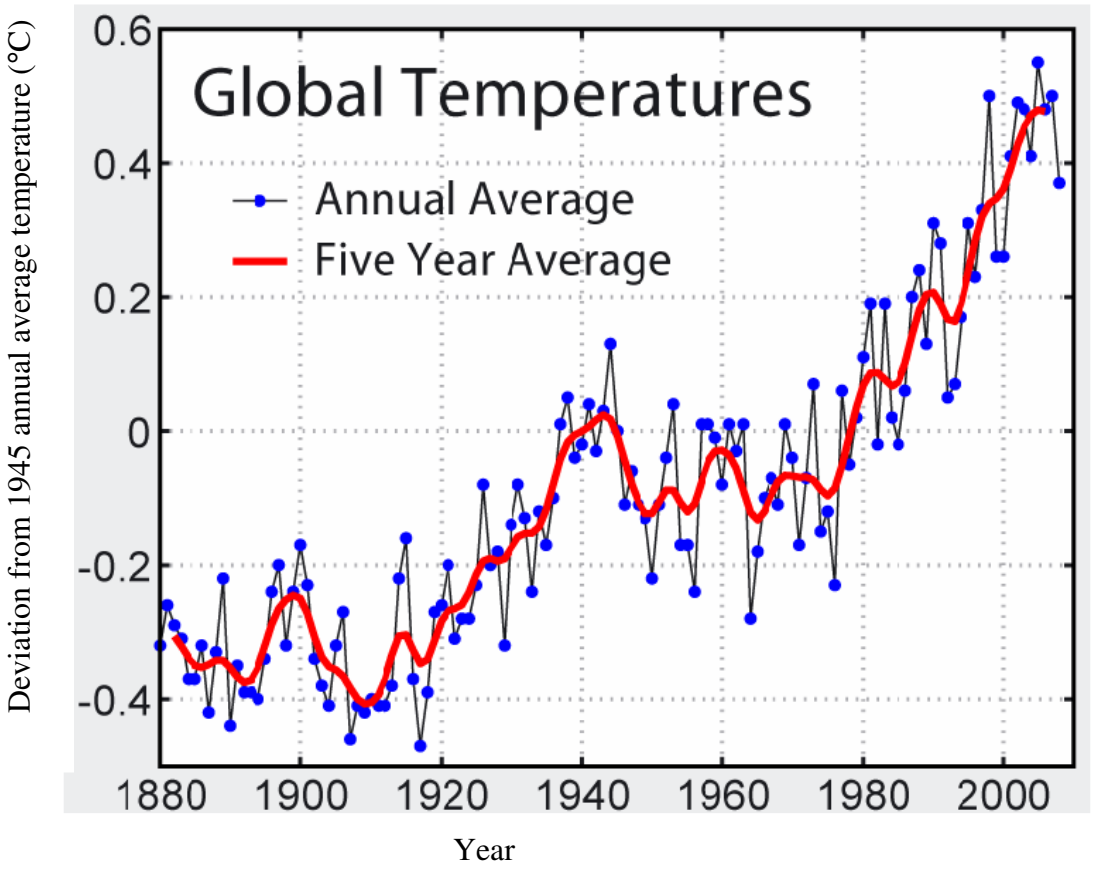
D.



E.



The following information relates to Questions 11, 12 and 13



Data from http://en.wikipedia.org/wiki/File:Instrumental_Temperature_Record.png

Question 11

The **sum** of annual average temperature deviations for the years 1978 to 1983 is closest to

A. 0.20

B. 0.32

C. 0.40

D. 0.58

E. 0.70

Question 12

The trend line for *five year average*, A, from 1978 (taken as $t = 0$ year) to 2006 is best modelled by

A. $A = 0.0186t - 36.83$

B. $A = 0.186t - 372.64$

C. $A = 0.186t + 0.37$

D. $A = 0.186t - 0.04$

E. $A = 0.0186t - 0.04$

Question 13
The correct trend line is used to construct a residual plot. The residual plot contains

A. points randomly scattered above and below zero

B. points in an increasing trend

C. points in a decreasing trend

D. points in some sort of pattern

E. points with no correlation

SECTION B Instructions

Answer **all** questions
A correct answer scores 1, an incorrect answer scores 0.
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No marks will be given if **more than one** answer is completed for any question.

