The Mathematical Association of Victoria
FURTHER MATHEMATICS
2008 Trial written examination 1: Solutions – Multiple choice

Solutions
Core: Data analysis

1. D	2. C	3. A	4. E	5. B
6. A	7. D	8. C	9. A	10. D
11. B	12. A	13. E		

Module 1: Number patterns

1. B	2. A	3. B	4. E	5. E
6. C	7. E	8. B	9. A	

Module 2: Geometry and trigonometry

1. D	2. C	3. C	4. E	5. A
6. D	7. C	8. E	9. B	

Module 3: Graphs and relations

1. D	2. A	3. A	4. E	5. C
6. D	7. B	8. E	9. A	

Module 4: Business-related mathematics

1. B	2. A	3. B	4. D	5. C
6. C	7. E	8. B	9. A	

Module 5: Networks and decision mathematics

1. D	2. C	3. A	4. B	5. C
6. E	7. B	8. D	9. A	

Module 6: Matrices

1. D	2. E	3. B	4. E	5. C
6. A	7. C	8. B	9. A	

SOLUTIONS – continued

Question 1 Answer D

The measures of centre
Affected by outliers: mean.
Not affected by outliers: median and mode.

The measures of spread
Affected by outliers: range and standard deviation.
Not affected by outliers: interquartile range.

Therefore median & interquartile range are not affected by outliers.

Question 2 Answer C

Using the first seven digits from 0 to 6 to represent a win for Hewey Weyton,

Numbers	Result
0,1,2,3,4,5,6	WIN (70% chance)
7,8,9	LOSE (30% chance)

Random Number Table
9 9 1 5 4 7 0 3 9 2 7 9 2 3 0 9 1 0 5 8

and from the random numbers given, there are a total of 12 digits that represent a win.

Question 3 Answer A

Discrete numerical as data is collected from having COUNTED the number of wickets taken by a bowler.

Nominal categorical - your favourite television station eg Channel Seven (a label).

Continuous numerical - speed of a car captured by a speed camera (measured).

Continuous numerical - weight of a car.

Ordinal categorical -finishing place in a 100 metres sprint (label with order e.g. 1^{st}, 2^{nd}, 3^{rd} and so on).

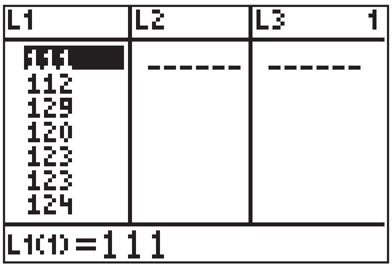
Question 4 Answer E

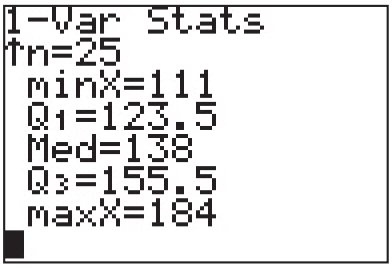
Ten pin scores are numerical discrete data usually in the three figure values of 100 or more and so each of the intervals is of discrete values only e.g. 110 to 119.

Score x	Frequency f
110-119	3
120-129	6
130-139	5
140-149	4
150-159	2
160-169	1
170-179	3
180-189	1

Question 5 Answer B

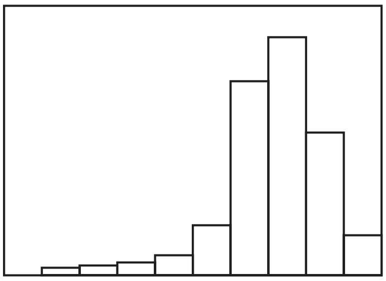
Using the graphics calculator the following screens give the correct summary of a median of 138 pins, interquartile range of 32 pins and a range of 73 pins.





Question 6 Answer A

The histogram shown below has a negatively skewed distribution. There are fewer pieces of the data in the lower values. No outliers are evident and without given values cannot be tested mathematically.



Question 7 Answer D

For the data set given the 5 key parameters are

$x_{\min} = 5, Q_1 = 7.5, Q_2 = 8, Q_3 = 9.5, x_{\max} = 15$

Interquartile range (IQR).

