INSIGHT

Trial Exam Paper

2009

FURTHER MATHEMATICS

Written examination 2

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
3	3	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference that may be annotated (can be typed, handwritten or a textbook), one approved graphics calculator (memory DOES NOT have to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring blank sheets of paper or white out liquid/tape into the examination.

Materials provided

- The question and answer book of 29 pages, with an answer sheet for the multiple-choice questions.
- A separate sheet with miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions

- Write your **name** in the box provided on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- Unless otherwise indicated, diagrams in this book are **not** drawn to scale.

At the end of the examination

- You may keep this question book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

Working space

SECTION A

Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** question within the modules selected. You do not need to give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Core: Data Analysis

Question 1

The data shows the heights of the males and females who reached the 4th round of the Australian Open tennis tournament in 2009.

Male (cm)	Female (cm)
185	178
183	171
193	178
180	173.5
191	183
188	173
185	175
188	180
188	169
180	170
183	164
188	180
198	175
198	165
196	180
185	176

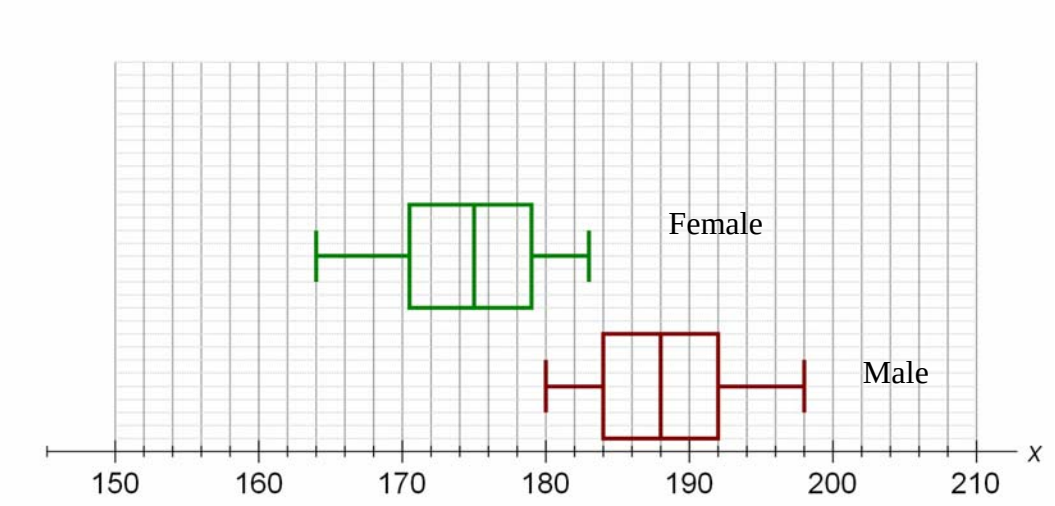
Table 1

Complete the table below. Express your answers to 2 decimal places.

	Mean	**Standard deviation**
Male	188.06	
Female		5.52

1 + 1 = 2 marks

Below are boxplots for the male and female heights.



a. The range of the male tennis players is ☐

1 mark

b. Explain the similarities and differences between the male and female heights for the above sample.

_____

_____

_____

2 marks

c. Calculate the standardised score for a player who is 182cm correct to 2 decimal places if the player is:

i. Male

_____

_____

ii. Female

_____

_____

1 + 1 = 2 marks

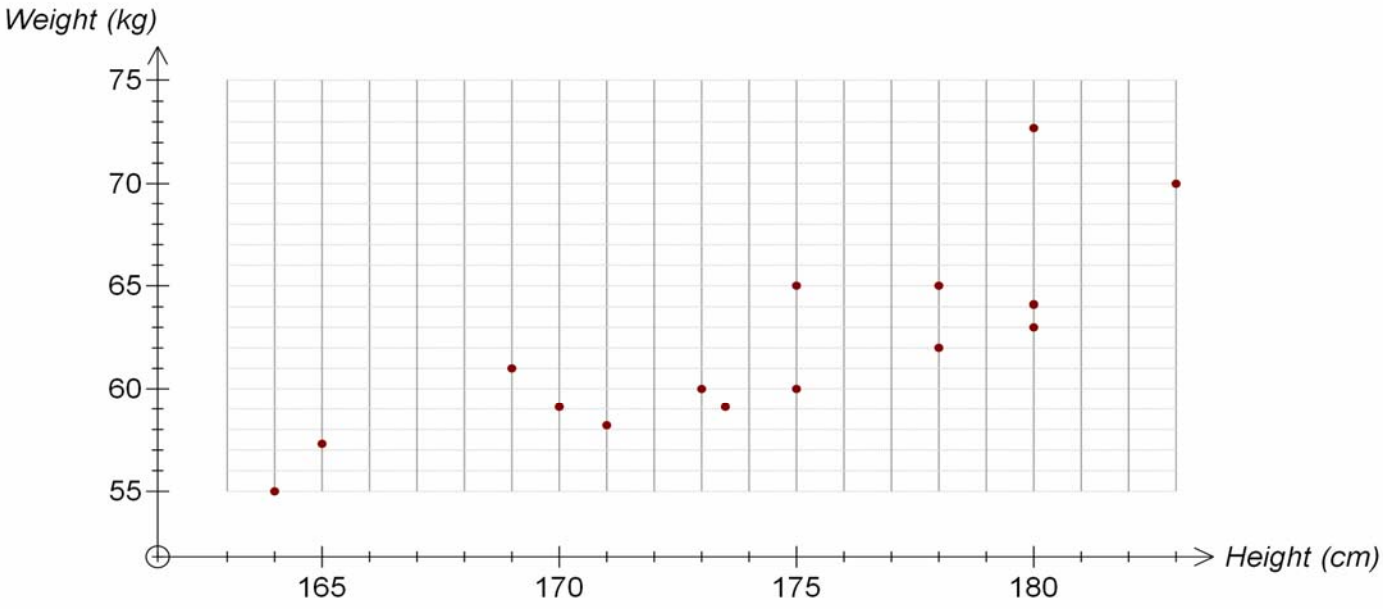
Question 2

The table below shows the weight and height of female tennis players who reached the 4th round of the Australian Open tennis tournament.

Player	Height (cm)	Weight (kg)
A	178	62
B	171	58.2
C	178	65.0
D	173.5	59.1
E	183	70.0
F	173	60.0
G	175	60.0
H	180	72.7
I	169	61
J	170	59.1
K	164	55
L	180	64.1
M	175	65
N	165	57.3
O	180	63
P	176	68.2

Table 2

A scatterplot for the data in Table 2 is shown below:



a. Complete the scatterplot by adding player P with a cross (X).

1 mark

The least squares regression equation is:

Weight = 0.7 × Height – 60.06

Pearson's product moment correlation co-efficient is 0.805.

b. On average, for each extra cm increase in height, the least squares regression equation predicts that there will be an increase of ☐ kg in weight.

1 mark

c. To the nearest whole percent, ☐ % of the variation in weight can be explained by the variation in height.

1 mark

d. Complete the residual analysis table below by finding the missing predicted and residual values, correct to 2 decimal places:

Player	Height (cm)	Predicted Weight	Residual value
A	178	64.54	
B	171		–1.44
C	178	64.54	0.46
D	173.5	61.39	2.29
E	183	68.04	1.96
F	173	61.04	–1.04
G	175	62.44	–2.44
H	180	65.94	6.76
I	169	58.24	2.76
J	170	58.94	0.16
K	164	54.74	0.26
L	180	65.94	–1.84
M	175	62.44	2.56
N	165	55.44	1.86
O	180		–2.94
P	176	63.14	

2 marks

e. Explain why the sum of the residual column will be close to 0.

_____

_____

1 mark

Question 3

a. In an attempt to improve the predictive power of the model, an inverse y transformation is carried out.

Find the equation of the least squares regression line, in terms of the variables, after the transformation has been carried out.

_____

_____

_____

1 mark

b. Explain whether the transformation has improved the predictive model. Use appropriate statistics to support your answer.

_____

_____

_____

1 mark

END OF SECTION A
TURN OVER

SECTION B

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected on the answer sheet provided.
Indicate the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.
Choose the response that is **correct** for the question.
One mark will be awarded for a correct answer; no marks will be awarded for an incorrect answer.
Marks **are not** deducted for incorrect answers.
No marks will be awarded if more than one answer is completed for any question.

