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Trial Exam Paper

2008

FURTHER MATHEMATICS

Written examination 2

Worked solutions

This book presents:

- worked solutions, giving you a series of points to show you how to work through the questions.
- mark allocations
- tips and guidelines

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Core

Table 1 below shows the average daily price (in cents per litre) of unleaded petrol in Melbourne, Brisbane and Adelaide respectively over 15 randomly chosen days in February.

Table 1

Average Petrol Prices (cents per litre) in February		
Melbourne	Brisbane	Adelaide
139	132	129
138	129	129
133	128	130
144	125	129
141	126	129
139	132	128
137	130	128
134	126	128
140	123	127
136	124	128
134	130	128
132	126	128
130	127	134
134	138	147
144	136	131

Source: AIP Research www.aip.com.au/pricing/retail.htm

Question 1

a. Complete Table 2 below by calculating the standard deviation of the average daily petrol price for Adelaide during February. Write your answer correct to one decimal place.

Table 2

City	Melbourne	Brisbane	Adelaide
Mean	137.0	128.8	130.2
Standard deviation	4.2	4.3	**4.9**

CORE – continued

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Solution

standard deviation = 4.9

1 mark

On a particular day in Melbourne the average petrol price is 141 cents per litre.

b. Calculate the standard price (z score) relative to this sample of petrol prices. Write your answer correct to two decimal places.

Worked solution

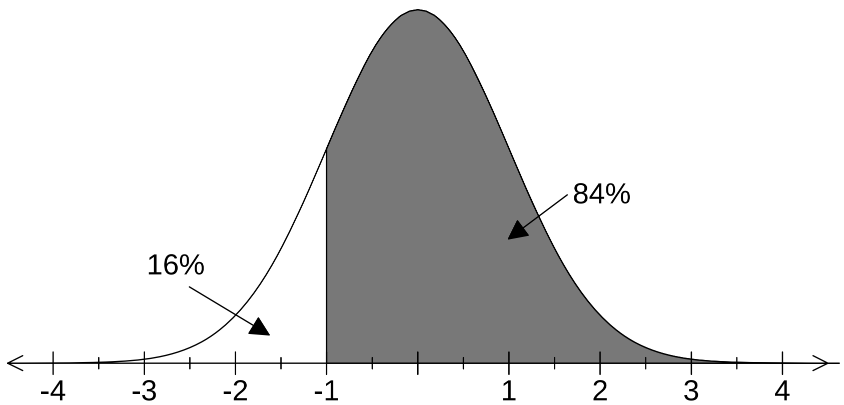
$$z = \frac{141 - 137}{4.2}$$
$$= 0.95$$

1 mark

The average petrol prices of Melbourne during February are normally distributed. On a particular day in February the standardised petrol price is -1.

c. Approximately what percentage of days in February will the petrol price be more than this day?

Worked solution



84%

1 mark

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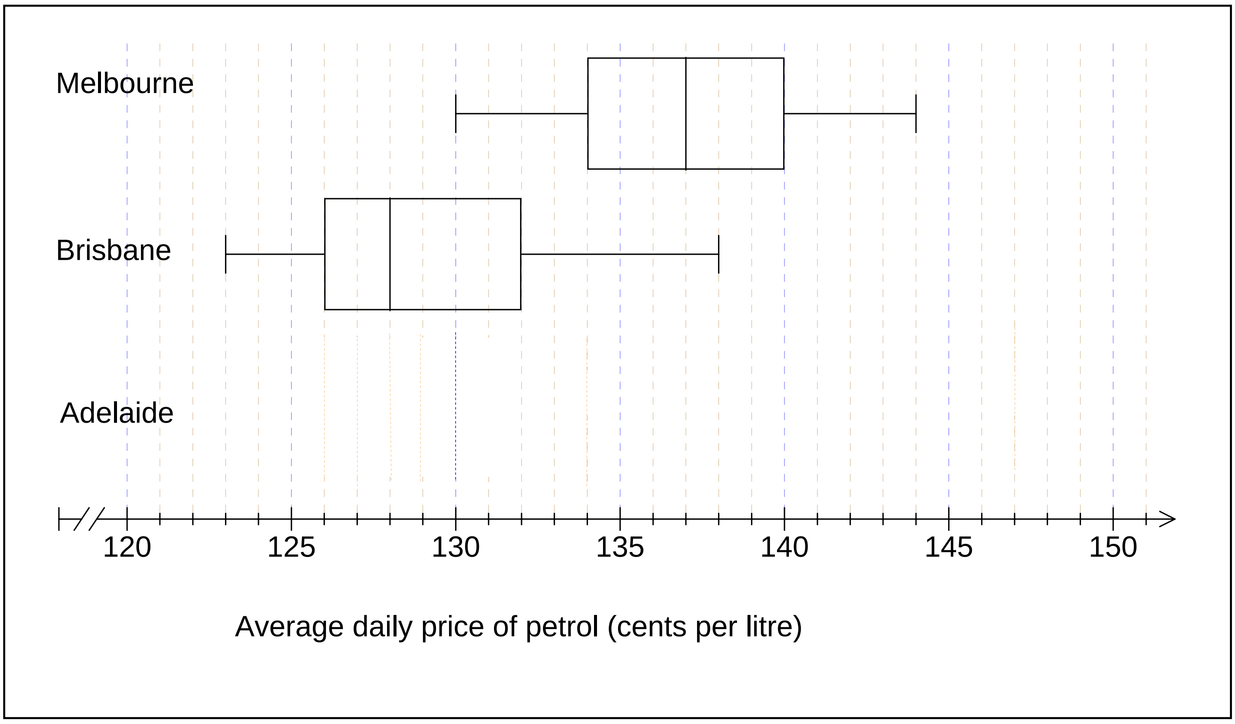
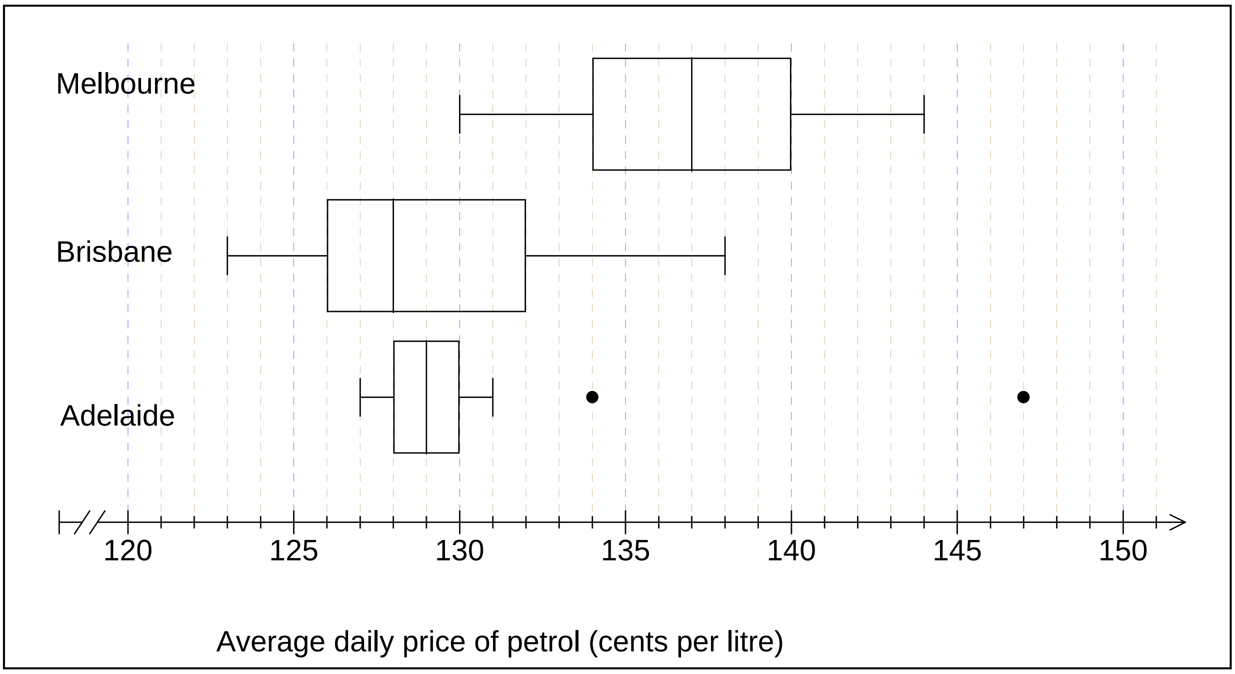
Using the data from Table 1, boxplots have been constructed to display the distributions of average daily petrol prices in February, 2008 for Melbourne and Brisbane as shown below.



d. Complete the display by constructing and drawing a boxplot that shows the distribution of unleaded petrol prices in Adelaide during February.

Worked solution



2 marks

Mark Allocation

- 1 mark for outliers at 134 and 147 shown
- 1 mark for correct box and whisker plot (127, 128, 129, 130, 131)

CORE – continued

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e. *Compare* the distribution of petrol prices in the three cities in terms of shape, centre and spread.

