The Mathematical Association of Victoria
FURTHER MATHEMATICS
2008 Trial written examination 2: Solutions – Extended Answer

Core: Data analysis
Question 1

a. i. The relationship between *additional households* and *additional cars* is positive; as the *additional households* increase, the *additional cars* also increase. **[A1]**

ii. The strength of the relationship appears to be strong as the points on the scatterplot are closely grouped together. Moderate strength could also be acceptable as the scale is very large. **[A1]**

iii. The form of the relationship is linear. **[A1]**

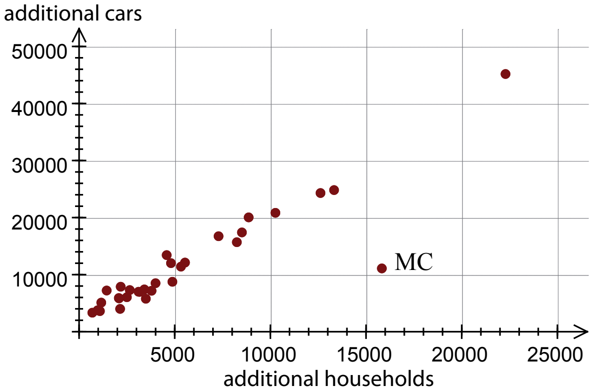
b. *additional cars*

= 1.87 × *additional households* + 1966

Variables correct **[A1]**
Figures correctly placed **[A1]**

c. Interpretation of the slope:
For each increase of one in *additional households* there is an increase of 1.87 in *additional cars*. **[A1]**

d.



Point plotted in approximately the correct position **[A1]**

e. The least-squares regression line would lean towards the point MC so the slope would decrease slightly. **[A1]**

Question 2

a.

<table>
<tr><th colspan="2">Driver deaths 2006</th><th colspan="2">Gender</th></tr>
<tr><th colspan="2"></th><th>Male</th><th>Female</th></tr>
<tr><td rowspan="2">Age Group</td><td>18– 25 years</td><td>$\frac{26}{113} \times \frac{100}{1} = 23$</td><td>$\frac{9}{41} \times \frac{100}{1} = 22$</td></tr>
<tr><td>> 25 years</td><td>77</td><td>78</td></tr>
<tr><td></td><td>Total</td><td>100</td><td>100</td></tr>
</table>

[M1]

b. There is only a difference of 1% in the proportions of 'male driver deaths that were young male driver deaths' and 'female driver deaths that were young female driver deaths' so the statement is not supported by the data. **[H1]**

c. Overall percentage of driver deaths that were young driver deaths

$= \frac{35}{154} \times \frac{100}{1} = 23\%$ **[A1]**

d. If young drivers are 15% of the drivers then we would expect that young driver deaths are 15% of the driver deaths. Young driver deaths are 23% of the overall driver deaths, a difference of 8%, so they are over-represented in driver deaths. **[A1]**

Question 3

a. There appears to be a downward trend.

Trend **[A1]**

b. If 2007 is Year 18 then 2010 will be Year 21. Substituting this in the equation gives:
Percentage of driver deaths that are young driver deaths
$= 33.7 - 0.49 \times 21 = 23.4\%$

Year correct **[A1]**
Substitution **[H1]**

Further Mathematics Exam 2: Solutions – Extended Answer
Module 1: Number patterns

Question 1

a. $t_1 = a = 7$ and $t_{13} = a + 12d = 91$
Substituting $a = 7$ in $a + 12d$ gives
$7 + 12d = 91$
$12d = 84$
$d = 7$; there is a yearly increase of 7% **[A1]**

b. 2009 is term 15
$t_{15} = a + 14d = 7 + 14 \times 7 = 105$
105% of the population would have internet access in 2009 using this model. **[A1]**

Question 2

a. Substituting in $t_n = ar^{n-1}$ gives
$t_{13} = 7 \times r^{12}$ [= 91] **[A1]**

b. $7 \times r^{12} = 91$
$r^{12} = 91$
So $r = \sqrt[12]{13} \approx 1.238$

For $r^{12} = 13$ **[A1]**

c. 2008 is the 14th term
$= 7 \times 1.238^{13} = 112.32$
(112.69 if using the calculator value for r)

Either 112 or 113 **[A1]**

Question 3

Both the arithmetic and geometric models will give the percentage of the population with internet access as greater than 100% from 2009, so these models are not feasible.