SECTION B

Module 1: Number Patterns

Question 1

Prue is a keen runner. To prepare for an upcoming fun run, she runs laps of the local oval. On the first day she runs 4 laps, on the second day she runs 7 laps and on the third day she runs 10 laps.

a. If this sequence continues, how many laps will Prue run on the 7th day?

_____

_____

1 mark

b. The sequence above, t_n, can be described in terms of n as: $t_n = 1 + d \times n$

The value of d is ☐

1 mark

c. On what day will Prue first run 31 laps?

_____

_____

_____

1 mark

d. Find how many laps Prue will run in total in between the 8th and 12th days, inclusive.

_____

_____

_____

2 marks

Ramon is also training for the fun run with the number of laps, C, that he runs each day in terms of n following the sequence $C_n = -3n + 46$

e. Find the day on which Prue will first run more laps than Ramon.

_____

_____

_____

1 mark

Question 2

Stephen rides his bicycle to deliver parcels. The numbers of parcels that he has delivered in the first 3 months of his job are 100, 120 and 144 respectively.

a. Show mathematically that the sequence described is a geometric sequence.

_____

_____

_____

1 mark

b. Write the difference equation which describes this pattern.

_____

_____

_____

1 mark

c. If this pattern continues, calculate the number of deliveries in the 6th month, rounded to the nearest whole delivery.

_____

_____

_____

1 mark

d. In the latest contract negotiation, Stephen needs to be delivering more than 360 parcels in a month to achieve a bonus. At his current rate of deliveries, in which month of delivery will Stephen achieve his bonus?

_____

_____

_____

1 mark

Question 3

The value of a property over the first 3 years is given below:

Year 1	$230 500
Year 2	$283 500
Year 3	$339 680

A difference equation that gives the value of the property in a particular year is in the form of $V_{n+1} = b + aV_n,\ V_1 = 230\,500$

a. Determine the value of *a* and *b*.

2 marks

b. What will the value of the property be in the 5th year, to the nearest dollar?

_____

_____

_____

1 mark

c. During which year will the value of the investment reach triple its original value?

_____

_____

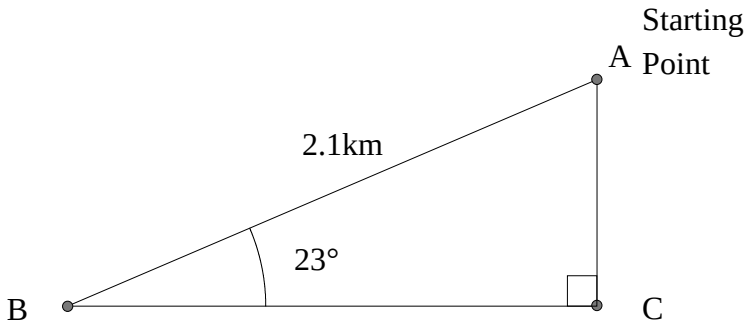
_____

2 marks

Module 2: Geometry and Trigonometry

Question 1

Peter's running club, the Jennali Joggers, maps out the following course for its annual fun run. The runners begin at Point A, then run to Point B, then turn and run due east to Point C, then return to Point A to finish the run.



a. Find the magnitude of angle BAC.

_____

_____

1 mark

b. Find the total length of the course to 1 decimal place.

_____

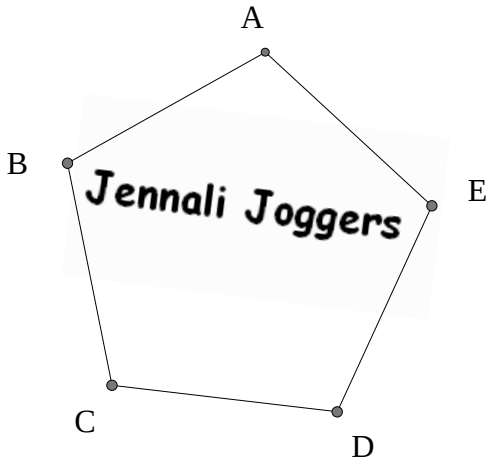
_____

_____

2 marks

Question 2

Peter is organising the club badges for the new members of the club. The design of the badge is shown below. It is in the shape of a regular pentagon:



a. Find the size of angle ABC.