Shape ____

Centre ____

Spread ____

Solution

Shape
Melbourne displays a symmetrical distribution of average daily petrol prices. Adelaide also has a symmetrical distribution with two outliers present at 134 and 147c/litre. Brisbane's petrol prices are positively skewed.

Centre
The median petrol price in Melbourne was the most expensive at 137c/litre in comparison to the cheapest median price in Brisbane at 128c/litre. Adelaide was also relatively less expensive at 129c/litre.

Spread
Adelaide had the most inconsistent petrol price with a range of 20c/litre in comparison to Brisbane's price range of 15c/litre and Melbourne at 14c/litre. The interquartile range however was the most consistent in Adelaide at 2c/litre, whereas the other two cities were 6c/litre.

3 marks

Mark Allocation

- 1 mark for each correct answer

Question 2

The graph below shows the daily petrol prices for Melbourne in February 2008.

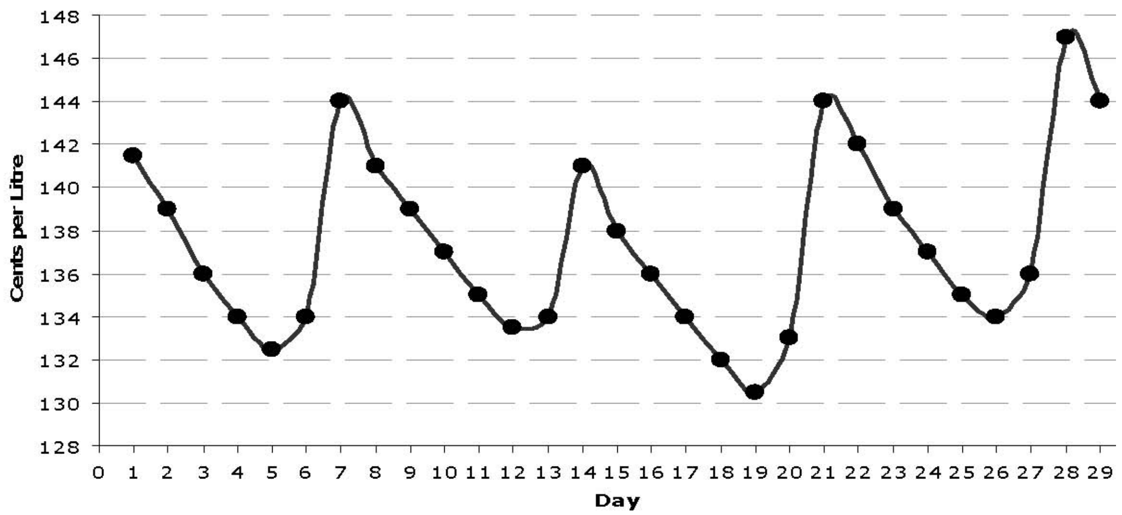

Figure 1: *Melbourne average daily petrol price, February 2008*

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a. Comment on the features of the graph.

Solution

There is *seasonal* variation.

1 mark

Table 3 below shows the averaged daily price of unleaded petrol in Melbourne during February.

Table 3

Week	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
1	141.5	139	136	134	132.5	134	144
2	141	139	137	135	133.5	134	141
3	138	136	134	132	130.5	133	144
4	142	139	137	135	134	136	147

b. The seasonal indices for this data are shown below. Calculate the missing seasonal index figure and complete the table below.

	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
Seasonal Index	**1.01**	1.01	1.00	0.98	0.97	0.98	1.05

Worked solution

$x + 1.01 + 1.00 + 0.98 + 0.97 + 0.98 + 1.05 = 7$

$x + 5.99 = 7$

$x = 1.01$

1 mark

c. Use the appropriate seasonal indices and the actual petrol prices in Table 3 to complete the table of deseasonalised petrol prices for February 2008 below.

Table 4

Deseasonalised Petrol Prices (cents per litre)							
Week	**Friday**	**Saturday**	**Sunday**	**Monday**	**Tuesday**	**Wednesday**	**Thursday**
1	138	138	136	137	137	137	137
2	137	138	137	138	138	137	134
3	135	135	134	135	135	136	137
4	138	138	137	138	139	139	**140**

CORE – continued

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Worked solution

$$\frac{\text{seasonal price}}{\text{seasonal index}} = \frac{147}{1.05} = 140 \text{ c/litre}$$

1 mark

The equation of the least squares regression line for the deseasonalised data is given by

$$\textbf{\textit{Deseasonalised Petrol Price}} = 0.05 \times Day + 136.34$$

(where *Day* = 1 is February 1st, 2008 which is a leap year)

d. Use this equation to draw the line of the deseasonalised petrol prices on the graph shown in Figure 1.

Solution

line should start at 136c/litre at 1st Feb and finish at 138c/litre at 29th Feb

1 mark

e. Complete the following sentence by filling in the box.

From the regression equation we can conclude that the petrol price increases on average by [ ] cents per litre every day.

Solution

0.05 cents per litre every day.

1 mark

f. Predict the deseasonalised petrol price, correct to two decimal places, for 10th March using this equation.

Worked Solution

substituting Day = 39 gives

Deseasonalised petrol price

$= 0.05 \times 39 + 136.34$

= 138.29 c/litre

1 mark

g. Hence, use the appropriate seasonal index to obtain a forecast for Monday 10th March. Give your answer correct to two decimal places.

Worked Solution

Seasonal petrol price

$= 138.29 \times 0.98$

= 135.52 c/litre

1 mark
Total 15 marks

Mark allocation

- 1 mark if correct seasonal index is used on answer to 2f to obtain correct answer

END OF CORE

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Module 1: Number Patterns

Andrew’s swim time is recorded during a rigorous training program. The time to the nearest second for the first three laps is shown in the table below.

Lap	1	2	3
Time (seconds)	50	48.5	47

Question 1

Andrew’s trainer believes that the swim time will form a decreasing arithmetic sequence.

a. Show that Andrew’s trainer is correct.

Worked Solution

$t_2 - t_1 = 48.5 - 50 = -1.5$

$t_3 - t_2 = 47 - 48.5 = -1.5$

$\therefore t_2 - t_1 = t_3 - t_2 = -1.5$

1 mark

Tip

- *There is a common difference, therefore the sequence is arithmetic.*

Mark allocation

- 1 mark – must show both differences for mark

b. An expression for Andrew’s swim time in the n th lap can be written as $A_n = b - 1.5n$. Determine the value of b.

Worked Solution

substituting $a = 50$ and $d = -1.5$ in

$$\begin{aligned} t_n &= a + (n-1)d \\ &= 50 + (n-1)(-1.5) \\ &= 50 - 1.5n + 1.5 \\ &= 51.5 - 1.5n \end{aligned}$$

Hence, $b = 51.5$

1 mark

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c. Andrew's fastest swim time is 17 seconds for one lap of the pool. If he continues in this sequence, how many laps does he swim to achieve his fastest time?

Worked Solution

sub $t_n = 17$ in $t_n = 51.5 - 1.5n$

$$17 = 51.5 - 1.5n$$
$$\frac{17 - 51.5}{-1.5} = n$$
$$n = 23 \text{ laps}$$

1 mark

d. Find the total time he swam to complete his fastest and final lap. Give your answer in seconds correct to one decimal place.

Worked Solution

$$S_{23} = \frac{23}{2}[2 \times 50 + 22 \times -1.5]$$
$$= 11.5 \times [100 - 33]$$
$$= 770.5 \text{ seconds}$$

1 mark

Question 2

Betty's swim time for each lap follows a geometric sequence with a common ratio of 0.94. Betty swam the first lap of the pool in 65 seconds.

a. By what percentage does Betty's swim time decrease for each lap of the pool?

Worked Solution

common ration 0.94 means the sequence is decreasing by

$(1 - 0.94) \times 100\% = 6\%$

1 mark

b. Determine the time it takes, to the nearest second, for Betty to swim the 4^{th} lap.

Worked Solution

$$t_4 = 65 \times 0.94^3$$
$$= 53.98796$$
$$= 54 \text{ seconds}$$

1 mark

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c. Write an equation that gives Betty's swim time B_n for the n th lap of the pool.

Worked Solution

$B_n = 65 \times (0.94)^{n-1}$

1 mark

d. How much faster did Betty swim her 10th lap of the pool in comparison to her 9th lap? Give your answer in seconds correct to two decimal places.

Worked Solution

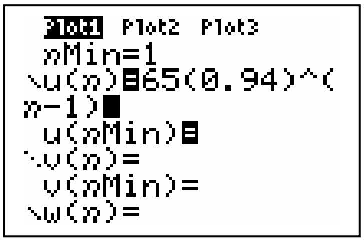
Betty took less time in the tenth lap

$B_9 - B_{10}$

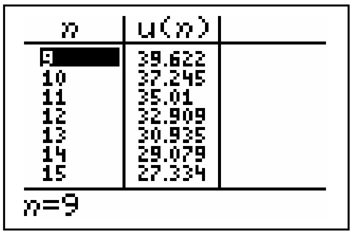
$= 65 \times (0.94)^8 - 65 \times (0.94)^9$

$= 2.38$ seconds

Alternatively the sequence can be generated on the calculator



The 9th and 10th term can be found in the table



n	u(n)
9	39.622
10	37.245
11	35.01
12	32.909
13	30.935
14	29.079
15	27.334

n=9

$B_9 - B_{10} = 39.622 - 37.245$

$= 2.38$ seconds

1 mark

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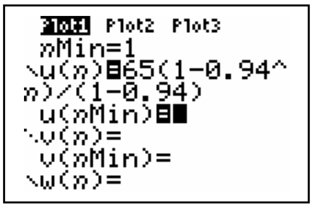
e. How many laps did Betty complete in the first 5 minutes of her swim?