Greater than 100% **[A1]**

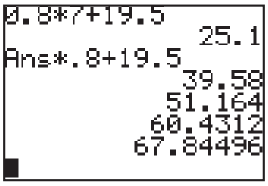
Question 4

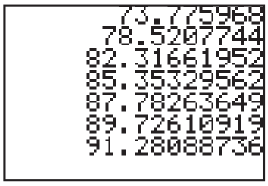
a. $t_2 = 0.8 \times t_1 + 19.5 = 0.8 \times 7 + 19.5 = 25.1$

Working shown **[M1]**
Answer correct **[A1]**

b. Using the calculator is the best option

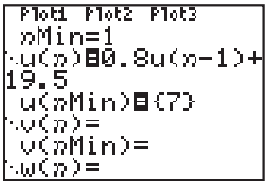
i. By iteration and counting the terms as you go:

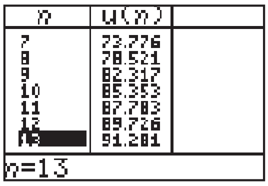




The 13th term is 91.3

ii. Using the sequence mode:





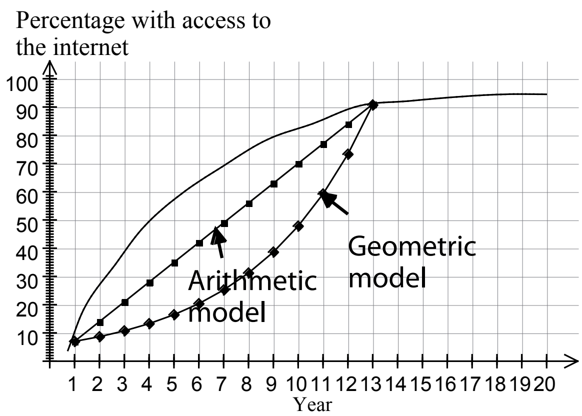
The 13th term is 91.3 **[A1]**

c. Scrolling down the table, or using iteration,will show a maximum value of 97.5 from term 56 onwards. Alternatively: Considering that $t_{n+1} = t_n$ when the maximum value is reached gives the equation

$$t_n = 0.8t_n + 19.5$$
$$0.2t_n = 19.5$$
$$t_n = \frac{19.5}{0.2} = 97.5$$

[A1]

d.



General shape correct, going through points of intersection **[A1]**

Tapering to less than 100% **[A1]**

Question 5

a. The sequence of new customers is geometric with $a = 1800$ and

$r = \frac{1620}{1800} = \frac{1458}{1620} = 0.9$

The total number of new customers in 10 weeks is the sum of the first 10 terms of the sequence.

Using $S_n = \frac{a(1-r^n)}{1-r}$

$$S_{10} = \frac{1800(1-0.9^{10})}{1-0.9} = 11723.78 \approx 11724$$

Geometric with $r = 0.9$ **[A1]**
Geometric calculation **[H1]**

b. It is possible to find the sum to infinity for this geometric sequence as $-1 < r < 1$ (excluding 0).

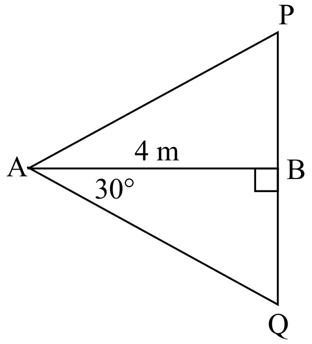
Using $S_\infty = \frac{a}{1-r}$; $S_\infty = \frac{1800}{1-0.9} = 18\,000$ customers. **[A1]**

Further Mathematics Exam 2: Solutions – Extended Answer
Module 2 : Geometry and trigonometry

Question 1

a. The hexagon is a regular hexagon so the angle PAQ at the centre is equal to $\frac{360°}{6} = 60°$ **[A1]**

b.



In the right-angled triangle ABQ the angle BAQ = 30°

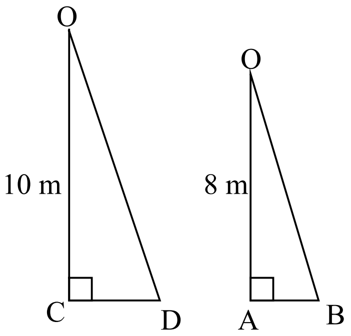
$\tan 30° = \frac{BQ}{4}$

$BQ = 4\tan 30° \approx 2.3094$ m

$PQ = 2 \times 2.3094 \approx 4.619$ m **[M1][A1]**

Question 2

a.