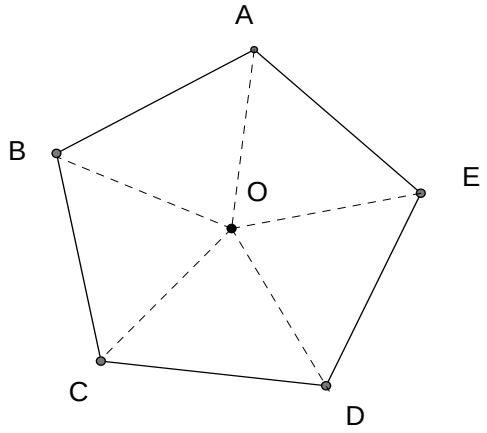
_____

_____

_____

1 mark

b. Given that each side has a length of 4 cm, show that the length of line BO is 3.4 cm.



_____

_____

_____

1 mark

c. Find the area of the club badge (give your answer correct to one decimal place).

_____

_____

2 marks

d. If the diagram is a 1: 2.5 model, what is the area of the badge in real life?

_____

_____

1 mark

Question 3

In preparation for the fun run, Peter runs on a bearing of 064 T for 2.8 km to the base of a cliff, then changes direction and walks on a bearing of 138T for a further 6.1 km to the edge of a lake.

a. Find how far Peter will have to run to return to his starting point, in kilometres, to 2 decimal places.

_____

_____

_____

1 mark

b. Find the bearing Peter will have to run to return to his starting point, to the nearest minute.

_____

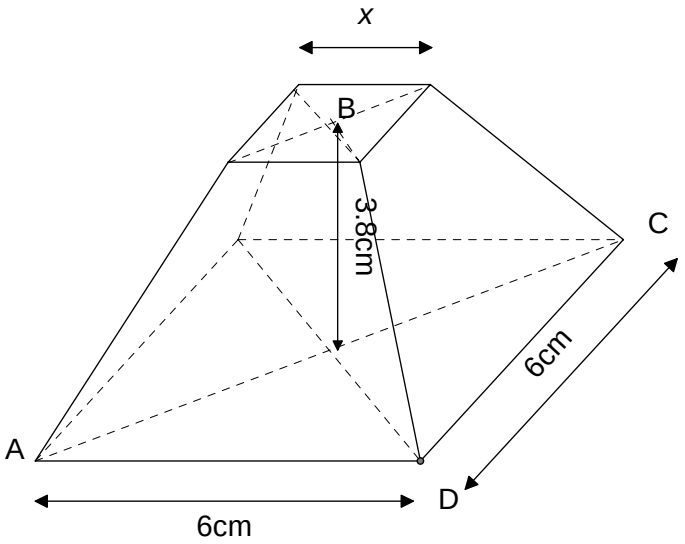
_____

_____

2 marks

Question 4

The trophy that is to be given to the first place runner is shown below. A square based pyramid has been removed from the top of the trophy. The removed section was 1.4 cm in height and had a volume of 5.6 cm^3.



a. Find the value of x, correct to one decimal place.

1 mark

b. Find the value of angle ABC to the nearest degree.

2 marks

c. Find the volume of the trophy to the nearest cm^3.

1 mark

Module 3: Graphs and Relations

Question 1

The table of values below gives the intake of money at a local market against the number of hours after the opening time of 9.00 am.

Hours (x)	Money intake (y)
2	$2
3	$6.25
4	$16
5	$31.25

The relationship between hours (x) and money intake (y) is $y = kx^3$.

a. Find the value of k.

_____

_____

_____

1 mark

b. After how many hours did the money intake reach $250?

_____

_____

_____

1 mark

Question 2

The Back Yard Pottery Company makes mugs to sell at the local market. The cost of producing the mugs is a fixed cost of $380 plus $8 for each mug made.

a. Find the total cost of producing 35 mugs in a week.

_____

_____

1 mark

b. Write an equation for the cost (C) of producing x mugs.

_____

_____

1 mark

c. Write an equation to calculate the profit (P) in terms of the number of mugs (x).

_____

_____

1 mark

d. Find the number of mugs that would need to be sold to achieve a profit of $100.