$IQR = Q_3 - Q_1 = 9.5 - 7.5 = 2.0$

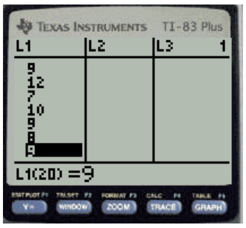
The test for outliers is

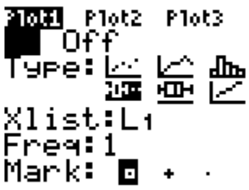
$Q_3 + 1.5 \times IQR = 9.5 + 1.5 \times 2 = 12.5$ and

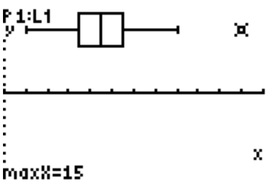
$Q_1 - 1.5 \times IQR = 7.5 - 1.5 \times 2 = 4.5$

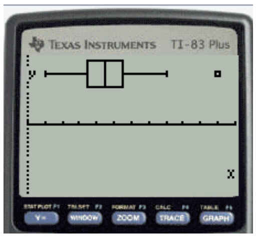
So there is only one score outside the accepted range of 4.5 and 12.5 and that was data value of 15.

Alternatively use a graphics calculator









Question 8 Answer C

The independent variable is hours of daylight.

$\bar{x} = 6$ hours $s_x = 1.0$ hours

The dependent variable is weight of watermelon

$\bar{y} = 3.0$ kg $s_y = 0.4$ kg

Gradient, m

$$m = \frac{rs_y}{s_x} = \frac{0.8 \times 0.4}{1.0} = 0.32$$

Y-intercept, c

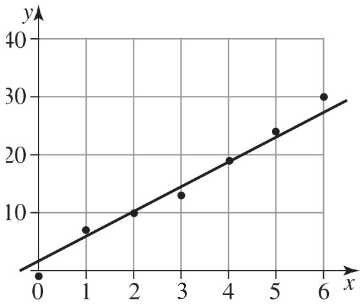
$c = \bar{y} - m\bar{x} = 3.0 - 0.32 \times 6.0 = 1.08$

Relationship

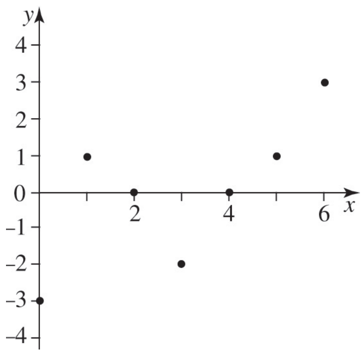
$y = c + mx$

weight of watermelon (*kg*) = 1.08 + 0.32 × *hours of day*

Question 9 Answer A



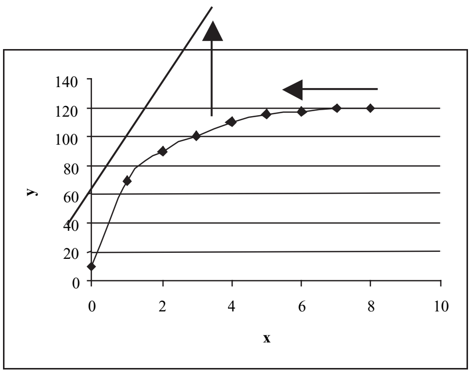
The first point near the origin is below the line of best fit shown and therefore the residual is a negative value as given in the residual plot **A**.



Question 10 Answer D

For a 5-point moving median, the last two points of the smoothed values come from:
Weeks 5 to 9 in order from lowest to highest are 570, 620, 630, 660 and 680 with the median of 630.
Weeks 6 to 10 in order from lowest to highest are 570, 620, 660, 680 and 720 with the median of 660.

Question 11 Answer B



Examining the large values of x, to linearise requires the x values to be compressed towards the suggested straight line. This could be either

$\log x$ or $\dfrac{1}{x}$.

Similarly, examining the large values of y, to linearise requires the y values to be stretched towards the suggested straight line. This could be y^2.

Question 12 Answer A

For 1200 grams of food then

$$\text{Amount of dog food (grams)} = \frac{-6000}{\text{Weight of dog (kg)}} + 1800$$

$$1200 = \frac{-6000}{\text{Weight of dog (kg)}} + 1800$$

$$-600 = \frac{-6000}{\text{Weight of dog (kg)}}$$

$$\text{Weight of dog (kg)} = \frac{-6000}{-600} = 10\text{kg}$$

Question 13 Answer E

The relationship given has

- the dependent variable as the amount of food in grams.
- the independent variable as the **reciprocal** of the weight of dog in kilograms

A measure of the variation is the coefficient of determination which is the square of the correlation coefficient.

$$\begin{aligned} r &= 0.9381 \\ r^2 &= 0.9381^2 \\ &= 0.880 \\ &= 88\% \quad . \end{aligned}$$

Thus 88% of variation in amount of food can be explained by the variation in the **reciprocal of** the weight of the dog.