The sides of the similar triangles OCD and OAB are in the ratio 10 : 8 = 5 : 4

So CD : AB = 5 : 4

$\frac{CD}{AB} = \frac{5}{4}$ Hence $\frac{CD}{4} = \frac{5}{4}$; CD = 5m

Ratio **[A1]**

Correct working of ratio **[H1]**

b.

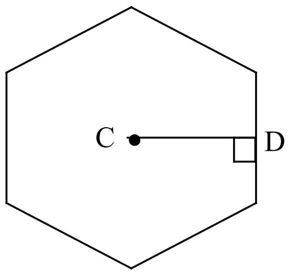
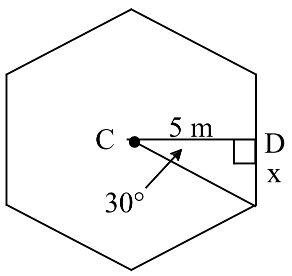


Figure 4

Any line drawn from the centre to the middle of one side **[A1]**

c. CD = 5 m

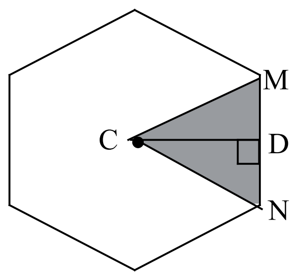


$\tan 30° = \frac{x}{5}$ so $x = 5\tan 30° \approx 2.8868$ m

Side of the hexagon = $2 \times 2.8868 \approx 5.774$ m

[A1]

d.

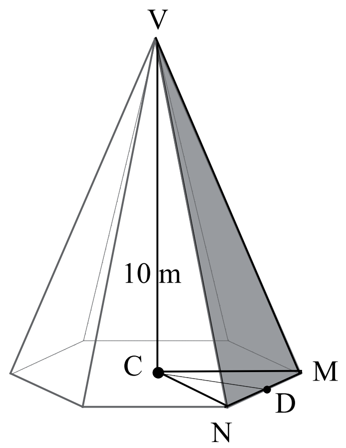


Area of shaded triangle = ½ × base × height
= ½ × 5.7735 × 5
= 14.4337..

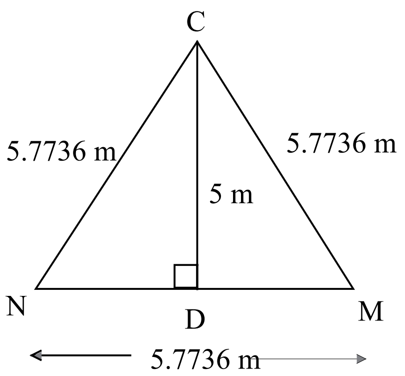
Area of hexagon = 6 × 14.4337...
= 86.6025... m^2
≈ 86.60 m^2

Area of triangle **[M1]**
Correct answer **[A1]**

e.



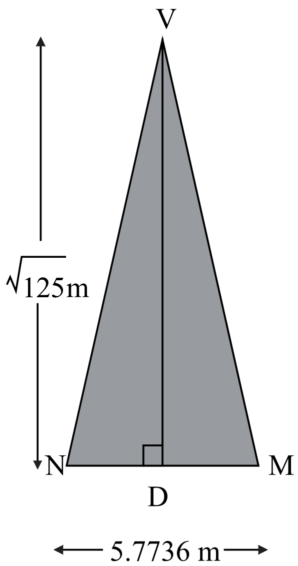
The triangle CMN is an equilateral triangle with side length 5.7736 m and an altitude, CD, of 5 m.



The altitude, VD, of the triangle VNM can be found using Pythagoras' rule.

$VD^2 = 5^2 + 10^2 = 125$

$VD = \sqrt{125} \approx 11.1803$



Area of the triangle VNM
= ½ × base × perpendicular height
= ½ × 5.7736 × $\sqrt{125}$
≈ 32.28 m^2

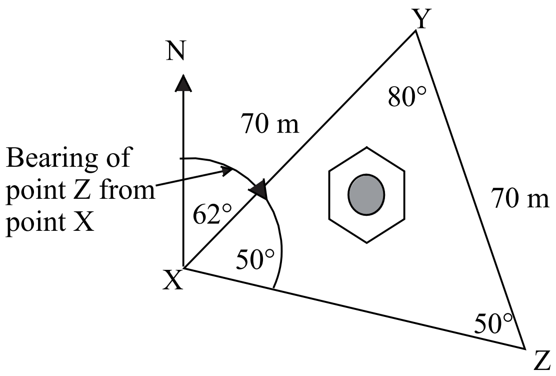
Alternatively Heron's formula can be
used to calculate the area, although it is a more tedious method:

$VN^2 = 10^2 + 5.7736^2$
VN ≈ 11.5471
VM is the same length as VN
$s = \frac{1}{2}(11.5471 + 11.5471 + 5.7736) = 14.4339$
and
Area = $\sqrt{s(s-11.5471)(s-11.5471)(s-5.7736)}$
≈ 32.28 m^2

Correct measurements **[A1]**
Length of VD **[M1]**
Calculation of area **[H1]**

Question 3

a.



The triangle XYZ is an isosceles triangle so the angle YXZ is equal to 50° and the angle XYZ is equal to 180° – 2×50° = 80°

XZ can be found using the cosine rule:

$$XZ = \sqrt{70^2 + 70^2 - 2 \times 70 \times 70 \cos 80^\circ}$$
$$\approx 89.99 \text{ m}$$

Angles correct **[A1]**
Length XZ correct **[A1]**

b. The bearing of point Z from point X is the angle that XZ makes with the North direction line.

Bearing = 62° + 50° = 112° **[H1]**

Further Mathematics Exam 2: Solutions – Extended Answer
Module 3 : Graphs and relations

Question 1

a. The cost equation for incandescent globes is $B = 0.98 + 0.015x$ **[A1]**

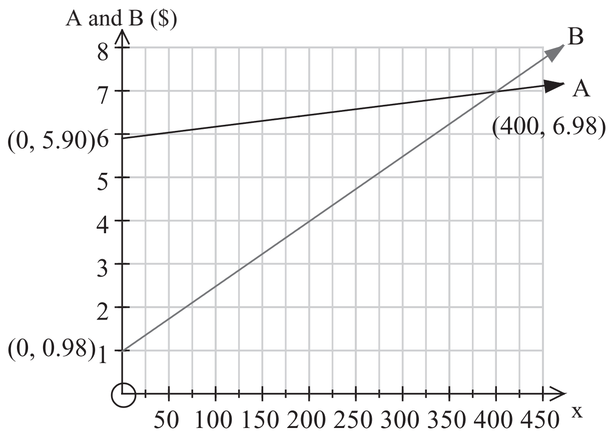
b. Equate the cost equations and solve for x

$$\begin{aligned} A &= B \\ 5.90 + 0.0027x &= 0.98 + 0.015x \\ 5.90 - 0.98 &= 0.015x - 0.0027x \\ 4.92 &= 0.0123x \\ x &= \frac{4.92}{0.0123} = 400 \end{aligned}$$

After 400 hours the costs are the same.

Equating costs **[M1]**
Correct answer **[A1]**

c. When $x = 400$ the cost is
$0.98 + 0.015\times400 = 6.98$



Graph of B **[H1]**
Graph labelled correctly **[H1]**

d. Cost, A = $5.90 + 0.0027 \times 8000$
= \$27.50 **[A1]**

e. The cost of running a light with an incandescent light globe for 8000 hours will require 8 light globes plus running costs.

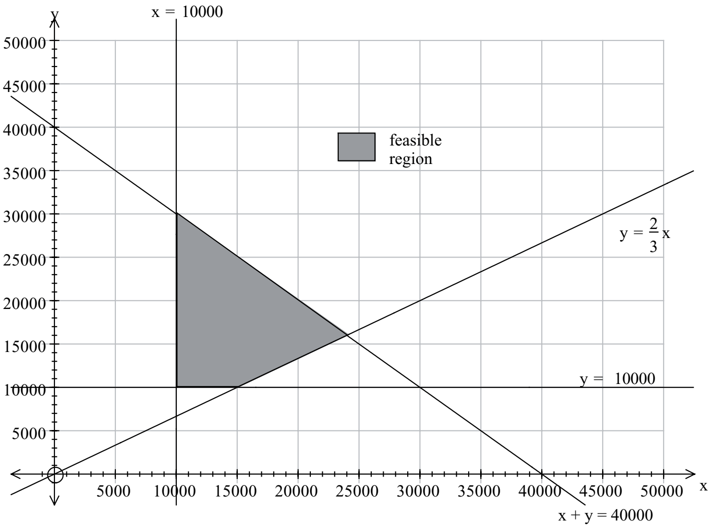
Cost = $8 \times 0.98 + 0.015 \times 8000$
= \$127.84 **[A1]**

Question 2

a. 'a maximum of 40 000 light globes in a day' means the total of all types of light globe is less than or equal to 40 000.