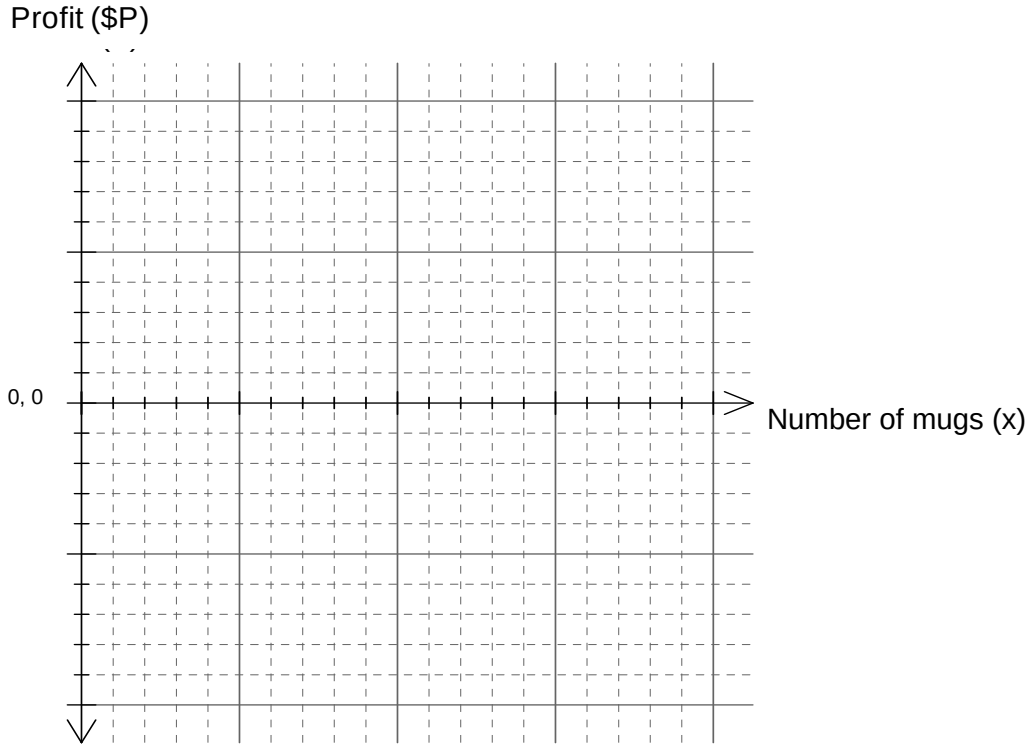
_____

_____

_____

1 mark

e. On the axis below, draw the graph of the rule for the total profit for selling x mugs. Include an appropriate scale.



1 mark

f. Find the number of mugs that would need to be sold to break even.

_____

1 mark

Question 3

The pottery company has decided to start to manufacture ceramic vases. They are to come in two sizes: small and large. To meet demand, the manufacturer has agreed to produce a maximum of 74 vases. They have also decided that they have enough material to make no more than 50 small vases and 35 large vases, and they are to manufacture at most three times as many small vases as large vases. The manufacturer must make at least one vase of each type.

Let x be the number of small vases and y be the number of large vases.

a. Using the above information, generate the 3 remaining constraints.