Worked Solution

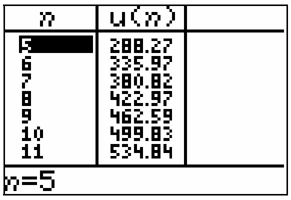
There are 300 seconds in 5 minutes. So Betty's total swim time is 300 seconds

Method 1: Using the calculator

Enter the sum equation to generate n laps over the total time swum



scroll down the table to find 300 seconds



Betty had fully completed **5 laps** in the first five minutes.

Method 2: Using algebra

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$300 = \frac{65(1-0.94^n)}{1-0.94}$$

$$300 = \frac{65(1-0.94^n)}{0.06}$$

$$0.94^n = 1 - \frac{300 \times 0.06}{65}$$

$$0.94^n = 0.723$$

$$n = \frac{\log_{10}(0.723)}{\log_{10}(0.94)}$$

$$n = 5.24$$

Betty fully completed 5 laps of the pool in the first 5 minutes.

1 mark

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f. If Betty swims 25 laps of the pool, calculate the time it takes her to complete the last 10 laps. Give your answer in seconds correct to two decimal places.

Worked Solution

Using the sum formula and table in the calculator

$S_{25} = 852.68$ seconds to swim 25 laps and $S_{15} = 655.1$ seconds to swim the first 15 laps

The time taken to swim the last ten laps is

$$S_{25} - S_{15}$$
$$= 852.68 - 655.1$$
$$= 197.58 \text{ seconds}$$

2 marks

Mark allocation

- 1 mark for subtracting from S_{25}
- 1 mark for correct answer

Question 3

The drink machine at the swim centre contains 400 drinks. Each day 8% of the drinks are sold and at the end of each day the machine is stocked with 20 new drinks.

The number of drinks in the machine, D_n, at the beginning of the n th day is modelled by the difference equation $D_{n+1} = 0.92D_n + 20$, where $D_1 = 400$

a. Find the number of drinks, to the nearest whole number, at the beginning of day 3.

Worked Solution

$$\begin{aligned} D_2 &= 0.92 \times D_1 + 20 \\ &= 0.92 \times 400 + 20 \\ &= 388 \\ D_3 &= 0.92 \times D_2 + 20 \\ &= 0.92 \times 388 + 20 \\ &= 376.96 \end{aligned}$$

There are 377 drinks

1 mark

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b. Show that the number of drinks at the beginning of each day does not follow an arithmetic or a geometric sequence.

Worked Solution

$D_3 - D_2$
$= 377 - 388$
$= -11$

$D_2 - D_1$
$= 388 - 400$
$= -12$

$D_3 - D_2 \neq D_2 - D_1$

therefore sequence is not arithmetic

$\frac{D_3}{D_2} = \frac{377}{388}$ and $\frac{D_2}{D_1} = \frac{388}{400}$

$\frac{D_3}{D_2} \neq \frac{D_2}{D_1}$

Therefore the sequence is not geometric

1 mark

c. For many days 8% of the drinks in the machine are sold and 20 drinks are restocked. Show a calculation explaining why there will never be fewer than 250 drinks in the machine.

Worked Solution

If there were 250 drinks then

$D_{n+1} = 0.92 \times 250 + 20$
$= 250$

or

8% of 250 = 20. The amount of drinks sold each day is the same as the number restocked

1 mark

d. How many drinks should be restocked in the machine each day so that the number remains stable, that is, so that there are 400 drinks in the machine each day?

Worked Solution

$400 = 0.92 \times 400 + x$
$400 = 368 + x$
$x = 32$ drinks

or

8% of 400 = 32 drinks removed must equal the number of drinks restocked.

1 mark
Total 15 marks

END OF MODULE 1

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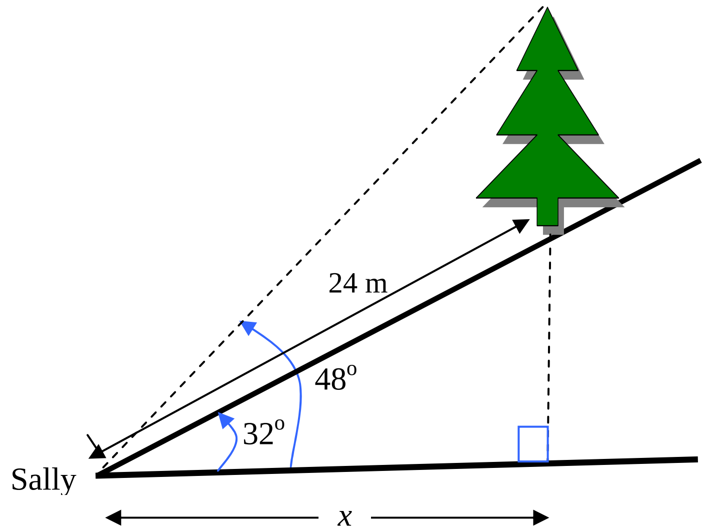
Module 2: Geometry and Trigonometry

A tree stands on a hillside of slope 32^o (from the horizontal). Sally stands at the bottom of the hill 24 m from the tree and measures the angle of elevation to the top of the tree to be 48^o as shown in the diagram.



Question 1

a. Show that the horizontal distance from Sally to the base of the tree, *x*, is 20.35 metres.

Worked Solution

$\cos 32^o = \frac{x}{24}$

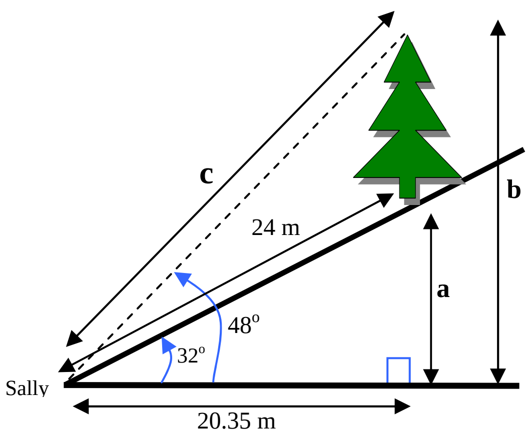
$x = 24\cos 32^o$

$x = 20.35$ metres

1 mark

b. Find the height of the tree, in metres, correct to two decimal places.

Worked Solution



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Method 1

Tree height $= b - a$

$= 20.35\tan 48^{\circ} - 20.35\tan 32^{\circ}$

$= 9.88$ metres

Method 2

$\cos 48^{\circ} = \dfrac{20.35}{c}$

$c = \dfrac{20.35}{\cos 48^{\circ}}$

$c = 30.42$

$h^2 = 24^2 + 30.42^2 - 2(24)(30.42)\cos 16^{\circ}$

$h^2 = 97.74$

$h = 9.89$ metres

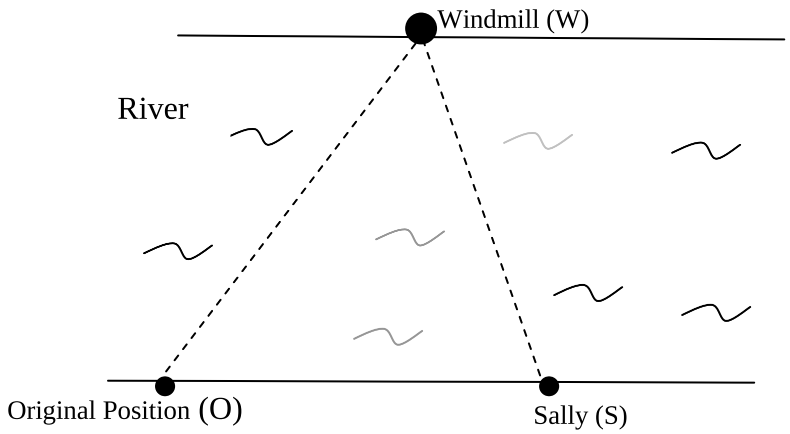
2 marks

Mark allocation

- 1 mark for substituting values into correct formula
- 1 mark for correct answer

Sally walks to a river that flows due East. She stops and looks across to the opposite river bank to see a windmill that has a bearing of 023°T. After walking downstream 40 m, Sally stops to find that the windmill is now on a bearing of 345°T.

DIAGRAM
NOT DRAWN
TO SCALE



Question 2

a. Show that the magnitude of angle OWS is 38°.