Further Mathematics Exam 1: Solutions – Multiple choice
Module 1: Number patterns

Question 1 Answer B

$a = 125$, $d = -2$ and $n = 50$,

Substituting in $t_n = a + (n - 1)d$
$t_{50} = 125 + (50 - 1) \times -2$
$= 125 - 98$
$= 27$

Question 2 Answer A

$a = 5$ and $t_3 = a \times r^2 = 20$
$5 \times r^2 = 20$
$r^2 = 4$
$r = \pm 2$
$t_4 = ar = 5 \times 2^3 = 40$
or $t_4 = 5 \times (-2)^3 = -40$

Question 3 Answer B

If $t_{n+1} = 1.5t_n + b$; $t_1 = 10$ then $t_2 = 1.5t_1 + b$

Substituting $t_1 = 10$ *and* $t_2 = 12$
$12 = 1.5 \times 10 + b$
$12 - 15 = b$
$b = -3$

Question 4 Answer E

If the sum to infinity is a finite number then the sequence must be geometric with the common ratio, r, being a value between –1 and 1 (excluding 0).

This means that the terms will be decreasing in value but not at a constant rate. The only graphs where the terms decrease in value are Answers B (constant rate) and Answer E where $-1 < r < 0$.

Question 5 Answer E

If $t_{n+2} = 3t_{n+1} + t_n - 6$ then substituting the values for t_1 and t_2 gives
$t_3 = 3t_2 + t_1 - 6 = 3 \times 10 + 5 - 6 = 29$

Substituting the values for t_2 and t_3
$t_4 = 3t_3 + t_2 - 6 = 3 \times 29 + 10 - 6 = 91$

Question 6 Answer C

The sequence is 12, 14, 16,......52; an arithmetic sequence with $a = 12$, $d = 2$ and $t_n = 52$

Substituting in $t_n = a + (n - 1)d$ gives
$52 = 12 + 2(n - 1)$
$40 = 2(n - 1)$
$20 = n - 1$
$n = 21$
Hence $t_{21} = 52 = l$, the last term.

The sum of 21 terms can be found using
$S_n = \frac{n}{2}(a + l)$
$S_{21} = \frac{21}{2}(12 + 52) = 672$

Question 7 Answer E

The amount of Sam's salary each year forms the terms of a geometric sequence with $a = 42\,000$ and $r = 1 + \frac{6}{100} = 1.06$

The total for 8 years is the sum of the first 8 terms of this sequence.

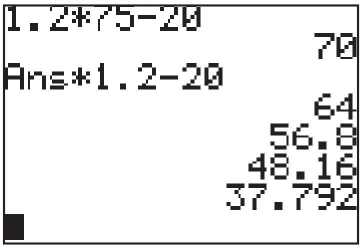
Substituting in $S_n = \dfrac{a(r^n - 1)}{r - 1}$ gives

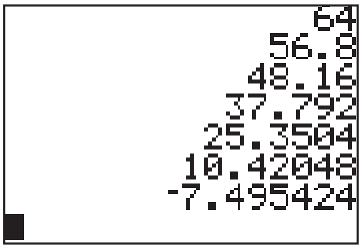
$$S_8 = \frac{42\,000(1.06^8 - 1)}{1.06 - 1} \approx 415\,694$$

Question 8 Answer B

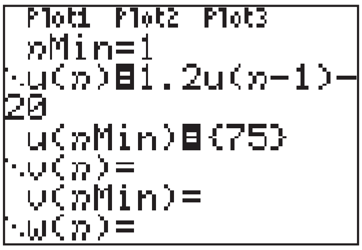
Using your calculator is the best method to answer this question.

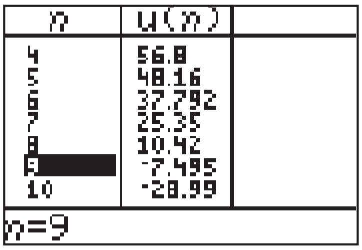
Either using iteration and counting:





Or using the sequence function and ψσ (Table):



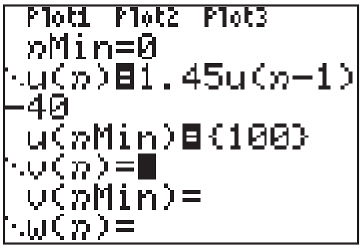


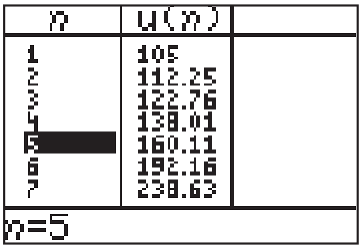
Question 9 Answer A

The herd will increase by 45% each year; an increase of 45% means a multiplying factor of
$1 + \frac{45}{100} = 1.45$