The constraint is $x + y \leq 40\,000$ **[A1]**

b. and c.



Boundary line for constraint **[H1]**

Region **[H1]**

d. Profit = $0.16x + 0.15y$ **[A1]**

e. The extreme points are (10 000, 10 000), (10 000, 30 000), (15 000, 10 000) and the intersection point of $x + y = 40\,000$ and

$y = \frac{2}{3}x$

Substitute $y = \frac{2}{3}x$ into $x + y = 40\,000$

$x + \frac{2}{3}x = 40\,000$

$\frac{5}{3}x = 40\,000$

$x = \frac{120000}{5} = 24\,000$

And hence $y = 16\,000$;

The point is (24 000, 16 000)

Point	Profit($)
(10 000, 10 000)	0.16 × 10 000 + 0.15 × 10 000 = 3100
(10 000, 30 000)	0.16 × 10 000 + 0.15 × 30 000 = 6100
(15 000, 10 000)	0.16 × 15 000 + 0.15 × 10 000 = 3900
(24 000, 16 000)	0.16 × 24 000 + 0.15 × 16 000 = **6240**

Maximum profit ($6240) occurs when the manufacturer makes 24 000 incandescent globes and 16 000 CFL globes. **[M1][A1]**

Question 3

From the shape of the graph the model is of the form

$y = \frac{k}{x}$ or $y = \frac{k}{x^2}$

From the table, it can be seen that xy = constant = 24 , so the graph is of the form

$y = \frac{k}{x}$ or $y = kx^{-1}$

Hence $k = 24$ and $n = -1$

k value **[A1]**
n value **[A1]**

Further Mathematics Exam 2: Solutions – Extended Answer
Module 4: Business-related mathematics

Question 1

a) Pre-GST price = Sale price ÷ 1.1.
= $12 500 ÷ 1.1
= $11 364 **[A1]**

b) Deposit amount = rate × sale price
= 24% × $12 500
=$3000 **[A1]**

c) $8 stamp duty per $200 lots
For a $12 500 there are 63 lots of $200.

multiples of $200 = $\frac{\$12\,500}{\$200}$ = 62.5 or rounded up to 63.

Stamp Duty = 63 × $8 = $504 **[A1]**

d) loan amount = purchase price + stamp duty – deposit
= $12 500 + $504 – $3000
= $10 004 **[A1]**

e) The monthly instalment on loan repayments where

Principal, P = $10 004
Interest rate, r = 9.5% pa
Loan period, T = 3 years

Interest charged $I = \frac{\mathrm{Pr}T}{100}$

$= \frac{10\,004 \times 9.5 \times 3}{100}$

= $2851.14

[M1]

Total repayments = Loan amount + Interest charged
= $10 004 + $2851.14
= $12 855.14

monthly instalment = $\frac{\text{Total repayments}}{\text{number of repayments}}$

$= \frac{\$12\,855.14}{36 \text{ months}}$

= $357.09
= $357 per month

[M1][A1]

Question 2

Calculate the average inflation rate for the past fourteen years.

A = price at the end = \$1.45 per litre
P = original price = \$0.85 per litre
T = time in years = 14 years
R = compound factor = $1 + \frac{r}{100}$
r = interest rate per annum

$$A = PR^T$$

$$\$1.45 = \$0.85 \times R^{14}$$

$$R^{14} = \frac{\$1.45}{\$0.85}$$ **[M1]**

$$R = \sqrt[14]{1.70588}$$

$$R = 1.0388857$$

$$R = 1 + \frac{r}{100}$$

$$1.0388857 = 1 + \frac{r}{100}$$ **[M1][A1]**

$$r = 3.89\%\ pa$$

Question 3

a) the amount of depreciation to the nearest dollar in the first two years.