Module 1: Number patterns

Question 1
Among the following 4 series, $2+2.6+3.9+5.2+\ldots$, $2-2.6+3.9-5.2+\ldots$, $2+2.4+3.6+4.8+\ldots$, $2-2.4+3.6-4.8+\ldots$, the number of geometric series is

A. 4

B. 3

C. 2

D. 1

E. 0

Question 2
An arithmetic sequence has 108 terms. The last two terms are 67 and 71.5. The sum of the middle 54 terms is

A. –9139.5

B. 9139.5

C. –16861.5

D. 16861.5

E. –18279

Question 3

$\frac{1}{2}, \frac{1}{a}, \frac{1}{4}, \ldots\ldots$ is an arithmetic sequence. The value of a is

A. 3

B. $\frac{5}{2}$

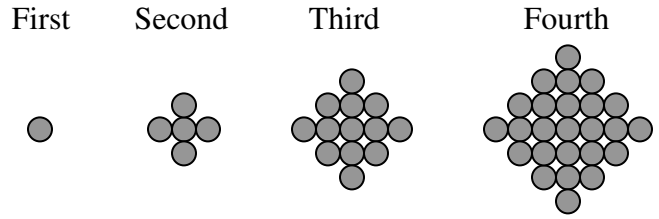
C. $\frac{7}{2}$

D. $\frac{8}{3}$

E. $\frac{9}{4}$

The following information relates to Questions 4 and 5

Ten-cent coins are used to form patterns.



Question 4

If t_n is the number of coins in the n^{th} pattern, then the number of coins in the $(n + 1)^{th}$ pattern is

A. $t_n + 4n$

B. $t_n + 3n$

C. $t_n + 2(n+1)$

D. $t_n + 3(n+1)$

E. $t_n + 4(n+1)$

Question 5

The $value of the tenth pattern is

A. 14.50

B. 15.30

C. 18.10

D. 22.10

E. 22.50

Question 6

The recurring decimal 11001.10011001..... is the value of $10000+\frac{a}{1-r}$ when