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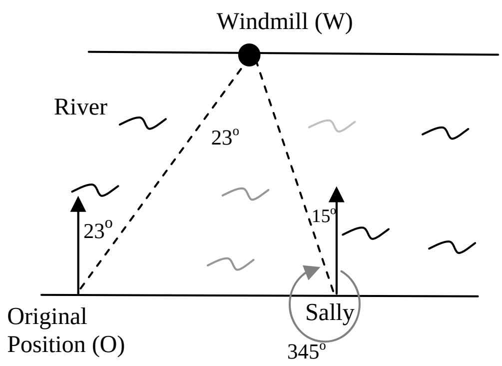
Worked Solution



The alternate angle at the original position plus the alternate angle at Sally's position means that the angle at OWS = 23^o + 15^o = 38^o

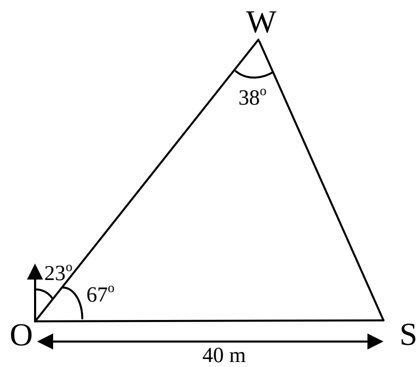
Method 2: Using complementary angles

$$\angle OWS = 180^0 - \angle SOW - \angle WSO$$
$$= 180^0 - 67^0 - 75^0$$
$$= 38^0$$

1 mark

b. Find the distance, in metres, correct to one decimal place from Sally to the windmill, SW.

Worked Solution



Using the Sine Rule

$$\frac{SW}{\sin 67^o} = \frac{OS}{\sin 38^o}$$

$$SW = \frac{40 \sin 67^o}{\sin 38^o}$$

$$SW = 59.8 \text{ metres}$$

2 marks

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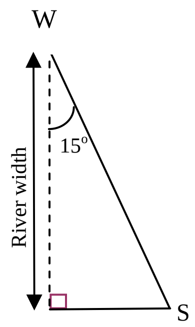
c. Hence, find the width of the river. Give your answer in metres correct to one decimal place.

Worked Solution

$$\cos 15^o = \frac{\text{river width}}{\text{SW}}$$

$$\text{river width} = 59.8\cos 15^o$$

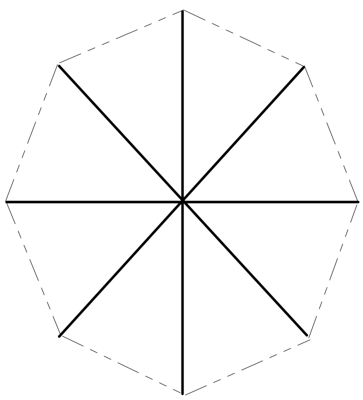
$$= 57.8 \text{ metres}$$



2 marks

Question 3

The windmill has 8 blades. The ends of the blades form a regular octagon as shown in the diagram. Each blade is 2 metres long.



a. Show that the angle at the centre, between the blades, is 45°.

Worked Solution

$$\text{angle} = \frac{360^\circ}{8}$$

$$= 45^\circ$$

1 mark

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b. Determine the area of the octagon correct to one decimal place.

Worked Solution

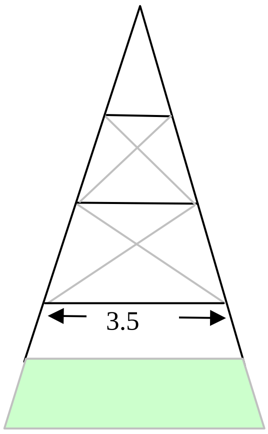
$$\text{Area} = 8 \times \frac{1}{2} \times 2 \times 2 \sin 45^{\circ}$$
$$= 11.3 \text{ m}^2$$

2 marks

Mark allocation

- 1 mark for substituting values into correct solution
- 1 mark for correct answer

The structure that holds the windmill is made of a square based pyramid. Each side is triangular with three horizontal supporting struts as shown on the diagram. The horizontal struts are in the ratio 2: 3: 4. The longest horizontal strut is measured to be 3.5 metres.



c. Find the length of the middle strut in metres, correct to 3 decimal places.

Worked Solution

$3:4$

$x:3.5$

$$\therefore 4x = 3 \times 3.5$$
$$x = \frac{3 \times 3.5}{4}$$
$$x = 2.625 \text{ metres}$$

Method 2

Ratio 4:3 = 1: 0.75

$$\therefore x = 3.5 \times 0.75$$
$$= 2.625 \text{ metres}$$

1 mark

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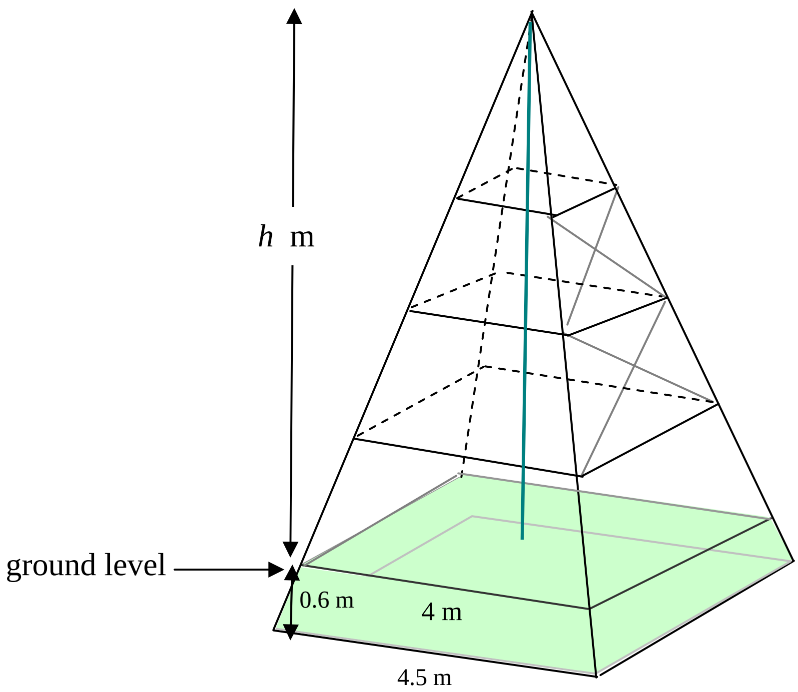
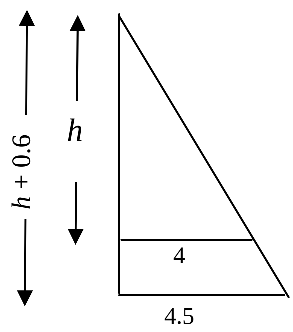
The windmill is supported by a structure in the shape of a square based pyramid. It is reinforced with concrete 0.6 m deep. The upper surface of the concrete has a length of 4 metres and the base of the concrete has a length of 4.5 metres.



d. Use similar triangles to show that the height, h, of the structure above ground level is 4.8 metres.

Worked Solution



$$\frac{h}{4} = \frac{h+0.6}{4.5}$$

$$4.5h = 4h + 2.4$$

$$0.5h = 2.4$$

$$h = 4.8 \text{ metres}$$

1 mark

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e. Determine the volume of the concrete, in cubic metres, correct to two decimal places.

Worked Solution

Method 1

Volume of Structure plus concrete

$= \frac{1}{3} \times area\ of\ base \times height$

$= \frac{1}{3} \times 4.5^2 \times 5.4$

$= 36.45\ m^3$

Volume of Structure only

$= \frac{1}{3} \times area\ of\ base \times height$

$= \frac{1}{3} \times 4^2 \times 4.8$

$= 25.6\ m^3$

Volume of concrete

$= 36.45 - 25.6$

$= 10.85\ m^3$

Method 2

Using ratios

Structure only: Structure plus concrete

4.8 : 5.4

= 8 : 9 Length ratio

$8^3 : 9^3$ Volume ratio

$= 512 : 729$

Therefore concrete ratio is 729 – 512 = 217

Actual volume of structure

$= \frac{1}{3} \times area\ of\ base \times height$

$= \frac{1}{3} \times 4^2 \times 4.8$

$= 25.6\ m^3$

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Set up volume ratio equation

Concrete : Structure

$217 : 512$

$x \quad : 25.6$

$512x = 25.6 \times 217$

$x = \frac{25.6 \times 217}{512}$

$x = 10.85\ m^3$

2 marks
Total 15 marks

END OF MODULE 2

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Module 3: Graphs and Relations

Question 1

A company, Cleanozone, designs and manufactures various models of rainwater tanks. The new *Slimline* model requires \$400 worth of materials to make each tank. It costs \$12 000 per year to provide the manufacturing facilities, regardless of the number of tanks that are produced. It is possible for the facilities to make up to 150 tanks per year. The total cost of manufacturing x tanks per year is given by the equation

$$C = 400x + 12\,000\,,\ 0 \le x \le 150$$

a. Find the total cost of manufacturing 100 tanks.