Constraint 1 $y \leq 35$

Constraint 2 $x \geq 1$

Constraint 3 $y \geq 1$

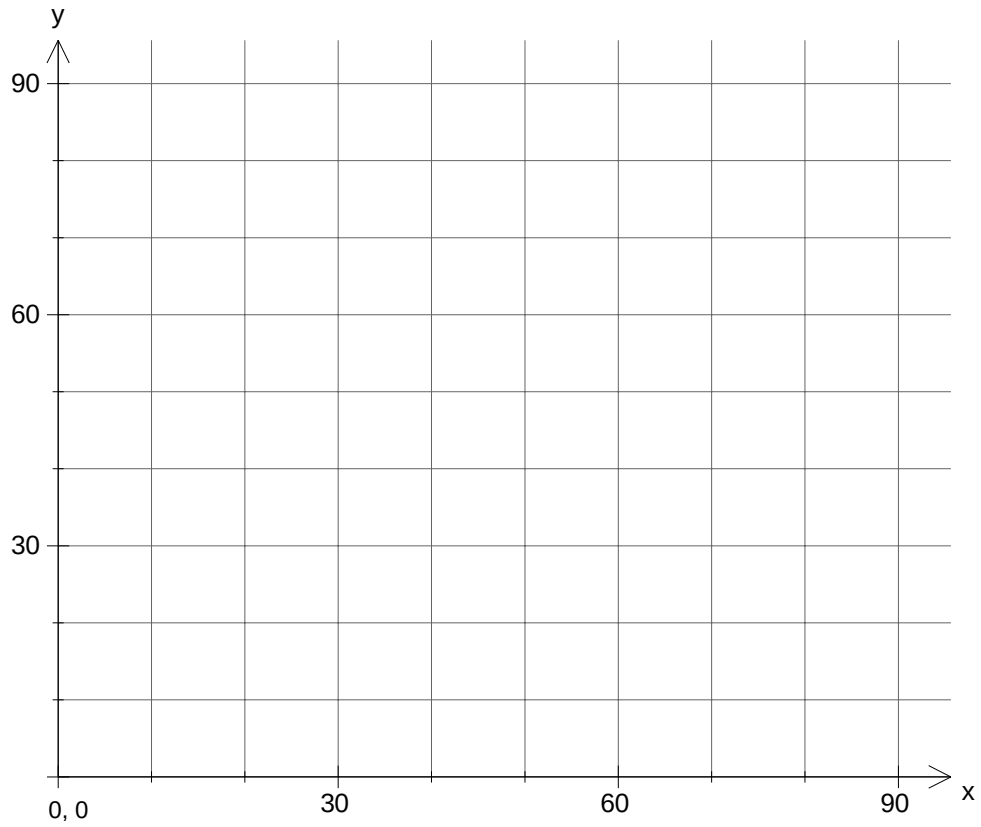
Constraint 4 ☐

Constraint 5 ☐

Constraint 6 ☐

3 marks

b. On the set of axes below, graph the 5 constraints and show the feasible region.



2 marks

c. The profit on the large vase is $10 and the profit on the small vase is $18. Write an objective function for the manufacturer's profit, P dollars.

_____

_____

1 mark

d. Find the number of each type of vase that the manufacturer should make in order to make a maximum profit, while still satisfying the constraints.

_____

_____

1 mark

Module 4: Business-related Mathematics

Question 1

The local hardware store, Nuts and Bolts, has their yearly sale. A drill that was normally $425 is reduced to $395.

a. What is the percentage discount? Write your answer to one decimal place.

_____

_____

1 mark

b. What will be the value of the drill in 3 years, to the nearest dollar, if the reducing balance depreciation rate is 30% (given that the original value is taken as the discounted price)?

_____

_____

1 mark

Question 2

Two employees of Nuts and Bolts, Peter and Sam, each negotiate different contracts for the purchase of their car. Sam's car costs $21 000. He pays a deposit of $3 000 and makes 40 monthly repayments of $700 using a hire-purchase contract.

Peter's car costs only $15 000. His hire-purchase agreement is a deposit of $1 000, with 24 equal monthly repayments. He will pay a flat rate of interest of 21% per annum.

a. Calculate the total interest that Sam will pay.

_____

_____

1 mark

b. Calculate the total amount that Peter will pay for his car.

_____

_____

_____

_____

1 mark

c. Which employee is paying a higher rate of interest for his loan? Show working to justify your choice.

2 marks

d. Determine the effective interest rate per annum for Peter's contract. Give your answer correct to one decimal place.

1 mark

Question 3

Another employee of Nuts and Bolts, Phillipa, has been saving for a holiday. She has $6 500 saved and is considering a number of options.

a. The first option is a term deposit of 12 months that pays 8.1% per annum with the interest payable at the end of the month. What will be the value of her investment at the end of the year, to the nearest dollar?

2 marks

b. Phillipa's holiday is going to cost $9 000. She would like to reach her target by the end of the year. A second option is to invest the money into a management fund that pays interest at the end of each month and earns 5.7% p.a. How much will she need to contribute to the fund each month, to the nearest cent, so that she can pay for her holiday?

_____

_____

_____

2 marks

Question 4

Nuts and Bolts have been approached by a local charity to set up a perpetuity in the form of a scholarship. They decide on the following arrangement:
An investment of $120 000 will be offered. The perpetuity is to provide $4 500 per year to disadvantaged students from the local area.

a. What will the interest rate be, if compounded monthly, that will require the perpetuity to make the yearly payment?

