2008 Further Mathematics Trial Exam 2 Solutions
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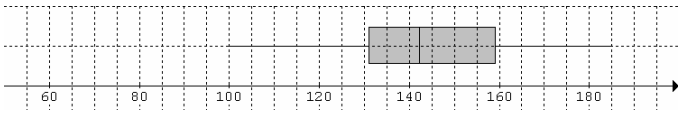
Core – Data analysis

Q1a The mean height ≈ 142,
95% are in the interval $142 \pm 2\sigma \approx 142 \pm 36$, $\therefore \sigma \approx 18$.

Q1b $z = -2$ corresponds to the height that is 2σ lower than the mean height, i.e. around 107.

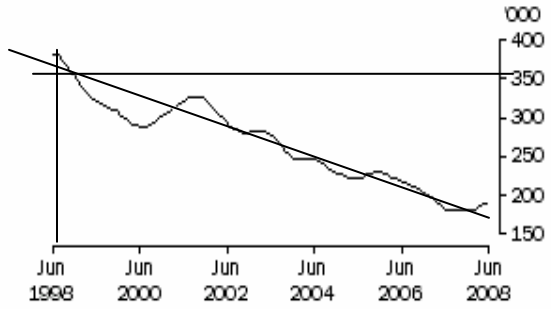
Q1c Median ≈ 142, $Q_1 \approx 131$, $Q_3 \approx 159$, min ≈ 100, max ≈ 185.



Q2a $0 < time < 10$ or $0 \le time \le 10$.

Q2b $-20.5 \times 12 + 435 = 189$ thousands.

Q2c



Number of males looking for full-time work $= -20 \times$ time $+ 370$.

Q2d In June 2010, number of males looking for full-time work $= -20 \times 12 + 370 = 130$ thousands.

% of unemployed males $\approx \dfrac{130}{189} \times 100\% \approx 69\%$.

Q3a Non-linear upward trend. $r = 0.8883$.

Q3b

Log d	0.2553	0.3802	0.7634	0.7924	0.8195	0.8921
h (m)	2.0	2.9	3.6	4.0	4.9	4.0

0.9345	1.0934	1.2833	1.4014	1.5079	1.7126
4.5	6.4	7.5	9.3	7.4	9.7

$h = 5.4531 \times \log d + 0.1381$.

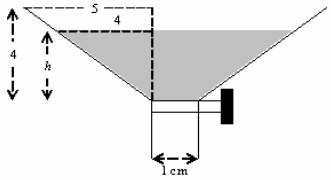
Q3c $r^2 = 0.9053$ of variability is explained by $h = 5.4531 \times \log d + 0.1381$.

Q3d When $d = 60$ cm, $h = 5.4531 \times \log 60 + 0.1381 = 9.8345$ m.

Module 2: Geometry and trigonometry

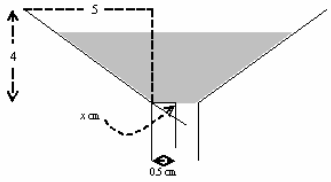
Q1a $r = 4.5$ cm, $A = \pi r^2 = 63.6$ cm^2

Q1b



$\dfrac{h}{4} = \dfrac{4}{5}$, $h = \dfrac{16}{5} = 3.2$.

Q1c



$\dfrac{x}{5} = \dfrac{4}{0.5}$, $\therefore x = 0.4$. Volume of water

$= \dfrac{1}{3}\pi(4.5)^2(3.2 + 0.4) - \dfrac{1}{3}\pi(0.5)^2(0.4) = 76.2$ cm^3. (Not 76.3)

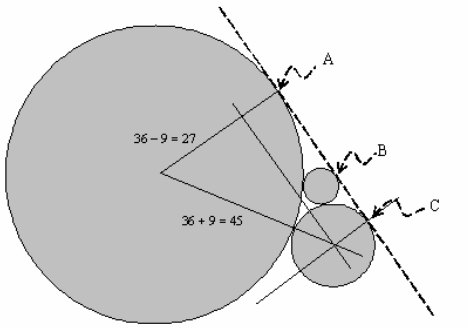
Q2a Side lengths are $a = 4.0 + 9.0 = 13$, $b = 9.0 + 36 = 45$ and $c = 36 + 4.0 = 40$. $s = \dfrac{1}{2}(a + b + c) = 49$.