A. $a=11$

B. $a=100$

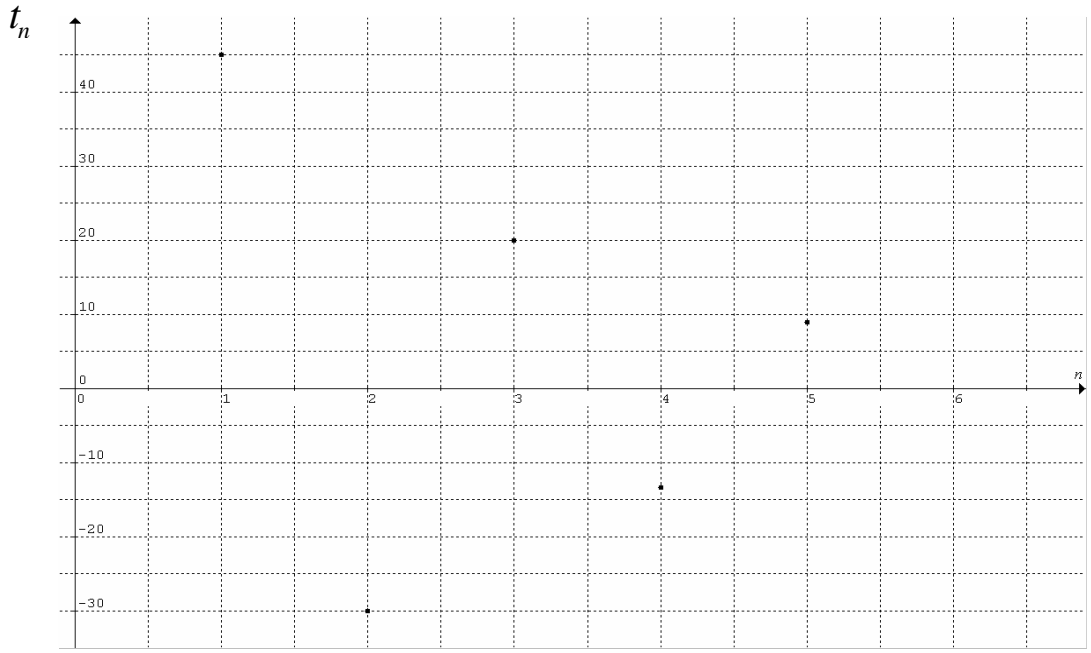
C. $a=1001$

D. $a=1100$

E. $a=11001$

Question 7

The first five terms of a geometric sequence is represented by the following graph.



The difference equation defining the sequence is

A. $t_{n+1}=45\left(-\frac{2}{3}\right)^n$

B. $t_{n+1}=45\left(\frac{2}{3}\right)^n$

C. $t_{n+1}=\frac{2}{3}t_n$

D. $2t_{n+1}+3t_n=0$

E. $3t_{n+1}+2t_n=0$

Question 8

The first three terms of a sequence defined by $u_{n+1} = a + bu_n$ are 20, 10 and 25. The values of a and b are respectively

A. 10 and 1.5

B. 15 and −0.25

C. 25 and −0.75

D. 30 and −1

E. 40 and −1.5

Question 9

A superball falls to the ground and bounces off the ground. In each **bounce** the distance travelled is 5 cm less than the preceding **falling** distance. If the distance travelled in the third **bounce** is 98 cm, the **total** distance (cm) travelled from the moment the ball is dropped till the end of the 18^{th} bounce is

A. 2358

B. 2440

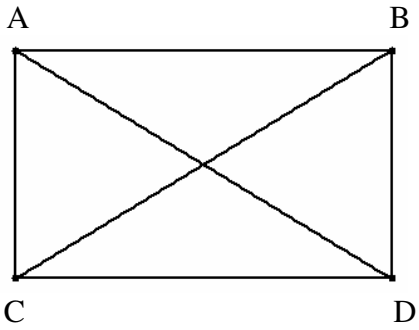
C. 2448

D. 2460

E. 3458

Module 5: Networks and decision mathematics

Question 1



An adjacency matrix that could be used to represent the network above is

A.
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \end{array}$$

B.
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 3 & 3 & 3 \\ 3 & 0 & 3 & 3 \\ 3 & 3 & 0 & 3 \\ 3 & 3 & 3 & 0 \end{bmatrix} \end{array}$$

C.
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

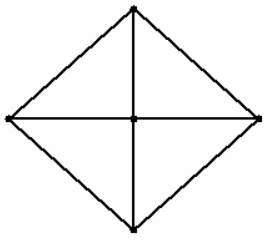
D.
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

E.
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \\ 2 & 3 & 0 & 1 \\ 1 & 2 & 3 & 0 \end{bmatrix} \end{array}$$

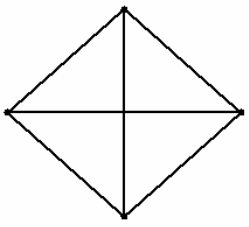
Question 2

Which one of the following graphs is **NOT** a planar graph?

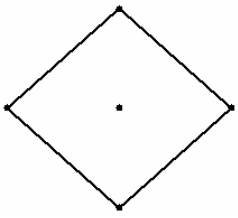
A.



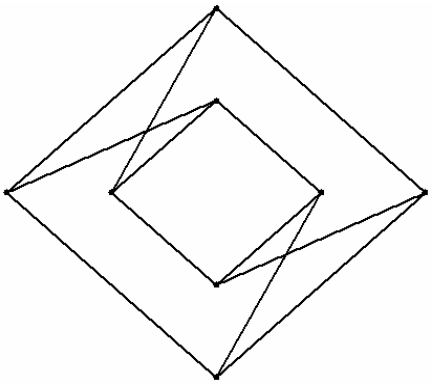
B.