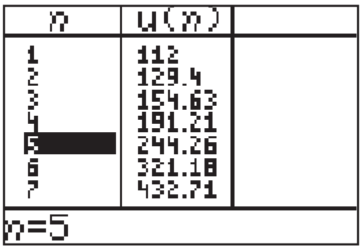
Using the calculator to find which of the models (can be any of alternatives A, B, D or E) produces a fifth term of approximately 160:

Answer A

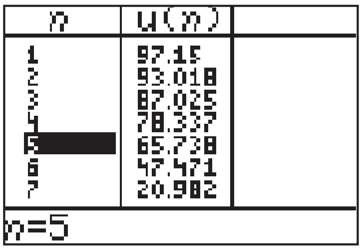




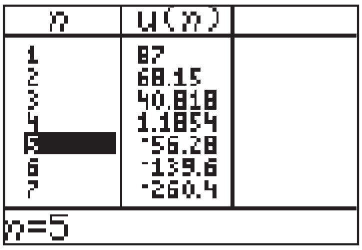
Answer B



Answer D

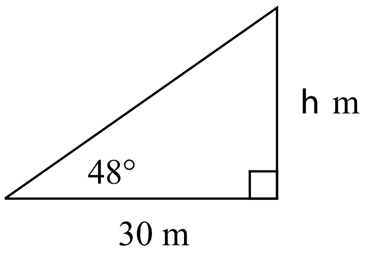


Answer E



Further Mathematics Exam 1: Solutions – Multiple choice
Module 2: Geometry and trigonometry

Question 1 Answer D



$\tan 48° = \frac{h}{30}$

$h = 30\tan48° = 33.32\text{m}$

Question 2 Answer C

The area enclosed = ½ (area of a circle of radius 26 cm) – ½(area of a circle of radius 12 cm)

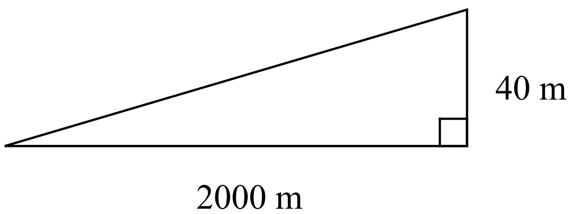
$= \frac{1}{2} \times \pi \times 26^2 - \frac{1}{2} \times \pi \times 12^2$

$\approx 835.66 \text{ cm}^2$

Question 3 Answer C

The point X is on the 40 m contour and the point Y is on the 80 m contour so the vertical distance between X and Y is 40 m

The horizontal distance between X and Y is 2 km = 2000 m

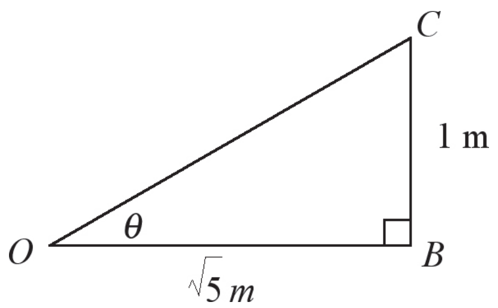


The average slope = $\frac{rise}{run} = \frac{40}{2000} = 0.02$

Question 4 Answer E

In triangle *OAB*

$OB = \sqrt{1^2 + 2^2} = \sqrt{5}$



In triangle *OBC*, $\tan\theta = \frac{1}{\sqrt{5}}$

Question 5 Answer A

The total surface area = TSA of the rectangular box – area of a circle radius r + $\frac{1}{2}$ (surface area of a sphere of radius r)

= [2 × 4 × 4 (top and bottom) + 4 × 4 × 2 (four sides)] – $\pi r^2 + \frac{1}{2} \times 4\pi r^2$

= [32 + 32] – $\pi r^2 + 2\pi r^2$

= 64 + πr^2 cm^2

Question 6 Answer D

The volume of a prism

= area of the triangular cross-section × length

Using *area of a triangle* = $\frac{1}{2}$ *absinC*

Volume of prism

= ($\frac{1}{2}$ × 32 × 24sin56°) × 40

≈ 12 734.02 cm^3

Question 7 Answer C

Triangles *OPS* and *OQR* are similar and their lengths are in the ratio 5 : 7

The areas of the triangles *OPS* and *OQR* will be in the ratio $5^2 : 7^2 = 25 : 49$