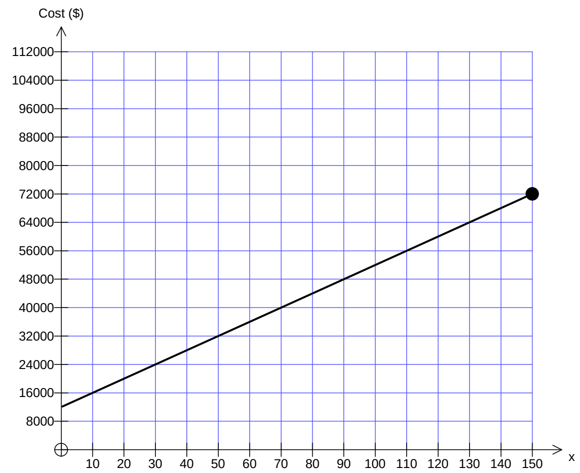
Worked Solution

$$\begin{aligned} C &= 400 \times 100 + 12000 \\ &= 40000 + 12000 \\ &= 52000 \end{aligned}$$

1 mark

b. Sketch the graph of the cost equation on the set of axes below.

Worked Solution



Straight line with y-intercept (0, 12 000) and correct end-point (150, 72 000)

1 mark

Cleanozone are able to sell the tanks to retailers. The first 40 tanks sell for \$500 each but the remaining 110 bring in \$700 each.

c. The revenue made from selling 40 tanks is \$20 000. Calculate the revenue made from selling 100 tanks.

Worked Solution

$40 \times 500 + 60 \times 700 = \$62\,000$

1 mark

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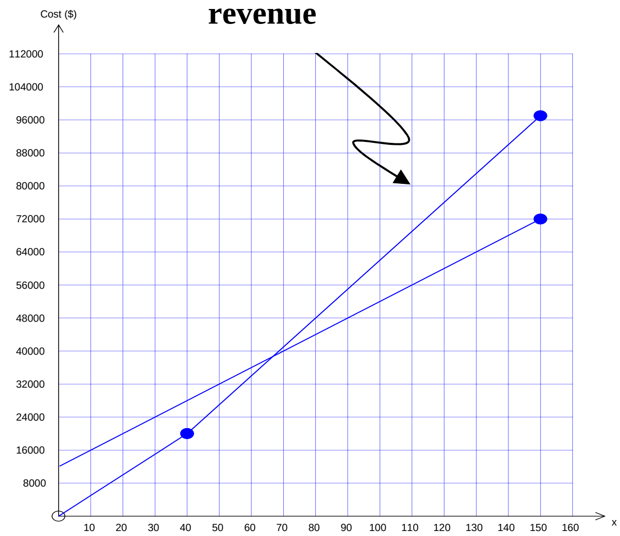
d. Sketch the revenue on the above axes.

Worked Solution



Revenue line has two line segments. From (0, 0) to (40, 20 000) to (150, 97 000)

1 mark

The revenue, R, dollars, from selling x *Slimline* tanks is given by the function:

$$R = \begin{cases} 500x & ;\ 0 \le x \le 40 \\ 700x + k & ;\ 40 \le x \le 150 \end{cases}$$

e. Show that the value for k is –8000.

Worked Solution

substitute a point (x, R) from second segment in $R = 700x + k$

e.g. $20\ 000 = 700 \times 40 + k$

$20\ 000 = 28\ 000 + k$

$\therefore k = -8\ 000$

1 mark

f. Find the least number of *Slimline* tanks that need to be sold for Cleanozone to make a profit.

Worked Solution

Revenue = Cost

$700x - 8\ 000 = 400x + 12\ 000$

$300x = 20\ 000$

$x = 66.67$

$\therefore 67$ tanks to make a profit

1 mark

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Question 2

Manufacturing a tank involves two main processes: welding and testing. The table below shows the time available in a week to manufacture two types of water tanks.

	Domestic (hours)	**Garden (hours)**	**Time available (hours)**
Welding	4	5	97
Testing	2	4	62

Let x be the number of domestic tanks and
y be the number of garden tanks are made each week.

This information can be expressed as Inequalities 1 and 2.

- Inequality 1: $4x + 5y \leq 97$
- Inequality 2: $2x + 4y \leq 62$

a. Which line (Line A or Line B) in Graph 1 below forms the boundary of the region defined by inequality 1?

Worked Solution

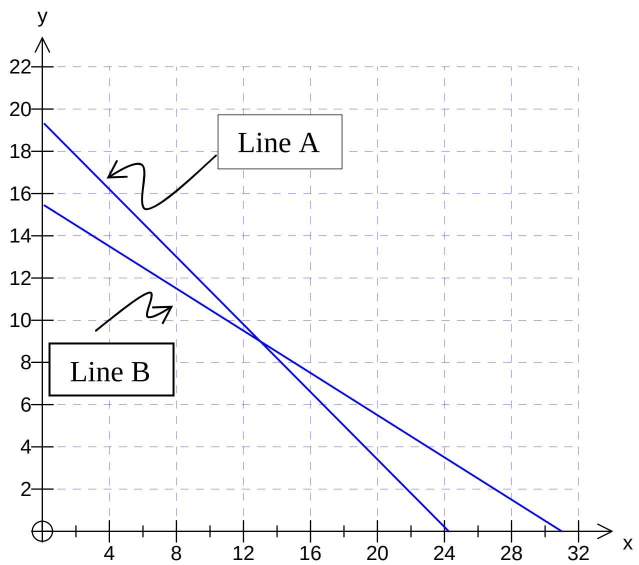
The boundary line for Inequality 1 is given by $4x + 5y = 97$

Making y the subject gives $y = \frac{-4x}{5} + \frac{97}{5}$

The y-intercept for the boundary line defined by inequality 1 is $\frac{97}{5} = 19.6$

Therefore Line A forms the boundary of inequality 1

1 mark



Graph 1

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b. Write down the co-ordinates of the point of intersection of Line A and Line B in Graph 1

Worked Solution

Method 1

Using elimination method, solve

$4x + 5y = 97 \quad eq(1)$

$2x + 4y = 62 \quad eq(2)$

$2 \times eq(2) - eq(1)$ gives

$3y = 27$

$\therefore y = 9$

sub in eq(2)

$2x + 4 \times 9 = 62$

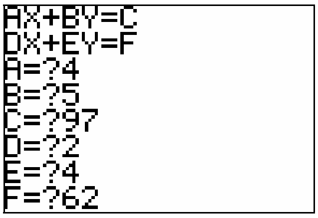
$2x = 26$

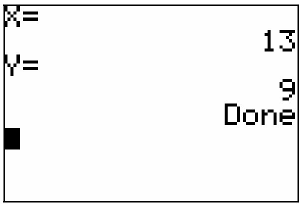
$x = 13$

Point of intersection is (13, 9)

Method 2

Use the SIMULT2 program on calculator





1 mark

c. Due to demand, the company must produce at least 7 domestic tanks and at least 5 garden tanks in a week.

Write the two corresponding inequalities,

- Inequality 3:
- Inequality 4:

Worked Solution

Inequality 3 $x \geq 7$

Inequality 4 $y \geq 5$

1 mark

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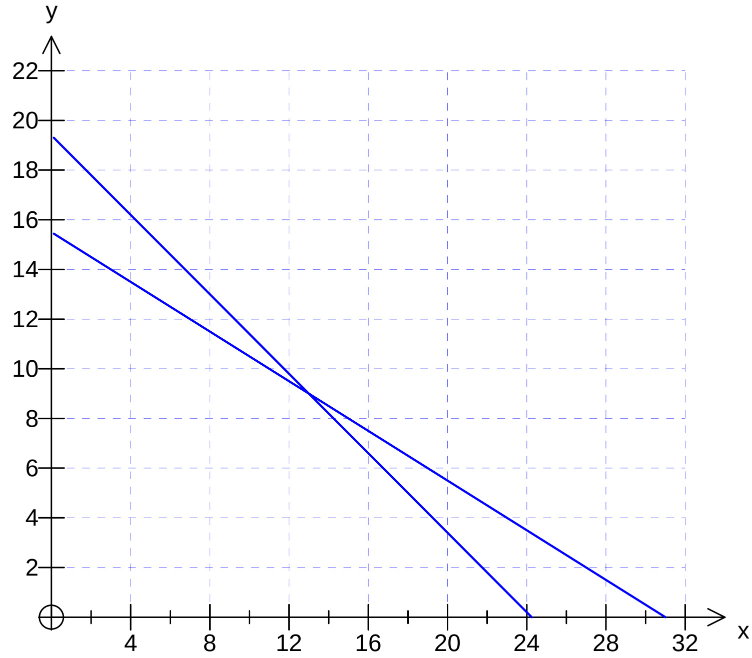
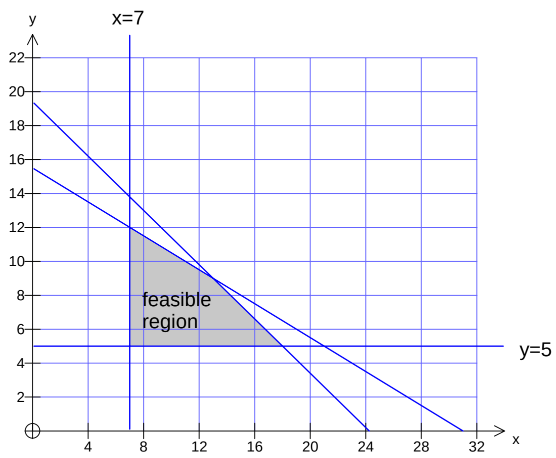
d. Using inequalities 1 to 4, construct and shade the feasible region for the production of the two types of tanks for one week on Graph 2 below.