BV = bookvalue = unknown

P = original price = \$12 500

T = time in years = 2 years

r = depreciation rate per annum = 22½%pa

$$R = \text{compound factor} = 1 - \frac{r}{100}$$

$$= 1 - \frac{22\,{}^1\!/_2}{100} = 0.775$$

$$BV = PR^T$$

$$= \$12\,500 \times 0.775^2$$ **[M1][A1]**

$$= \$7507.81$$

Depreciation = Purchase Price – Bookvalue

= \$12 500 – \$7507.81

= \$4992.19

= \$4992

Alternative method is iteration.

$$\text{Deprec 1st Yr} = 12\,500 \times \frac{22.5}{100}$$

= \$2812.50

BV = \$12 500 – \$2812.50

= \$9687.50

$$\text{Deprec 2nd Yr} = 9687.50 \times \frac{22.5}{100}$$

= \$2179.69

Total deprec = \$2812.50 + \$2179.69

= \$4992.19

= \$4992

b) BV = scrap value = \$5 000

P = original price = \$12 500

T = time in years = unknown

r = interest rate per annum = 22½%pa

R = compound factor = 0.775

$$BV = PR^T$$

$$\$5000 = \$12\,500 \times 0.775^T$$

$$0.775^T = \frac{\$5000}{\$12\,500}$$

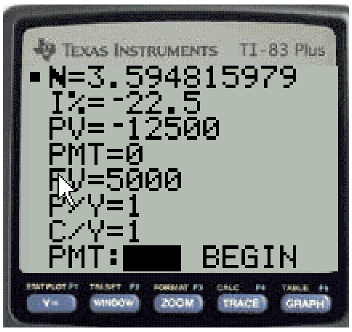
$$T \times \log(0.775) = \log(0.4)$$

$$T = \frac{\log(0.4)}{\log(0.775)} = 3.5948$$

To reach \$5000 it will take 4 years **[M1][A1]**

Alternative is to use TVM solver.

N = ?, I% = -22.5, PV = –12 500, PMT = 0,
FV = 5000, P/Y & C/Y = 1, and solved
N = 3.59 yrs



c) No, because reducing balance is a diminishing depreciation method where the book value is always 77.5% of previous year's book value. **[A1]**

Further Mathematics Exam 2: Solutions – Extended Answer
Module 5:Networks and decision mathematics

Question 1

a. A Hamiltonian circuit starts and finishes at the same vertex (**B**) and passes through each vertex once only.

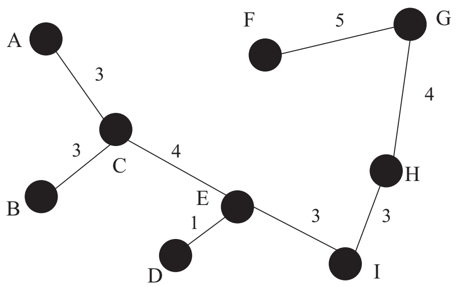
Possible circuits are

B-A-C-F-G-H-I-E-D-**B**
B-A-C-F-G-H-E-I-D-**B**
B-C-A-F-G-H-I-E-D-**B**
B-C-A-F-G-H-E-I-D-**B**
B-D-E-I-H-G-F-A-C-**B**
B-C-E-D-I-H-G-F-A-**B** **[A1]**

b. The shortest route from winery A to winery I passes through wineries A, C, E and I.
The length of the shortest route is therefore 3 + 4 + 3 = 10 km **[A1]**

c. i. The start of a minimal tree is the shortest edge which for the given network has a length of 1 km and wineries E and D. So E is a suitable location for an ambulance service to access all wineries along the minimum total length of roads to be upgraded. **[A1]**

ii. The two possible minimal-length spanning trees are shown below.



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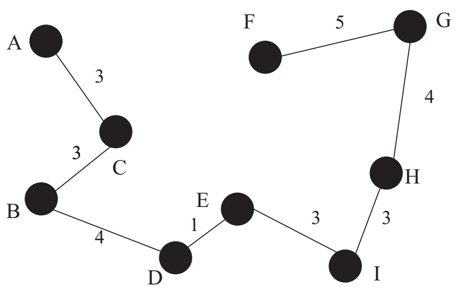
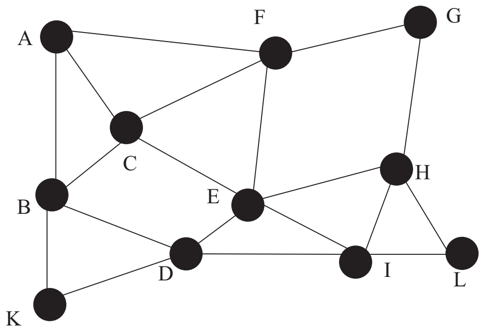


[A1]

iii. The minimum length of roads required to be upgraded to a sealed surface is the total distance of the minimum spanning tree.
1 + 4 + 3 + 3 + 3 +3 + 4 + 5 = 26 km **[A1]**

d. i. For an Euler path all vertices must have an even degree with at the most a pair of odd degree vertices.