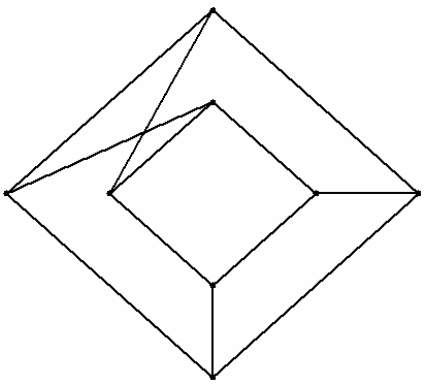
C.



D.



E.



Question 3
A connected planar graph has **even** number of faces. This graph could **NOT** have

A. 13 edges and 7 vertices

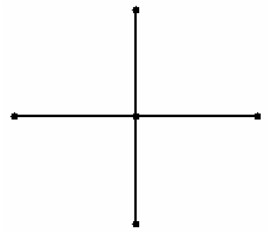
B. 11 vertices and 17 edges

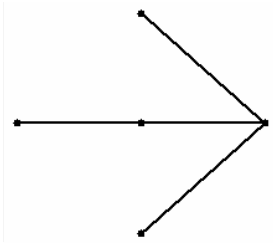
C. 12 edges and 6 vertices

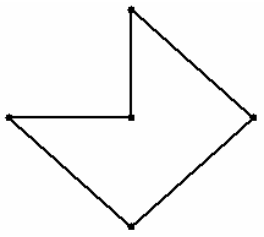
D. 3 vertices and 8 edges

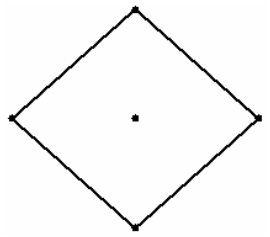
E. 7 edges and a vertex

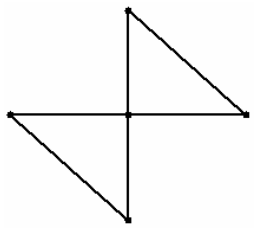
Question 4
Which one of the following graphs is a bipartite graph?

A.


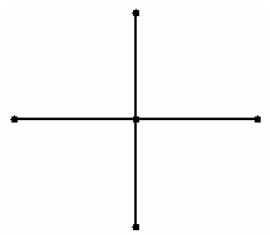
B.


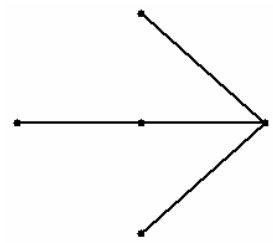
C.


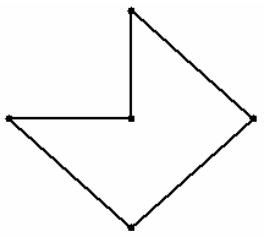
D.


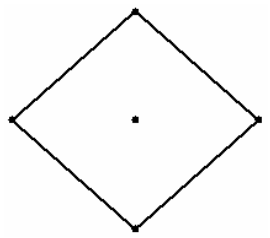
E.


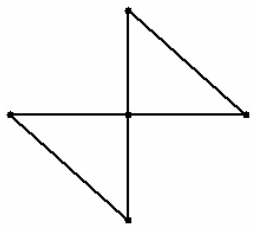
Question 5
Which one of the following graphs has an Euler path AND a Hamiltonian path?

A.