$A = \sqrt{s(s-a)(s-b)(s-c)} = 252$ m^2.

Q2b $45^2 = 13^2 + 40^2 - 2(13)(40)\cos\theta°$,

$\theta° = \cos^{-1}\left(\dfrac{13^2 + 40^2 - 45^2}{2(13)(40)}\right) \approx 104°$.

Q2c

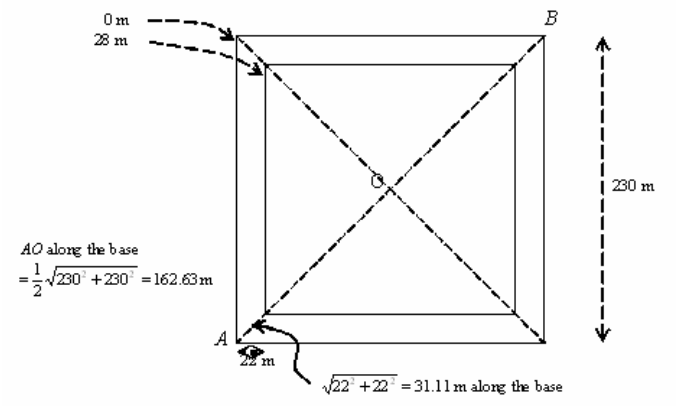


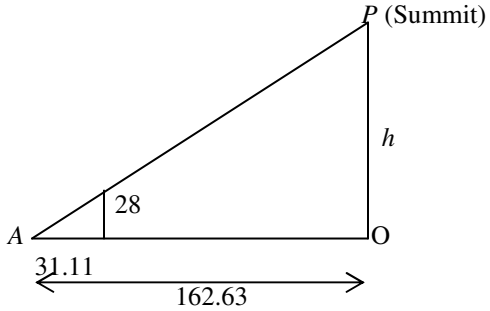
Distance $AC = \sqrt{45^2 - 27^2} = 36$ m.

Q2d Length scale factor $= \dfrac{1}{2}$, area scale factor $= \dfrac{1}{4}$,

$\therefore$ area $= \dfrac{1}{4} \times 252 = 63$ m^2.

Q3a





$\frac{h}{28}=\frac{162.63}{31.11}$, $h=146.37$,

$\therefore AP=\sqrt{162.63^2+146.37^2}=218.79$.

Distance from A to B over the summit $=218.79\times 2=437.6$ m.

Q3b $\tan\theta=\frac{28}{31.11}$, $\theta=\tan^{-1}\left(\frac{28}{31.11}\right)\approx 42°$.

Q3c Volume of pyramid $=\frac{1}{3}\times 230\times 230\times 146.37=2580991\text{ m}^3$.

Volume scale factor $=\frac{1}{2580991}$,

$\therefore$ length scale factor $=\sqrt[3]{\frac{1}{2580991}}$,

$\therefore$ area scale factor $=\left(\sqrt[3]{\frac{1}{2580991}}\right)^2=0.000053147$

Base area of the model $=230\times 230\times 0.000053147\approx 2.8\text{ m}^2$.

Module 3: Graphs and relations

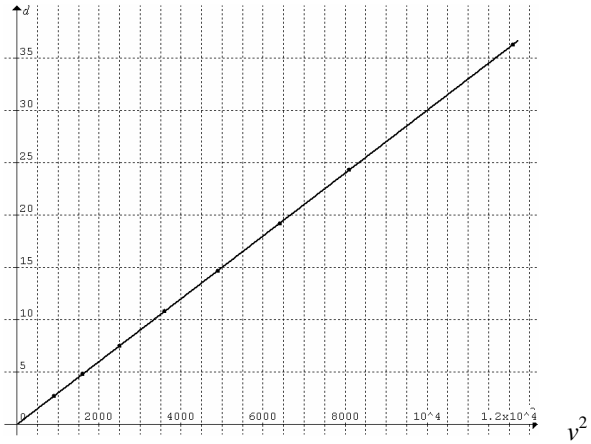
Q1a Speed = slope of graph $=\frac{15}{2.5}=6.0\text{ km h}^{-1}$.

Q1b $t=0$ to $t=4$

Q1c At $t=1.5$, distance apart $=6.5$ km.

Q1d At $t=1.5$, closest distance $=8.0-6.5=1.5$ km.