The original network had vertices with odd degrees at A,B,D,E,I and H. By adding two roads to each of the new wineries, K and L creates two new even degrees. Then connecting each of the four roads to B,D,I and H will convert them to even degrees. This leaves vertices A and E as the pair of odd degrees. **[A1]**

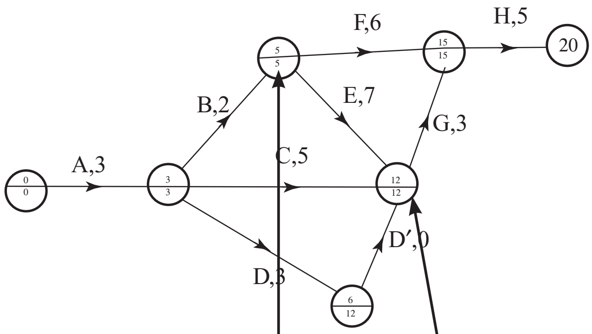
ii. If the winery tour was to follow an Euler path the start and/or finish are at vertices A and E. **[A1]**

Question 2

Preliminary Investigation

Forward and Backward scanning of the earliest and latest starting times would provide all of the answers. In the nodes, record the earliest starting time working from A to H recording the weeks in the upper semicircle. Then perform a backward scan recording the latest starting times in the bottom of the semicircles at each node.

a. Add the Dummy activity as shown. Direction must be clearly shown for full marks. **[A2]**



b. Completing the table using the values in the semicircles.

Activity	*Earliest start time*	*Latest start time*
A	0	0
B	3	3
C	3	7
D	3	9
E	5 **[A1]**	5
F	5	9 **[A1]**
G	12 **[A1]**	12
H	15	15

F is not on the critical path and so its latest time is 15 – 6 = 9^{th} week.

c. The critical path for the network is A, B, E, G and H. This is the path where the earliest start time and latest start time are the same or have **no slack**. **[A1]**

d. The completion time for the project is the final value which is 20. Alternatively add up all activities that are on the critical path.

3 + 2 + 7 + 3 + 5 = 20 weeks **[A1]**

e. C has slack time as it is not on the critical path.

The slack time for activity C is

Latest start – Earliest start – activity = slack time

12 – 3 – 5 = 4 weeks. **[A1]**

Further Mathematics Exam 2: Solutions – Extended Answer
Module 6 : Matrices

Question 1

a. Represent the season results as a 2 × 3 matrix

$$\begin{bmatrix} 10 & 1 & 7 \\ 8 & 2 & 8 \end{bmatrix}$$

[A1]

b. Represent the points awarded as a 3 × 1 matrix

$$\begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$$

[A1]

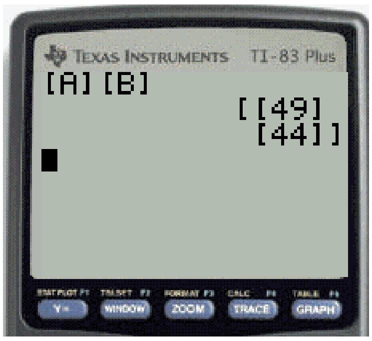
c. Using matrices, calculate the total premiership points for each team.

$$\begin{bmatrix} 10 & 1 & 7 \\ 8 & 2 & 8 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 49 \\ 44 \end{bmatrix}$$

[M1] [A1]

Eaglehawks have 49 premiership points. Magpiedemons have 44 premiership points.

Using a graphics calculator:



Question 2

a. x = adult ticket prices
y = children ticket prices
z = pensioner ticket prices

First Equation:
$245x + 310y + 76z = \$720.40$

Second Equation:
$120x + 44y = \$311.00$ **[A1]**

Third Equation:
$321x + 410y + 102z = \$945.80$ **[A1]**

b. The simultaneous equations as matrices.