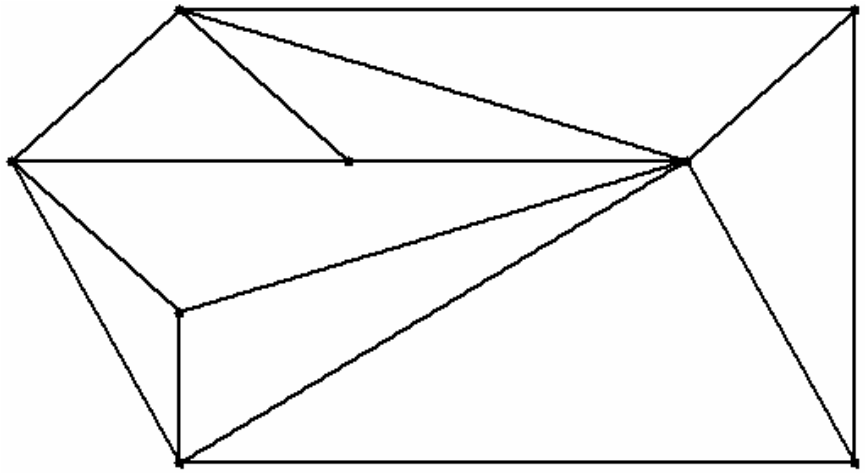
B.


C.


D.


E.


Question 6



A spanning tree of the above graph must have

A. exactly 8 edges

B. at least 8 edges

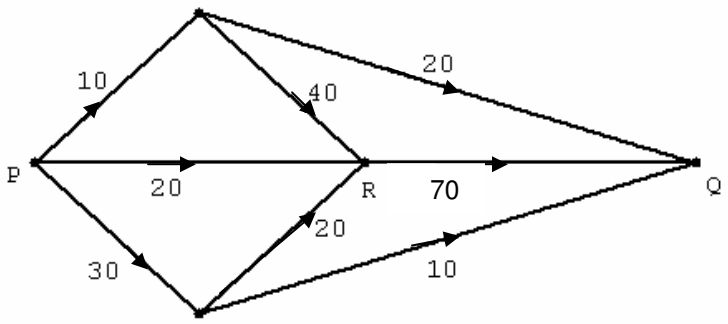
C. exactly 7 edges

D. at least 7 edges

E. at least 6 edges

Question 7

A network of roads from Town P to Town Q is shown below. The numbers on the edges indicate the capacity of flow of traffic along each section of road.



The capacity of flow of traffic is 70 in section RQ. **If the flow from P to Q is maximum**, the **difference** between the maximum and minimum flows of traffic in section RQ is

A. 5

B. 10

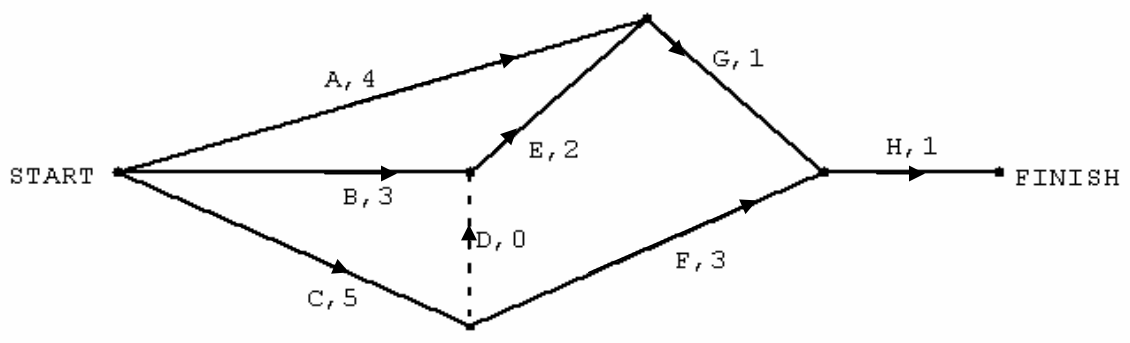
C. 15

D. 20

E. 25

Question 8

The digraph indicates the activities *A* to *H*, which are required to complete a project. The completion time (in hours) for each activity is shown next to each one.