_____

_____

2 marks

b. How much will the yearly grant be increased, to the nearest dollar, if the decision is made to compound the amount daily rather than monthly (considering that the interest rate will not change)?

_____

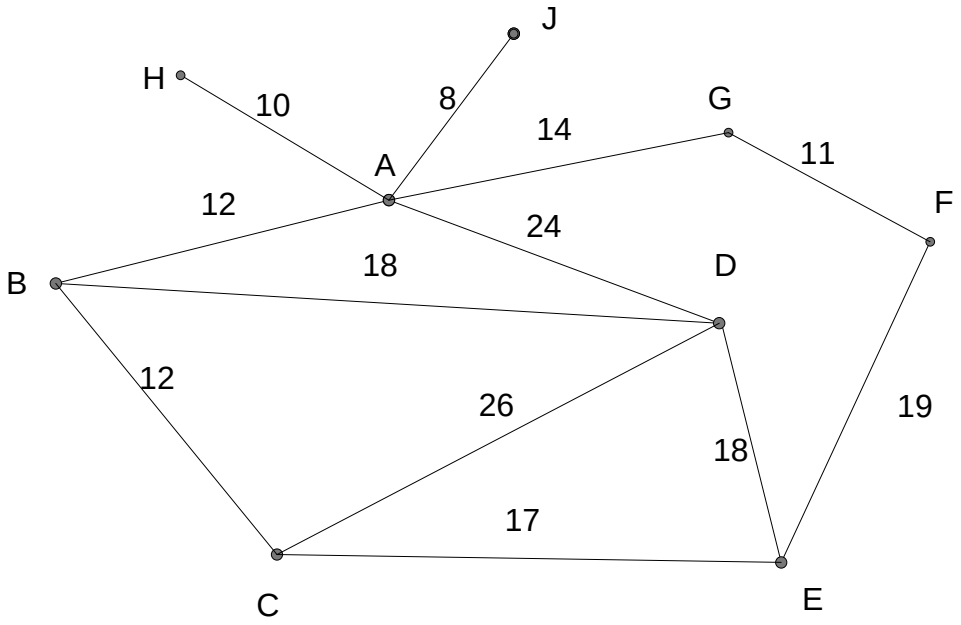
_____

2 marks

Module 5: Networks and Decision Mathematics

Question 1

Beth is a sales representative for a local food company. As part of her job, each week she travels to the towns in the network below. The paths between each town represent the roads that can be travelled. The numbers represent the distance between each town in kilometres.



a. Find the sum of the degrees of the vertices in the network.

_____

_____

1 mark

b. Beth is to travel from Town A to Town E. List the shortest path that Beth could take.

_____

_____

1 mark

c. What is the distance of the minimum spanning tree?

_____

_____

1 mark

d. During a particular day, Beth wishes to visit each town on the network to check with her suppliers without visiting the same town more than once. She does not need to start and finish at the same vertex.

i. What mathematical term describes Beth's task?

_____

_____

1 mark

ii. This task is not possible for Beth to complete. If she were to start at Vertex H, what is the maximum number of towns she could visit while attempting this task, without visiting the same town more than once?

_____

_____

1 mark

Question 2

The purpose of Beth's visit is to promote 4 new products. Towns A, B, C and D will sell one product each. They have each quoted Beth the cost of marketing each product in their town.

	Product 1	Product 2	Product 3	Product 4
A	$145	$140	$110	$160
B	$260	$300	$230	$130
C	$270	$250	$200	$130
D	$180	$210	$180	$170

a. Find a possible matching between town and products so that the total cost to Beth is at a minimum.

_____

_____

_____

_____

_____

2 marks

b. State the minimum cost.