Graph 2

Worked Solution



Correct line at x = 7

Correct line at y = 5

Feasible region shaded

3 marks

e. The company is able to make a profit of $140 on each domestic tank and $280 on each garden tank. Write an expression for the profit, P, in terms of *x* and *y*.

Worked Solution

$\text{Profit} = 140x + 280y$

1 mark

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f. Find the combination of domestic tanks and garden tanks the company should produce in a week to maximise their profit.

Worked Solution

Substitution in $Profit = 140x + 280y$

Vertices of feasible region	$P = 140x + 280y$
(7, 5)	2380
(7, 12)	**4340**
(13, 9)	**4340**
(18, 5)	3920

Maximum profit occurs along the line (7, 12) and (13, 9)

i.e. maximum occurs at

(7, 12), (13,9) and also (9, 11) and (11, 10)

i.e. **7 domestic** and **12 garden** tanks

or **13 domestic** and **9 garden** tanks

or **9 domestic** and **11 garden** tanks

or **11 domestic** and **9 garden** tanks

will produce the maximum profit.

Note: (9, 12) and (11, 9) are the only integer points along the line joining (3, 14) and (13, 9)

2 marks
Total 15 marks

END OF MODULE 3

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Module 4: Business-related Mathematics

Question 1

Wendy wants to buy a commercial oven for her pizza restaurant. Ovens Galore normally sells them for $18 000, but they have a discounted price of $17 280.

a. What is the percentage discount? Write you answer correct to one decimal place.

[ ] %

Worked Solution

Discount = 18 000 – 17 280 = $720

Percentage discount = $\frac{720}{18000} \times 100 = 4\%$

1 mark

b. Ovens Galore offers to sell the oven for the discount price of $17 280. The terms of the sale are $1 200 deposit and $515 per month for 36 months.

i. What is the total cost of the oven on these terms?

Worked Solution

Total Cost = 1 200 + 36 × 515 = $19 740

1 mark

ii. Show that the annual flat rate of interest charged is 5.1%.

Worked Solution

Total Interest charged

= $19 740 – $17 280 = $2 460

Therefore annual interest charged

$= \frac{\$2\,460}{3} = \$820$

Flat Interest Rate = $\frac{820}{17280 - 1200} \times 100$

$= \frac{820}{16080} \times 100$

$= 5.0995$

The annual flat rate of interest is 5.1%.

1 mark

Module 4: Business-related mathematics – continued

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iii. Determine the effective rate of interest per annum.Write your answer correct to one decimal place.

Worked Solution

Effective Rate = $\dfrac{2n}{n+1} \times$ flat rate

$= \dfrac{2 \times 36}{37} \times 5.1$

$=$ 9.92%

1 mark

iv. Explain why an effective interest rate differs from a flat interest rate.

Worked Solution

The effective rate of interest is different than the flat rate of interest because the effective rate takes into account the reducing balance of the loan as repayments are made.

1 mark

c. Wendy sees the same oven for sale at Hot Ovens Discount Store, also for $17 280. The terms of the sale there require no deposit and monthly repayments over three years at an interest rate of 6.4% per annum, calculated monthly on a reducing balance.

The monthly repayments can be determined using the annuities formula:

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}.$$

The loan is paid out in three years.

i. What are the values of n, P and A?

Worked Solution

n = 3 × 12 = 36

P = $17 280

A = 0

2 marks

Mark allocation

- One mark for two values correct
- One mark for third value correct

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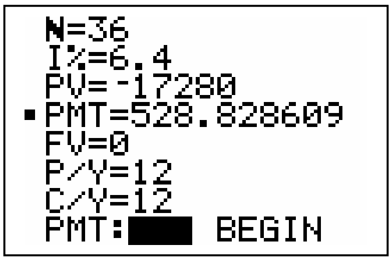
ii. What is the monthly repayment for this loan? Write your answer in dollars, correct to two decimal places.

Worked Solution

The TVM solver can be used.



The monthly repayment is $528.83

Method 2

The monthly repayment can also be determined algebraically:

$R = 1 + \frac{r}{100}$ where r = monthly interest rate

$$R = 1 + \frac{6.4}{1200}$$
$$= 1.005333$$

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$$

$$0 = 17280(1.005333)^{36} - \frac{Q(1.005333^{36} - 1)}{1.0053333 - 1}$$

$$0 = 20926.98714 - 39.5723Q$$

$$39.5723Q = 20926.98714$$

$$Q = \frac{20926.98714}{39.5723}$$

$$Q = 528.83$$

The monthly repayment is $528.83

1 mark

iii. What is the total cost of the oven from Hot Ovens Discount store on these terms? Write your answer correct to the nearest dollar.

Worked Solution

Total cost
= repayment amount × number of repayments
= 528.83 × 36
= 19 037.88
= $19 038 to the nearest dollar

1 mark

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d. Whose terms, Ovens Galore or Hot Ovens Discount Store, offer the lowest total cost for the oven? Justify your answer.

Worked Solution

Ovens Galore cost $19 740

Hot Oven Discounts cost $19 038

So the better deal is at Hot Oven Discounts where there is an overall saving of

19 740 – 19 038 = $702

1 mark

Question 2

Wendy purchases the oven with an initial value of $17 280. For tax purposes Wendy considers two methods of depreciating the value of the oven.

a. Suppose the value of the oven is depreciated using the reducing balance method over five years and reducing at a rate of 14% per annum. What is the depreciated value after five years? Write your answer correct to the nearest dollar.

Worked Solution

$$\text{Value} = \text{P}\left(1 - \frac{r}{100}\right)^{n}$$

$$= 17\ 280\left(1 - \frac{14}{100}\right)^{5}$$

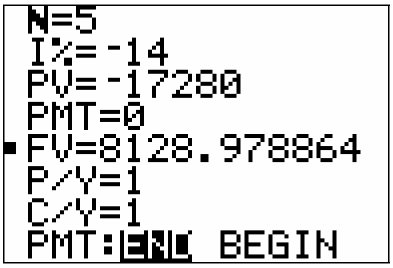
$$= 17\ 280\left(0.86\right)^{5}$$

$$= \$8\ 128.98$$

The depreciated value of the oven after 5 years is $8129

Method 2

Using TMV solver



The depreciated value of the oven after 5 years is $8129

2 marks

Mark allocation

- 1 mark correct for equation or correct TVM listing
- 1 mark for correct answer

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b. Alternatively, suppose that the machine is depreciated using the unit cost of depreciation method. Wendy sells 25 000 pizzas per year and the unit cost per pizza is 8 cents. Determine the depreciated value of the oven after five years. Write your answer correct to the nearest dollar.

Worked Solution

Value = 17 280 – 25 000 × 0.08 × 5
= $7 280

1 mark

c. Wendy wants the depreciated value of the oven after five years to be the same when calculated by both methods of depreciation. What would the unit cost per pizza have to be for this to occur? Write your answer in cents correct to two decimal places.

Worked Solution

Value = 17 280 – 25 000 × c × 5

= 17 280 – 125 000c

The value of the oven must be equal to $8129 from part a.

8 129 = 17 280 – 125 000c

125 000c = 17 280 – 8 129

125 000c = 9 151

$$c = \frac{9151}{125000}$$

= 0.073208

The unit cost per pizza has to be $0.073208 which is 7.32 cents

2 marks
Total 15 marks

Mark allocation

- 1 mark for setting up equation for c
- 1 mark for correct answer

END OF MODULE 4

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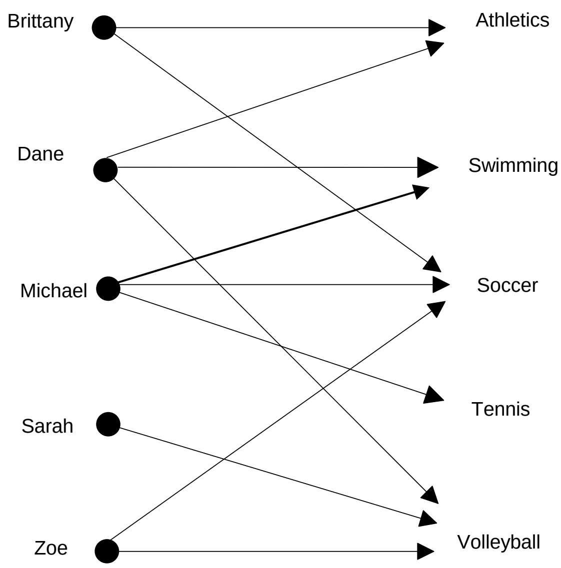
Module 5: Networks and Decision Mathematics

Question 1

A group of five school friends represent their school in five different sports. The information is displayed in the following bipartite graph.