The shortest time required to complete the project is

A. 1

B. 6

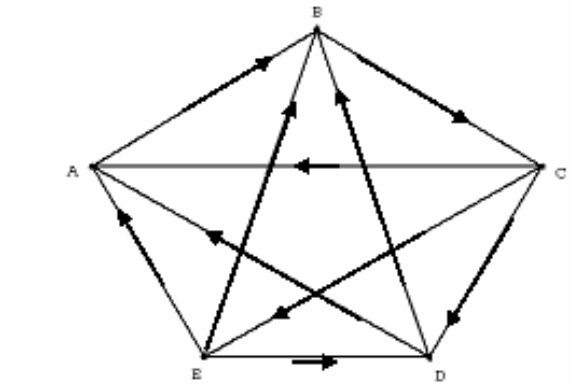
C. 7

D. 8

E. 9

Question 9

There are five players, A, B, C, D and E, in a chess competition. Each player played each other player once. The results are summarised in the graph below together with one of the corresponding dominance matrices with two missing entries. An arrow from B to C indicates that B defeated C.



$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & & 0 \end{bmatrix} \end{array}$$

The **sum** of the two missing entries is

A. 0

B. 1

C. 2

D. 3

E. 4

Module 6: Matrices

Question 1

$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 0 & 4 \end{bmatrix} - X \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ is defined if the order of matrix X is

A. 2×2

B. 2×3

C. 2×4

D. 3×3

E. 3×4

Question 2

For $\begin{bmatrix} a & 2 \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

A. $a = \dfrac{1}{2}$ and $b = \dfrac{1}{2}$

B. $a = -\dfrac{1}{2}$ and $b = \dfrac{1}{2}$

C. $a = \dfrac{1}{2}$ and $b = -\dfrac{1}{2}$

D. $a = -\dfrac{1}{2}$ and $b = -\dfrac{1}{2}$

E. real values a and b do not exist.

Question 3

Matrix X in the equation $\begin{bmatrix} -6 & -2 \\ -2 & 1 \end{bmatrix} = X \begin{bmatrix} -1 & -2 \\ 3 & 1 \end{bmatrix}$ is

A. $\begin{bmatrix} 0.2 & 0.4 \\ -0.6 & -0.2 \end{bmatrix}$

B. $\begin{bmatrix} -\frac{1}{7} & -\frac{2}{7} \\ \frac{3}{7} & \frac{1}{7} \end{bmatrix}$

C. $\begin{bmatrix} 0 & -2 \\ -1 & -1 \end{bmatrix}$

D. $\begin{bmatrix} 0 & \frac{10}{7} \\ \frac{5}{7} & \frac{5}{7} \end{bmatrix}$

E. not defined.

Question 4

A store sells dining suites, lounge suites and bedroom suites. The profit is $950 for a dining suite sold, $1200 for a lounge suite sold, and $1000 for a bedroom suite sold. The sales figures for the four quarters in 2008 are shown in the table.

	First quarter	Second quarter	Third quarter	Fourth quarter
Dining suites	15	20	18	23
Lounge suites	9	13	7	19
Bedroom suites	16	22	12	21

The total profit in selling dining suites, lounge suites and bedroom suites in 2008 is given by the matrix product

A. $\begin{bmatrix} 15 & 20 & 18 & 23 \\ 9 & 13 & 7 & 19 \\ 16 & 22 & 12 & 21 \end{bmatrix} \begin{bmatrix} 950 \\ 1200 \\ 1000 \end{bmatrix}$

B. $\begin{bmatrix} 950 \\ 1200 \\ 1000 \end{bmatrix} \begin{bmatrix} 15 & 20 & 18 & 23 \\ 9 & 13 & 7 & 19 \\ 16 & 22 & 12 & 21 \end{bmatrix}$

C. $\begin{bmatrix} 950 & 1200 & 1000 \end{bmatrix} \begin{bmatrix} 15 & 20 & 18 & 23 \\ 9 & 13 & 7 & 19 \\ 16 & 22 & 12 & 21 \end{bmatrix}$