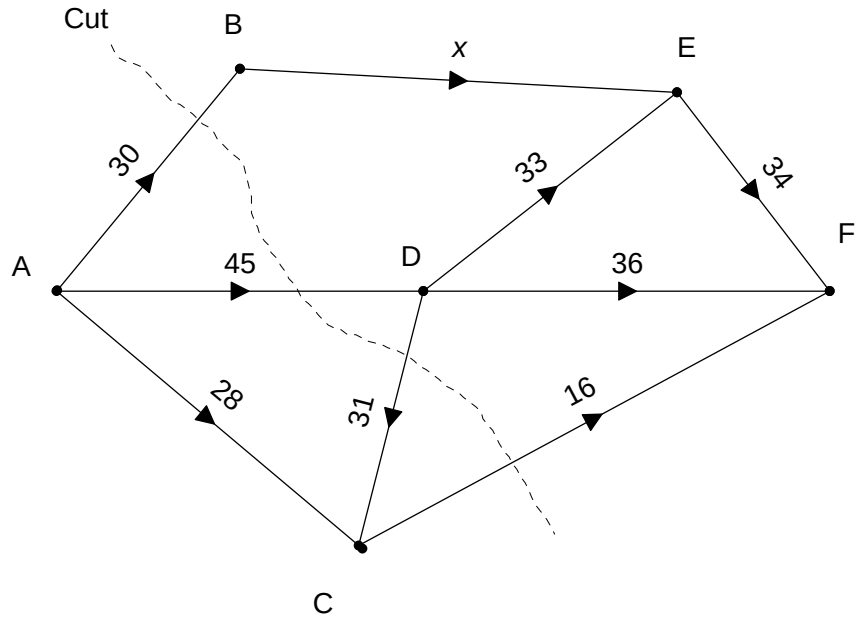
_____

_____

1 mark

Question 3

The network below shows the path through a National Park, with the values on the edges representing the number of people that are allowed on the track due to safety concerns.



a. What is the value of the cut shown above?

_____

_____

1 mark

b. Given that the maximum flow of the network is 70, find the value of x.

_____

_____

_____

1 mark

Question 4

While building one of the paths, the builders need to complete a number of activities in order to construct the tracks. The immediate predecessors and activity times (in days) are shown in the following table.

Activity	Predecessor	Duration
A	-	5
B	-	6
C	A	2
D	A	3
E	B	3
F	C, D	7
G	E	4
H	G, F	2
J	H	5

a. Complete the network diagram below, labelling each activity and the duration.

2 marks

b. Give the float time for Activity C.

_____

_____

_____

1 mark

c. Find the critical path.

_____

_____

1 mark

d. What is the earliest time in which all activities can be completed?

_____

_____

1 mark

SECTION B – continued

Module 6 – Matrices

Question 1

The matrix, G, represents the marks achieved by 4 students taking 3 separate tests. Each test is out of 10.

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 4 & 7 \\ 6 & 5 & 1 \\ 7 & 9 & 2 \end{bmatrix}$$

a. Find element (3, 2) and interpret the result.

_____

_____

1 mark

Matrix G is multiplied by a second matrix H: $\begin{bmatrix} 0 \\ 1.05 \\ 0 \end{bmatrix}$

b. Prove that the product matrix GH is defined.

_____

_____

1 mark

c. Solve the product matrix, GH, and explain your result.

_____

_____

2 marks

Question 2

The matrix A is given as $\begin{bmatrix} 2 & c-3 \\ 5 & c \end{bmatrix}$

a. Find the determinant of matrix A.

_____

_____

1 mark

b. Find the inverse of matrix A in terms of c.

2 marks

c. Find the value of c if the matrix A is to be singular.

_____

_____

1 mark

d. The result of the sum of matrix A and matrix B is $\begin{bmatrix} -2 & c \\ 2 & 2c+4 \end{bmatrix}$. Find matrix B.

1 mark

Question 3

a. Express the following simultaneous equations in matrix form:

$3x + 2y + 4z = 26$

$7x + y = 24$

$7x + 2z = 32$

1 mark

b. Solve the values of x, y and z to two decimal places.

2 marks

Question 4

In a survey of local road users in a large town, 25% of drivers are currently using the new tollway regularly to divert around a busy intersection.

Eight percent of the surveyed drivers plan to start using the tollway the following year, while 6% of the current users plan to stop using the tollway the following year, due to the high cost involved.

a. Represent this situation using a transition matrix.

1 mark

b. The owners of the tollway believe that in 8 years, if the pattern continues, at least 45% of the local road users will regularly be using their tollway. Support or reject their claim using appropriate matrix calculations.

2 marks

END OF QUESTION AND ANSWER BOOK