Each sport must be represented by one student at a school assembly.

a. Which student **must** represent swimming?

Solution

From the bipartite graph we can see that only Michael can represent tennis, leaving Dane to represent swimming

1 mark

b. Complete the table showing the sport that each student **must** represent.

Student	**Sport**
Brittany	
Sarah	
Zoe	

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Solution

Since Dane is representing swimming this leaves Brittany with athletics and Zoe with soccer, so Sarah must represent volleyball.

Student	**Sport**
Brittany	Athletics
Sarah	Volleyball
Zoe	Soccer

2 marks

Mark allocation

- 1 mark for any two correct
- 1 mark for all correct

Question 2

The five students decide to play a game of *one on one* basketball. Each student competes against each of the other four students one at a time. For each game there is a winner and a loser.

The results are shown in the **incomplete** dominance matrix. On the directed graph an arrow from Dane to Brittany shows that Dane won against Brittany.

Matrix 1

loser

$$\textbf{winner}\quad \begin{array}{c} \\ B \\ D \\ M \\ S \\ Z \end{array} \begin{array}{c} \begin{array}{ccccc} B & D & M & S & Z \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ & & & & 0 \end{bmatrix} \end{array}$$

Zoe lost to Michael, but won all the other games.

a. Complete the dominance Matrix 1, above, showing Zoe's results.

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Worked Solution

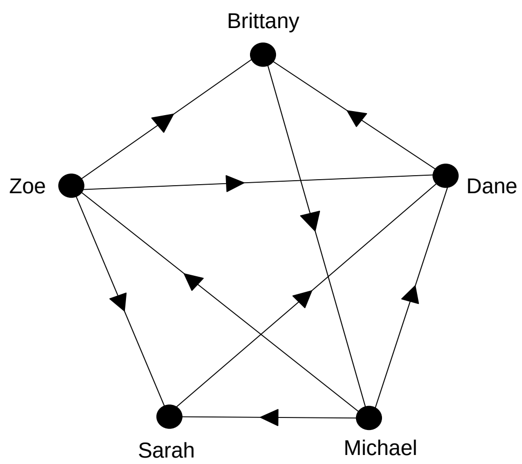
Since Zoe won the games against Brittany, Dane and Sarah a 1 is placed in all columns except Michael's, where a 0 shows that Zoe lost to him.

$$\begin{array}{c} \\ B \\ D \\ M \\ S \\ Z \end{array} \begin{array}{c} \begin{array}{ccccc} B & D & M & S & Z \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

1 mark

The results of the game are also represented in the **incomplete** directed graph below.

Directed Graph

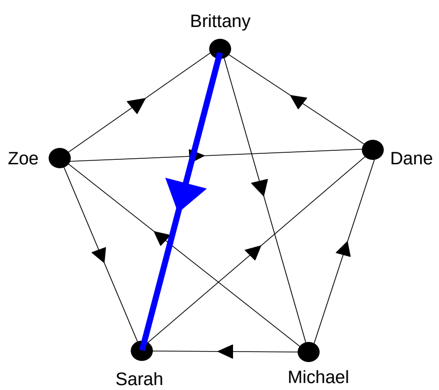


One of the edges of the graph is missing.

b. Using the information in Matrix 1, **draw** in the missing edge on the directed graph above and clearly show its **direction**.

Worked Solution

From the dominance matrix we can see that Brittany defeated Sarah. Hence we need to add an edge that is directed from Brittany to Sarah.



1 mark

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The results of each one on one basketball contest (one-step dominances) are summarized as follows.

Student	Dominance value (wins)
Brittany	2
Dane	1
Michael	3
Sarah	1
Zoe	3

c. Which two students are ranked equal first in this contest?

Worked Solution

Michael and Zoe are ranked equal first.

1 mark

In order to rank the students from first to last in the basketball contest, two-step (two-edge) dominances will be considered.

The following **incomplete** matrix, Matrix 2, shows two-step dominances.

Matrix 2

$$\begin{array}{c} \\ B \\ D \\ M \\ S \\ Z \end{array} \begin{array}{c} \begin{array}{ccccc} B & D & M & S & Z \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 2 & 2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & x & 1 & 1 & 0 \end{bmatrix} \end{array}$$

d. Explain the two-step dominance of Brittany (B) over Dane (D).

Solution

Brittany defeated Sarah and Michael, who each beat Dane.

1 mark

e. Determine the value of x in Matrix 2.

Solution

We need to find the two step dominance of Zoe over Dane. Using the directed graph we find only one, Z – S – D. So x = 1

Alternatively, square matrix 1 to give matrix 2

1 mark

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f. Taking into consideration both the one-step and two-step dominances, determine which student was ranked first and which was ranked last in the *one-on-one* basketball competition.

First

Last

Worked Solution

Let D_1 be the one-step dominance matrix and D_2 be the two-step dominance matrix.

Forming $D_1 + D_2$, we obtain

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 2 & 2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{array}{c} \\ B \\ D \\ M \\ S \\ Z \end{array} \begin{array}{c} \begin{array}{ccccc} B & D & M & S & Z \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 3 & 0 & 2 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 2 & 2 & 1 & 2 & 0 \end{bmatrix} \end{array} \quad \begin{array}{c} \text{Dominance} \\ 6 \\ 3 \\ 8 \\ 2 \\ 7 \end{array}$$

Adding the entries in each row we see that the rankings are

First Michael

Last Sarah

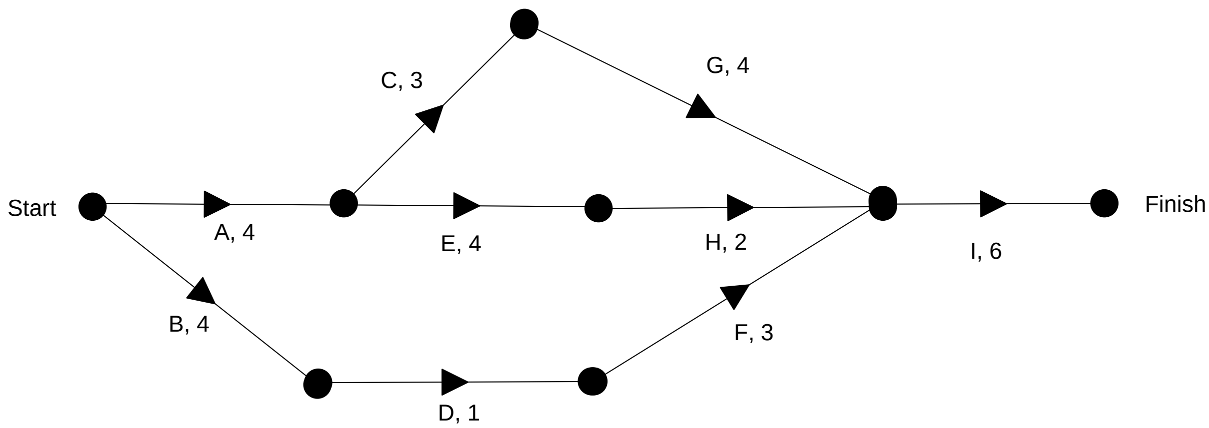
2 marks

Mark allocation

- 1 mark for each answer

Question 3

A new gym is to be built at the school. Nine activities have been identified for this building project. The directed network below shows the activities and their completion times in weeks.



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a. Determine the minimum time, in weeks, to complete this project.

Solution

17 weeks

1 mark

b. Determine the slack time, in weeks, for activity D.

Solution

7 – 4 = 3 weeks

1 mark

The builders of the gym are able to speed up the project. Some of the activities can be reduced at an additional cost. The activities that can be reduced in time are B C E F and G.

c. Which of these activities, if reduced in time individually, would not result in an earlier completion of the project?

Solution

B, E and F. While these activities are not on the critical path, crashing any of these will not affect the completion time of the project.

1 mark

The school council is prepared to pay an additional cost to achieve early completion. The cost of reducing the time for each activity is $3 000 per week. The maximum reduction in time for each one of the five activities B, C, E, F, and G is 2 weeks.

d. Determine the minimum time, in weeks, for the project to be completed now that certain activities can be reduced in time.

Solution

14 weeks

1 mark

e. Determine the minimum additional cost of completing the project in this reduced time.

Worked Solution

AEHI gives 14 weeks when E is reduced by 2 hours. ACGI gives 13 weeks when reducing C by 2 hours and G by 2 weeks.

Make ACGI a critical path as well, so reduce by 3 + 2 weeks overall = 5 × $3 000 = $15 000

1 mark
Total 15 marks

END OF MODULE 5

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Module 6: Matrices

Frank's Fruit Mart sells the following fruit packs:

	Type of Fruit		
	Apples	**Mangos**	**Bananas**
Standard Pack (S)	6	4	6
Family Fruit Pack (F)	12	12	24
Bulk Fruit Pack (B)	20	15	40

The cost price is:

- $7.10 for the Standard Pack,
- $22.20 for the Family Fruit Pack and
- $33.00 for the Bulk Fruit Pack.