If x is the area of triangle *OPS* (shaded) then

$x : 7.5 = 25 : 49$

$$\frac{x}{7.5} = \frac{25}{49}$$

$$x = \frac{25 \times 7.5}{49} \approx 3.83 \text{ m}^2$$

Question 8 Answer E

The similar cones have lengths in the ratio 2 : 5 so their volumes will be in the ratio

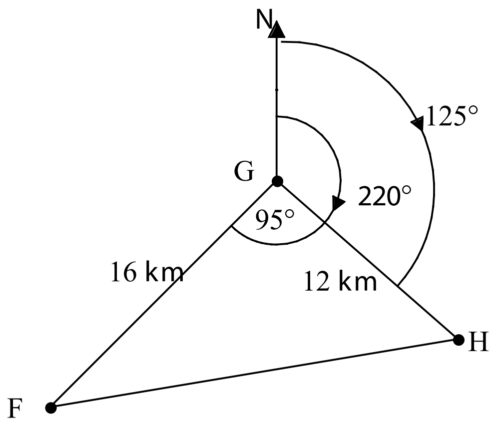
$2^3 : 5^3 = 8 : 125$

After removal of the top the remaining parts of the original 125 parts equals

125 – 8 = 117 parts.

As a percentage this is $\frac{117}{125} \times \frac{100}{1} = 93.6\%$

Question 9 Answer B



In triangle *FGH* the angle *FGH* is

220° – 125° = 95°

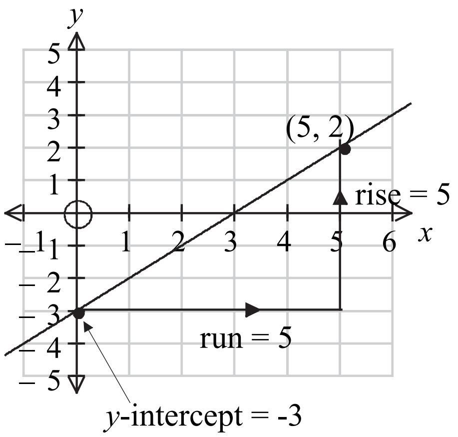
The distance between points *F* and *H* is found using the cosine rule:

$$FH = \sqrt{16^2 + 12^2 - 2 \times 16 \times 12 \cos 95°}$$

$$\approx 20.82 \text{ km}$$

Further Mathematics Exam 1: Solutions – Multiple choice
Module 3: Graphs and relations

Question 1 Answer D



Gradient = $\frac{rise}{run} = \frac{5}{5} = 1$

Question 2 Answer A

The straight line connecting *time* = 0 hours and *time* = 7 hours indicates a **constant rate** of increase. *Temperature* is increasing with *time* but the rate of increase is constant.

Question 3 Answer A

The average rate of change over the 12-hour period is equal to the gradient of the straight line joining the points (0, 14) and (12, 21)

Gradient = $\frac{rise}{run} = \frac{21-14}{12-0} = \frac{7}{12} \approx 0.583$

Question 4 Answer E

The regions to the left of the y-axis and below the x-axis are excluded so both the constraints $x \geq 0$; $y \geq 0$ apply. The points below $y = 6$ are also included so $y \leq 6$ is also a constraint.

The equation of the line going through (0, 3) and (3, 0) is $x + y = 3$ and the included points are where $x + y \geq 3$ or $y \geq 3 - x$

The equation of the other constraint is $y = 2x - 10$ so the constraint $y \geq 2x$ is not correct.

Question 5 Answer C

Rain fell on the days when the volume was increasing; where line segments have a positive gradient.

Rain fell on days 5, 11, 20, 22, 23, 24, 25; a total of 7 days.

Question 6 Answer D

profit = revenue – costs

The revenue from selling x articles at \$15 each is $15x$ dollars

$profit = 15x - (7250 + 6.4x) = 8.6x - 7250$

To 'make a profit' means 'profit' is greater than 0.

$8.6x - 7250 > 0$

$8.6x > 7250$

$x > \frac{7250}{8.6} \approx 843.02$

At least 844 articles need to be produced and sold.

Question 7 Answer B

The graph is a straight line going through the origin so it has an equation of the form $y = m(x^2)$ where m is the gradient of the straight line.

$$m = \frac{3}{2} \div \frac{3}{1} = \frac{3}{2} \times \frac{1}{3} = \frac{1}{2}$$

So the relationship between x and y is $y = \frac{1}{2}x^2$

If $x = 4$ then $y = \frac{1}{2} \times 4^2 = 8$

Question 8 Answer E

Booklets containing 25 pages cost \$0.20 per page so 25 × \$0.20 = \$5 each

100 booklets cost 100 × \$5 = \$500

Booklets containing 80 pages cost \$0.18 per page so 80 × \$0.18 = \$14.40 each.