D. $\begin{bmatrix} 950 & 1200 & 1000 \end{bmatrix} \begin{bmatrix} 76 \\ 48 \\ 71 \end{bmatrix}$

E. $\begin{bmatrix} 950 \\ 1200 \\ 1000 \end{bmatrix} \begin{bmatrix} 40 & 55 & 37 & 63 \end{bmatrix}$

Question 5

$P = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.9 & 0.9 \end{bmatrix}$, $Q = \begin{bmatrix} -0.2 & 0.3 & 0.9 \\ 0.7 & 0.2 & 0.1 \\ 0.5 & 0.5 & 0 \end{bmatrix}$, $R = \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.2 & 0.3 & 0.5 \\ 0.8 & 0.1 & 0.1 \end{bmatrix}$, $S = \begin{bmatrix} 0.4 & 0.6 \\ 0.3 & 0.2 \\ 0.3 & 0.2 \end{bmatrix}$

Which one of the above matrices could not be a transition matrix?

A. P only

B. Q only

C. Q and R only

D. P, R and S only

E. All

The following information relates to Questions 6 and 7

The following set of simultaneous equations can be expressed in matrix form using vector $[a \quad b \quad c]$.

$$b - a + c = 2$$
$$2c + a - 3b = 1$$
$$2a - 5b + 3 = 0$$

Question 6
The matrix equation is

A. $\begin{bmatrix} -1 & 1 & 2 \\ 1 & -3 & -5 \\ 1 & 2 & 0 \end{bmatrix} [a \quad b \quad c] = [2 \quad 1 \quad -3]$

B. $\begin{bmatrix} -1 & 1 & 1 \\ 1 & -3 & 2 \\ 2 & -5 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$

C. $[a \quad b \quad c] \begin{bmatrix} -1 & 1 & 1 \\ 1 & -3 & 2 \\ 2 & -5 & 0 \end{bmatrix} = [2 \quad 1 \quad -3]$

D. $[a \quad b \quad c] \begin{bmatrix} -1 & 2 & 1 \\ 1 & -5 & -3 \\ 1 & 0 & 2 \end{bmatrix} = [2 \quad -3 \quad 1]$

E. $\begin{bmatrix} -1 & 2 & 1 \\ 1 & -5 & -3 \\ 1 & 0 & 2 \end{bmatrix} [a \quad b \quad c] = [2 \quad -3 \quad 1]$

Question 7
Vector $[a \quad b \quad c]$ is

A. $[0 \quad 0.6 \quad 1.4]$

B. $[0.6 \quad 0 \quad 1.4]$

C. $[-1 \quad 1.6 \quad 1.4]$

D. $[1.6 \quad -1 \quad 1.4]$

E. $[-6.5 \quad -2 \quad 0]$

The following information relates to Questions 8 and 9

A city has four shopping centres P, Q, R and S competing for a fixed number of customers weekly. The following transition matrix is a good indication of the movements of customers among the four shopping centres from one week to the next week. For examples, 88% (0.88) of the customers of P in a particular week will return to P the following week; 2% (0.02) of the customers of Q in a particular week will go to P the following week.

A particular week

	P	Q	R	S	
	0.88	0.02	0.04	0.01	P
	0.05	0.92	0.05	0.01	Q
	0.03	0.03	0.85	0.01	R
	0.04	0.03	0.06	0.97	S

The following week

Let 3500, 2800, 3800 and 2500 be the number of customers of P, Q, R and S respectively in a particular week.

Question 8
The number of customers of Q

A. decreases by the same percentage from one week to the next.

B. increases by the same percentage from one week to the next.

C. decreases by a smaller percentage from one week to the next.

D. increases by a smaller percentage from one week to the next.

E. decreases by a greater percentage from one week to the next.

Question 9
In the long term, the percentage of all the customers of P, Q, R and S shopping at S is closest to

A. 25%

B. 40%

C. 55%

D. 70%

E. 85%

End of Exam 1