Question 1

a. The cost price of each apple, mango and banana is x, y and z dollars respectively. Write a matrix equation, of the form below, that you can solve to find the value of x, y and z.

$$\begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \quad \\ \\ \end{bmatrix}$$

Worked Solution

$$\begin{bmatrix} 6 & 4 & 6 \\ 12 & 12 & 24 \\ 20 & 15 & 40 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7.1 \\ 22.20 \\ 33 \end{bmatrix}$$

1 mark

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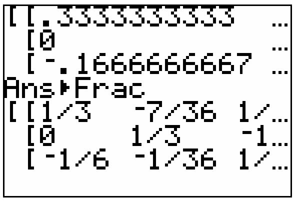
b. Write down an inverse matrix that can be used to solve these equations. Write the elements as fractions.

Worked Solution

The inverse of $\begin{bmatrix} 6 & 4 & 6 \\ 12 & 12 & 24 \\ 20 & 15 & 40 \end{bmatrix}$ is needed to solve the equations.

Using the calculator:



$$A^{-1} = \begin{bmatrix} \frac{1}{3} & -\frac{7}{36} & \frac{1}{15} \\ 0 & \frac{1}{3} & \frac{-1}{5} \\ -\frac{1}{6} & -\frac{1}{36} & \frac{1}{15} \end{bmatrix}$$

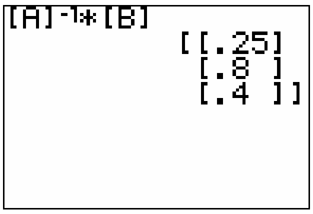
1 mark

c. Solve the equation and hence write down the cost price of an apple, a mango and a banana.

Worked Solution

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{7}{36} & \frac{1}{15} \\ 0 & \frac{1}{3} & \frac{-1}{5} \\ -\frac{1}{6} & -\frac{1}{36} & \frac{1}{15} \end{bmatrix} \begin{bmatrix} 7.7 \\ 24.45 \\ 36 \end{bmatrix}$$

Using the calculator:



An apple costs $0.25, a mango $0.80 and a banana $0.40 (answers can be in cents)

1 mark

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Question 2

The **selling price** of each type of pack is calculated by multiplying the cost price by a factor.

These factors are different for each pack.
For the Standard pack the selling price is 1.6 times the cost price
For the Family pack the selling price is 1.5 times the cost price and
For the Bulk pack the selling price is 1.4 times the cost price.

To calculate the selling price Frank's Fruit Mart have set-up a matrix equation of the form:

$$\begin{array}{c} \\ M \times \end{array} \begin{array}{c} \textit{Cost price} \\ \begin{bmatrix} 7.10 \\ 22.20 \\ 33.00 \end{bmatrix} \begin{matrix} S \\ F \\ B \end{matrix} \end{array} = \begin{array}{c} \textit{Selling price} \\ \begin{bmatrix} 11.36 \\ 33.30 \\ 52.80 \end{bmatrix} \begin{matrix} S \\ F \\ B \end{matrix} \end{array}$$

a. State the order of matrix M.

Worked Solution

Matrix M must have order 3 × 3

$(3 \times 3) \times (3 \times 1) = (3 \times 1)$

1 mark

b. Write down the matrix M.

Worked Solution

$$M = \begin{bmatrix} 1.6 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1.4 \end{bmatrix}$$

2 marks

Mark allocation

- 1 mark for correct figures on the diagonal
- 2 marks for all correct

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Question 3

Frank's Fruit Mart has three outlets for their fruit packs; stores P, Q and R.
The table below shows the number of fruit packs that were sold during one week at each of these outlets.

	Outlet		
	P	**Q**	**R**
Standard pack	25	15	8
Family pack	38	23	5
Bulk pack	11	20	12

Given that Profit = selling price – cost price

a. Set up a matrix equation that will enable Frank's Fruit to find the matrix P that represents the profit made on each of the types of fruit packs for this week.

Worked Solution

$$P = \begin{bmatrix} 25 & 38 & 11 \\ 15 & 23 & 20 \\ 8 & 5 & 12 \end{bmatrix} \left(\begin{bmatrix} 11.36 \\ 33.30 \\ 52.80 \end{bmatrix} - \begin{bmatrix} 7.10 \\ 22.20 \\ 33.00 \end{bmatrix} \right)$$

2 marks

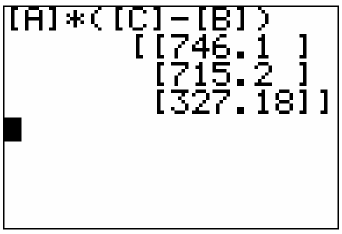
Mark allocation

- 1 mark for 3×3 matrix correct
- 1 mark for Profit matrix correct

b. Calculate the profit made by Frank's Fruit Mart on each of the types of fruit packs for this week.

Worked Solution

b. Using the calculator:



Frank's Fruit Mart makes $746.10 profit on the Standard packs, $715.20 profit on the Family packs and $327.18 profit on the Bulk packs.

1 mark

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Question 4

Frank's Fruit Mart is investigating the purchasing habits of its retail customers. Records show that:
Of the customers who purchased the standard pack this week

50% will purchase the standard pack next week
30% will purchase the family pack next week and
20% will purchase the bulk pack next week.

Of the customers who purchased the family pack this week

20% will purchase the standard pack next week
70% will purchase the family pack next week and
10% will purchase the bulk pack next week

Of the customers who purchased the bulk pack this week

30% will purchase the standard pack next week
40% will purchase the family pack next week and
30% will purchase the bulk pack next week.

a. Enter this information into transition matrix T as indicated below, expressing percentages as proportions.

$$\begin{array}{c} \textit{this week} \\ \begin{array}{ccc} S & F & B \end{array} \end{array}$$

$$T = \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} \begin{matrix} S \\ F \\ B \end{matrix} \ \textit{next week}$$

Worked Solution

$$\begin{array}{c} \textit{this week} \\ \begin{array}{ccc} S & F & B \end{array} \end{array}$$

$$T = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.7 & 0.4 \\ 0.2 & 0.1 & 0.3 \end{bmatrix} \begin{matrix} S \\ F \\ B \end{matrix} \ \textit{next week}$$

2 marks

Mark allocation
- 1 mark for 3×3 matrix given as proportions or percentages with at most 2 errors
- 2 marks for all correct as proportions

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During the first week of monitoring Frank's Fruit Mart there were 875 packs purchased in total. In the same week, 215 standard fruit packs and 512 family fruit packs were purchased.

b. Write this information in the form of an initial state column matrix, I_0.

Worked Solution

$$I_0 = \begin{bmatrix} 215 \\ 512 \\ 148 \end{bmatrix}$$

1 mark

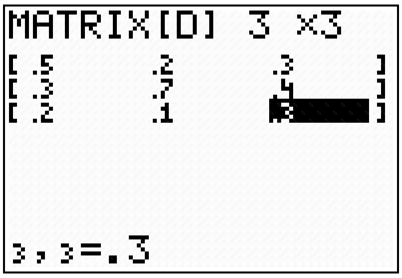
Assume that each customer purchases one pack each week and the pack they purchase depends entirely on their purchase in the previous week.

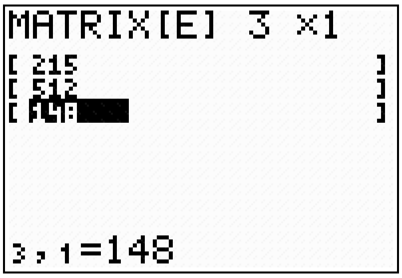
c. Determine the expected number of Standard, Family and Bulk fruit packs purchased in the **third** week. Give your answers to the nearest whole number.

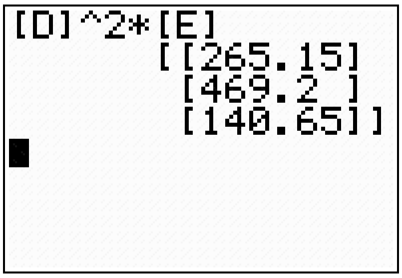
Worked Solution

The expected numbers are given by $T^2 \times I_0$

Using the calculator:







During week three, 265 customers are expected to purchase the Standard pack, 469 customers are expected to purchase the Family pack and 141 customers are expected to purchase the Bulk pack.

2 marks

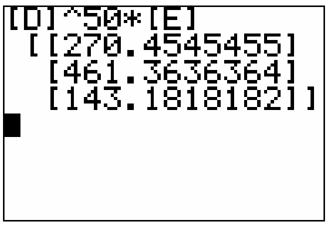
Mark allocation

- 1 mark for showing multiplication of their matrices $T^2 \times I_0$
- 2 marks for correct evaluation of column matrix from product of 2a and 2b answers

d. Of the 875 customers determine, in the long term, the number of Bulk fruit packs that are purchased in a particular week.

Worked Solution

Using a high power of T:



In the long term 143 Bulk fruit packs are expected to be sold. 1 mark

Total 15 marks

END OF SOLUTIONS BOOK

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