200 × \$14.40 = \$2880

A total of \$500 + \$2880 = \$3380

Question 9 Answer A

There are three extreme points in the feasible region:

$P(0, 8)$, $R(6, 0)$ and the point of intersection of $5x + 6y = 30$ (1) and $8x + 3y = 24$ (2)

Multiply (2) by 2 and subtract (1):

$11x = 18$; $x = \frac{18}{11} \approx 1.64$ Substitute in (1)

$$\frac{5 \times 18}{11} + 6y = \frac{330}{11}$$

$$y = \frac{330 - 90}{11 \times 6} = \frac{240}{66} = \frac{40}{11} \approx 3.64$$

Substituting these points into the objective function $Z = 10x + 3y$:

$P(0, 8)$ $Z = 10 \times 0 + 3 \times 8 = 24$

$R(6, 0)$ $Z = 10 \times 6 + 3 \times 0 = 60$

$Q(1.64, 3.64)$ $Z = 10 \times 1.64 + 3 \times 3.64 = 27.32$

The minimum value occurs at point P.

Alternatively

Using the sliding line technique:

The gradient of the objective function is $\frac{-10}{3}$ which is 'steeper' than the gradient of PQ which is $\frac{-8}{3}$. To minimise the objective function we move a line with gradient $\frac{-10}{3}$ towards the origin and P is the last of the extreme points that the line will go through. Hence the minimum value of Z will occur at P.

Further Mathematics Exam 1: Solutions – Multiple choice
Module 4: Business-related mathematics

Question 1 Answer B

Capital gain = Selling Price – Purchase Price – Cost of Purchase – Cost of Selling

Capital gain = 300 000 – 250 000 – 10 000 – 10 000 = \$30 000

As it was sold within first twelve months there is **no reduction** of the capital gain amount by half.

Question 2 Answer A

Sale price = 90% of Retail price
\$27.00 = 0.9 × Retail price

$$\text{Retail price} = \frac{27.00}{0.9}$$
$$= \$30.00$$

Question 3 Answer B

Given
Simple interest, $I = \$2.55$

Principal, $P = \$564$

Time, $T = {}^{1}\!/_{12}$ year (for 1 month)

Then using simple interest formula

$$r\% = \frac{100 \times I}{P \times T}$$
$$= \frac{100 \times 2.55}{564 \times {}^{1}\!/_{12}}$$
$$= \frac{255}{47}\%\text{pa}$$
$$= 5.43\%\text{pa}$$

Question 4 Answer D

Using the perpetuity formula

$Q = \frac{\Pr}{100}$ and given

Q = regular payment = \$4000

r = 8% pa

$$P = \frac{100 \times Q}{r}$$
$$= \frac{100 \times 4000}{8}$$
$$= \$50\,000$$

\$50 000 needs to be invested

Question 5 Answer C

For compound interest given
P = \$5000
r = 8% pa (compounded yearly)
T = 3 years
n = 3

Compound interest = Total amount – Principal
= 6298.56 – 5000
= \$1298.56

For simple interest
P = \$5000
r = 9% pa
T = 3 years

$$I = \frac{\Pr T}{100}$$
$$= \frac{5000 \times 9 \times 3}{100}$$
$$= \$1350$$

Difference in interest rate is

Simple interest – compound interest
= 1350 – 1298.56
= \$51.44

Question 6 Answer C

Total depreciation = Purchase Price – Scrap value
= 32000 – 3200
= 28800

For straightline depreciation

$$\text{Annual depreciation amount} = \frac{\text{Total depreciation}}{\text{number of years}}$$

$$= \frac{28800}{10} = \$2880\text{pa}$$

Straightline depreciation rate

$$= \frac{\text{Annual depreciation amount}}{\text{Purchase Price}}$$

$$= \frac{2880}{32000} \times \frac{100}{1}$$

= 9%pa

Question 7 Answer E

Mr Ive Haddit has 15 years (age 45 to 60) to grow his current amount of \$120 000 to \$800 000.