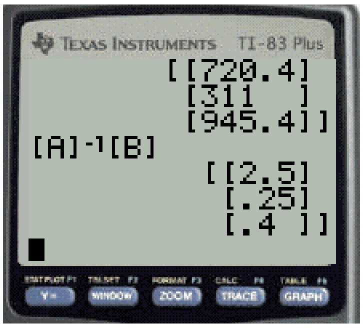
$$\begin{bmatrix} 245 & 310 & 76 \\ 120 & 44 & 0 \\ 321 & 410 & 102 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 720.40 \\ 311.00 \\ 945.80 \end{bmatrix}$$ **[M1]**

Evaluating as a 3 × 1 matrix

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 245 & 310 & 76 \\ 120 & 44 & 0 \\ 321 & 410 & 102 \end{bmatrix}^{-1}\begin{bmatrix} 720.40 \\ 311.00 \\ 945.80 \end{bmatrix}$$

$$= \begin{bmatrix} 2.5 \\ 0.25 \\ 0.4 \end{bmatrix}$$ **[M1]**

Using a graphics calculator



The price of adult tickets is \$2.50.
The price of child tickets is \$0.25
The price of pensioner tickets is \$0.40. **[A1]**

Question 3

a. The transition matrix for when Eaglehawks are the better side.

$$\begin{array}{cc} & \text{From} \\ & \begin{array}{cc} EH & MD \end{array} \\ \text{To} \begin{array}{c} EH \\ MD \end{array} & \begin{bmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{bmatrix} \end{array}$$

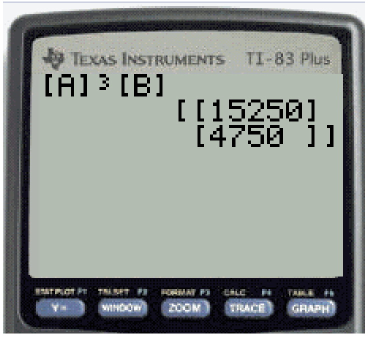
[A1]

b. An appropriate matrix equation.

$$\begin{array}{c} EH \\ MD \end{array}\overset{\begin{array}{cc} EH & MD \end{array}}{\begin{bmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{bmatrix}^3} \begin{bmatrix} 10\,000 \\ 10\,000 \end{bmatrix} = \begin{bmatrix} 15\,250 \\ 4750 \end{bmatrix}$$

[M1]

Use a graphics calculator to evaluate.



After 3 years of the Eaglehawks performing better than the Magpiedemons, there are 15 250 Eaglehawks' supporters and 4750 Magpiedemons. **[A1]**

c. The supporter base distribution equation matrix is

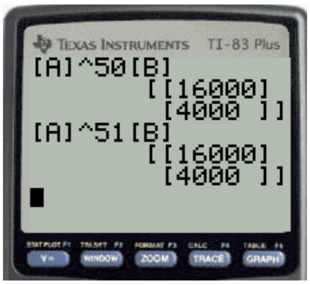
$$\begin{array}{c} EH \\ MD \end{array}\overset{\begin{array}{cc} EH & MD \end{array}}{\begin{bmatrix} 0.7 & 0.05 \\ 0.3 & 0.95 \end{bmatrix}^3} \begin{bmatrix} 10\,000 \\ 10\,000 \end{bmatrix} = \begin{bmatrix} 4818.75 \\ 15181.25 \end{bmatrix}$$

Rounding off to whole numbers there are expected to be 15 181 Magpiedemons' supporters and 4819 Eaglehawks' supporters if the Magpiedemons had three years of better performance. **[A1]**

d. For long term trends set n to a very large number say 50.

Eaglehawks performing well in a long run will have 16 000 supporters at maximum.

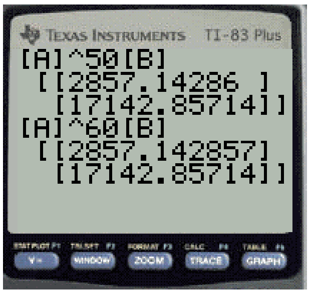
$$\begin{array}{c} \\ EH \\ MD \end{array}\begin{array}{c} \begin{array}{cc} EH & MD \end{array} \\ \begin{bmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{bmatrix}^{50} \end{array} \begin{bmatrix} 10\,000 \\ 10\,000 \end{bmatrix} = \begin{bmatrix} 16\,000 \\ 4000 \end{bmatrix}$$



Magpiedemons performing well in a long run will have 17 143 supporters at maximum.

$$\begin{array}{c} \\ EH \\ MD \end{array}\begin{array}{c} \begin{array}{cc} EH & MD \end{array} \\ \begin{bmatrix} 0.7 & 0.05 \\ 0.3 & 0.95 \end{bmatrix}^{60} \end{array} \begin{bmatrix} 10\,000 \\ 10\,000 \end{bmatrix} = \begin{bmatrix} 2857.14286 \\ 17142.85714 \end{bmatrix}$$

[M1]



Therefore Magpiedemons would be expected to have a larger supporter base if they were the better side for a long period. **[A1]**

END OF SOLUTIONS