Q2a



Q2b m km^{-2} h^2

Q2c Slope $=\frac{7.5}{2500}=0.003$, $d=0.003\times v^2$.

Q2d $d=0.003\times 100^2=30$ m.

Q3a $3x-y+2=0$ …… (1)

$5y+2=4x$ …… (2)

From (1), $y=3x+2$ …… (3)

Substitute (3) in (2), $5(3x+2)+2=4x$, $\therefore x=-\frac{12}{11}$ …… (4)

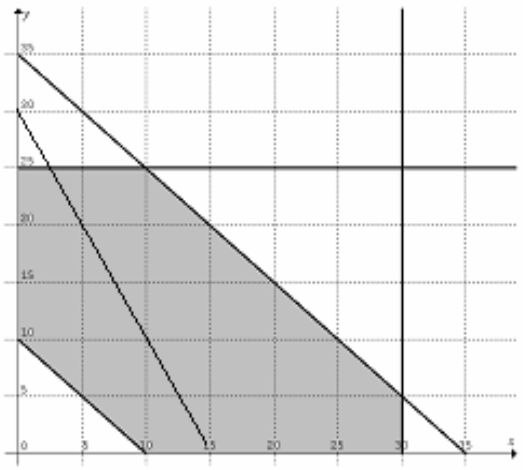
Substitute (4) in (3), $y=3\left(-\frac{12}{11}\right)+2=-\frac{14}{11}$.

Q3b $3x-\frac{15}{4}y+2>0$ and $5y+2\le 4x$.

$\therefore 4x-5y>-\frac{8}{3}$ and $4x-5y\ge 2$.

Hence $4x-5y\ge 2$.

Q4a $C=5050-60x-30y$, $\therefore y=-2x+c$ is the general equation for a graph of the cost function.



Q4b Minimum C at $(30,5)$.

$C=5050-60(30)-30(5)=3100$.

Module 4: Business-related mathematics

Q1a Total depreciation $= 18500 - 3300 = 15200$

$15200 = \dfrac{18500 \times r \times 4}{100}$, $r = 20.54$. $\therefore$ flat rate $= 20.54\%$ p.a.

Q1b $3300 = 18500 \times \left(1 - \dfrac{r}{100}\right)^4$, $\left(1 - \dfrac{r}{100}\right)^4 = \dfrac{3300}{18500}$,

$1 - \dfrac{r}{100} = \left(\dfrac{3300}{18500}\right)^{\frac{1}{4}} = 0.6499$, $r = 35.01$.

$\therefore$ reducing balance depreciation rate $= 35.01\%$ p.a.

Q1ci $V = 18500 - 3800t$

Q1cii $V = 18500 \times 0.6499^t$

Q2a End of first 6 months, $A = 3800\left(1 + \dfrac{8.25}{100 \times 2}\right) = 3956.75$.

End of next 6 months, $A = 3956.75\left(1 + \dfrac{8.75}{100 \times 2}\right) = 4129.86$.

Q2b Let $\$x$ be the value of \$4129.86 a year ago.

$\left(1 + \dfrac{4.6}{100}\right)x = 4129.86$, $\therefore x = 3948.24$.

Real gain $= 3948.24 - 3800 = 148.24$.

Real % gain $\dfrac{148.24}{3800} \times 100 = 3.90$.

Q3a 2006/07 average rate $= 8.32\%$,
2007/08 average rate $= 9.29\%$.
Average increase $= 9.29\% - 8.32\% = 0.97\%$.

Q3b 2006/07 interest $= \dfrac{250000 \times 8.32 \times 1}{100} = 20800$,

2007/08 interest $= \dfrac{250000 \times 9.29 \times 1}{100} = 23225$.

Total interest $= 20800 + 23225 = \$44025$.

Q3c $R = 1 + \dfrac{8.0}{100 \times 12} = 1.00666667$.

$A = PR^n - \dfrac{Q\left(R^n - 1\right)}{R - 1}$

$= 250000 \times 1.00666667^{24} - \dfrac{1950\left(1.00666667^{24} - 1\right)}{0.00666667} = 242652.24$

or by TVM Solver.

Total interest $1950 \times 24 + 242652.24 - 250000 = 39452.24$.

Amount of interest less $= 44025 - 39452.24 \approx \$4573$.

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