$A = \$800\,000$
$P = \$120\,000$
$n = 15 \text{ years} \times 12 = 180 \text{ months}$

$$r\% = \frac{5.8\%\,pa}{12} = 0.48\dot{3}\% \text{ per month}$$

$$R = 1 + \frac{r}{100} = 1.0048\dot{3}$$

$$A = PR^n + \frac{Q(R^n - 1)}{R - 1}$$

$$800\,000 = 120\,000 \times 1.0048\dot{3}^{\,180} + \frac{Q(1.0048\dot{3}^{\,180} - 1)}{1.0048\dot{3} - 1}$$

$Q = \$1798.34 \approx \$1798$

Alternative using TVM SOLVER

Question 8 Answer B

Loan Amount = *Purchase Price – Deposit*
=\$19 800 – \$5000
=\$14 800

Total Payment = 36 payments
= 36 × \$650
= \$23 400

Interest charged = *Total payments – Loan amount*
= \$23 400 –\$14 800
= \$8600

Interest Rate *(r)*

$$I = \frac{\Pr T}{100}$$

(transposing the formula)
(divide both sides by P × T)

$$r = \frac{I \times 100}{P \times T}$$

$$r = \frac{8600 \times 100}{14\,800 \times 3}$$

$r \approx 19.37\% = \text{FR}$

(c) $\text{ER} = \frac{2n}{n+1} \times \text{FR}$

$$= \frac{2 \times 36}{36+1} \times 19.37$$

$$= \frac{72}{37} \times 19.37$$

$\approx 37.69\%$

Question 9 Answer A

Using the annuity formula

$$A = P \times R^n - \frac{Q(R^n - 1)}{R-1}$$

Where

A = unknown
P = \$80 000
n = 5 years × 4 = 20 quarterly payments

$r\% = \frac{12\% pa}{4} = 3\%$ per quarter

$$R = \frac{8600 \times 100}{14\,800 \times 3}$$

Q = \$4240

A. $A = 80\,000 \times 1.03^{20} - \frac{4240(1.03^{20} - 1)}{1.03 - 1}$

Further Mathematics Exam 1: Solutions – Multiple choice
Module 5:Networks and decision mathematics

Question 1 Answer D

Count the number of edges at each vertex.

Vertex	*Degree*
A	2
B	4
C	2
D	2
E	3
F	2
G	3

The total number or sum is 18.

Question 2 Answer C

For planar graphs with 7 vertices and 10 edges, Euler's formula states

Vertices = edges – faces + 2
7 = 10 – faces + 2
faces = 10 + 2 – 7
faces = 5

Question 3 Answer A

The euler path where all **nine** edges are used once only is

A. G – B – C – F – B – A – E – G – D – E

Option B has 10 edges where A to E is repeated.
Options D and E have only eight edges.
Option C has edge A to E repeated.

Question 4 Answer B

Find the longest edge E – F (21).

Add the next longest edge to either E or F:
E – F – D(40).

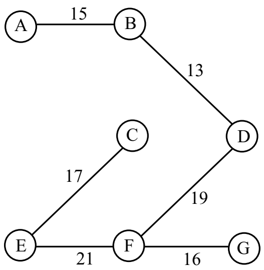
Add the next longest edge to E, F or D:
E – C (57).

Add the next longest edge to F, D or C:
E – F – G (73).

Add the next longest edge to D or C:
E – F – D – B (86).

Add the next longest edge to C or B:
E – F – D – B – A (101).

The maximum spanning tree has a length of 101.



Question 5 Answer C

A planar graph is where any crossings of edges are removed.

Question 6 Answer E

A one-stage adjacency matrix indicates number of edges leaving a node and its destiny vertex.

For example vertex A has no edges leaving and thus it is 0 for all vertices

$$\text{From}\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix}\overset{\begin{matrix} & & \textit{To} & & \\ A & B & C & D & E \end{matrix}}{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}}$$

Question 7 Answer B

The node from which activity E leaves, has only one activity leading into that node, that being activity B.

Question 8 Answer D

Activity H is preceded by F
Activity F is preceded by C and A.
Activity C is critical as it is preceded by B

So the path to activity H is B – C – F – H
The total hours are 3 + 9 + 11 = 23 hours.

Question 9 Answer A

The critical path is B – C – F – H – K with an earliest finish time of 42 hours.

Further Mathematics Exam 1: Solutions – Multiple choice
Module 6: Matrices

Question 1 Answer D

Order of matrix is stated as *row* × *column*. The matrix given has 3 rows and 2 columns and so the order is 3 × 2.

Question 2 Answer E

A diagonal matrix is a square matrix where the values of the elements in either diagonal are equal.

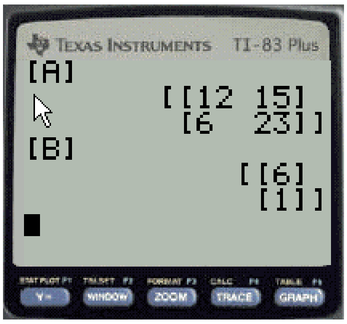
Question 3 Answer B

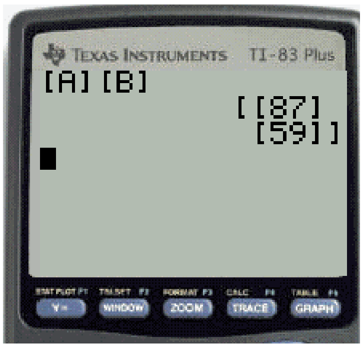
Matrix A is in fact the multiplicative inverse of matrix B which means $A \times B = I$ is true as is $AB = BA$. Given all three matrices are of the same order then addition is possible and for addition $A + B = B + A$ and $A + (B + I) = (A + B) + I$ are true.

However the subtraction of matrices is associative and thus $A - B = B - A$ is false.

Question 4 Answer E

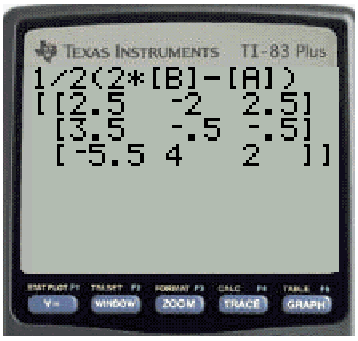
The number of points is represented by a 2 × 1 matrix and thus it is the product of a **2** × 2 × 2 × **1**.





Question 5 Answer C

Using the graphics calculator



Question 6 Answer A

The columns represent *from* and the rows represent *to*. For example for node A there is one line each leaving from A to B and C.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \left[\begin{array}{ccccc} \ & & & & \\ 1 & & & & \\ 1 & & & & \\ \ & & & & \\ \ & & & & \end{array}\right] \end{array}$$

Continuing the process results in

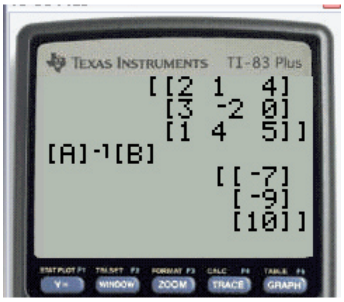
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \left[\begin{array}{ccccc} 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right] \end{array}$$

Question 7 Answer C

The matrix form of the simultaneous equations is

$$\begin{bmatrix} 2 & 1 & 4 \\ 3 & -2 & 0 \\ 1 & 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 17 \\ -3 \\ 7 \end{bmatrix}$$

Where $AX = B$ then $X = A^{-1}B$ and using a graphics calculator gives the following solution.



$x = -7 \quad y = -9 \quad z = 10$

Question 8 Answer B

$S_{steady\ state} = T^n \times S_0$ where n is very large.

$n \geq 50$ is recommended but $n = 40$ also gives a steady state condition.

$$S_{steady\ state} = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.8 & 0.3 \\ 0.2 & 0 & 0.4 \end{bmatrix}^{40} \begin{bmatrix} 100 \\ 50 \\ 80 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3 & 0.3 & 0.3 \\ 0.6 & 0.6 & 0.6 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 100 \\ 50 \\ 80 \end{bmatrix}$$

$$= \begin{bmatrix} 69 \\ 138 \\ 23 \end{bmatrix}$$

Question 9 Answer A

The matrix representation is

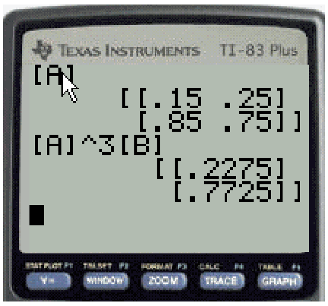
$$S_{4th\,day} = S_3 = \begin{bmatrix} 0.15 & 0.25 \\ 0.85 & 0.75 \end{bmatrix}^3 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Where $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is the initial state where first row is for dry day set to 0 and second row set to 1 as it is initially a wet day.

The fourth day is only 3 days from the initial day i.e.

S_0 = initial or 1st day
S_1 = 2nd day
S_2 = 3rd day
S_3 = 4th day

Using a graphics calculator gives



The 0.2275 or 22.75% in the first row represents the chance of it being a dry day. The 0.7725 or 77.25% in the second row represents chance of it being a wet day on the fourth day.

END OF SOLUTIONS