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FURTHER MATHEMATICS TRIAL EXAMINATION 2 SOLUTIONS 2008

Section A

Core - solutions

Question 1

a. The boxplot is negatively skewed.

(1 mark)

b. The interquartile range is

$$\begin{aligned}&Q_3 - Q_1\\&= 1300 - 1050\\&= 250\end{aligned}$$

(1 mark)

c. If a piece of data is less than

$$\begin{aligned}&Q_1 - 1\cdot 5 \times IQR\\&= 1050 - 1\cdot 5 \times 250\\&= 675\end{aligned}$$

then it is an outlier.
Since $650 < 675$, it is an outlier.

(1 mark)

Question 2

a. Enter the data into your calculator.

$$r = 0 \cdot 841219...$$
$$= 0.8412 \quad \text{correct to 4 decimal places}$$

(1 mark)

b. The coefficient of determination is given by

$$r^2 = 0 \cdot 7076497...$$
$$= 0.708 \quad \text{correct to 3 decimal places}$$

(1 mark)

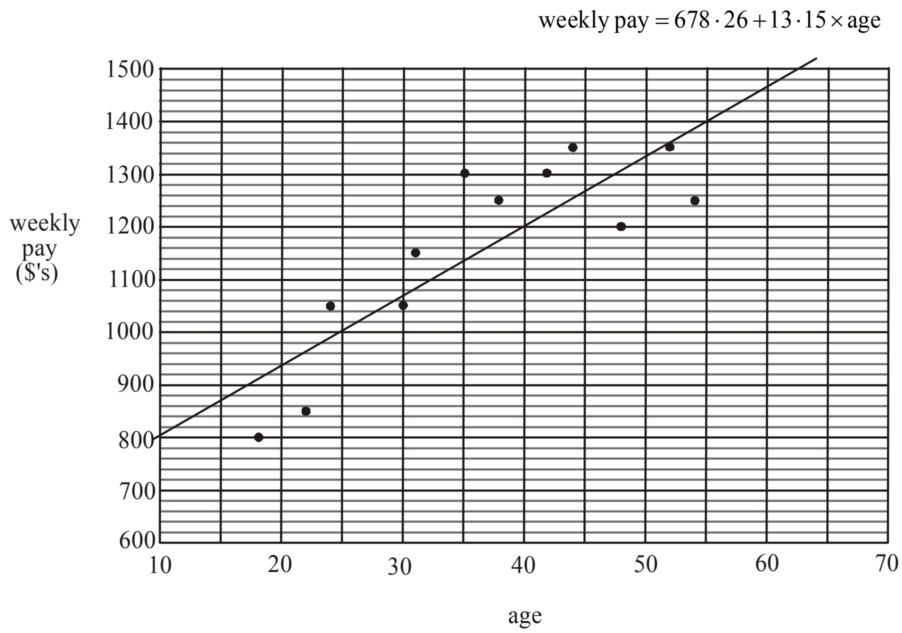
c. Since $r^2 = 0 \cdot 708$, 71% (to the nearest whole percent) of the variation in weekly pay can be accounted for by the variation in age.

(1 mark)

d. **i.** The equation of the least squares regression line is given by
$$\text{weekly pay} = 678 \cdot 26 + 13 \cdot 15 \times \text{age}$$

(1 mark) correct numbers
(1 mark) correct equation

ii.



(1 mark)

e. The actual weekly pay of the employee aged 54 is $1 250.
Using the least squares regression equation,
$$\text{weekly pay} = 678 \cdot 26 + 13 \cdot 15 \times \text{age}$$
when age = 54,
$$\text{weekly pay} = 678 \cdot 26 + 13 \cdot 15 \times 54$$
$$= 1388 \cdot 36$$
$$\text{residual} = \text{actual} - \text{predicted} \quad \text{(formula sheet)}$$
$$= 1250 - 1388 \cdot 36$$
$$= -138 \cdot 36$$

(1 mark)

f. The residual plot shows a pattern; albeit rough, of an inverted parabola. This suggests that the relationship may not be linear. If there were a random collection of points on the residual plot we could assume there was a linear relationship.

(1 mark)

g. A transformation that might be used to linearise the data would be the $\log(x)$ transformation or the $\frac{1}{x}$ transformation or the y^2 transformation.

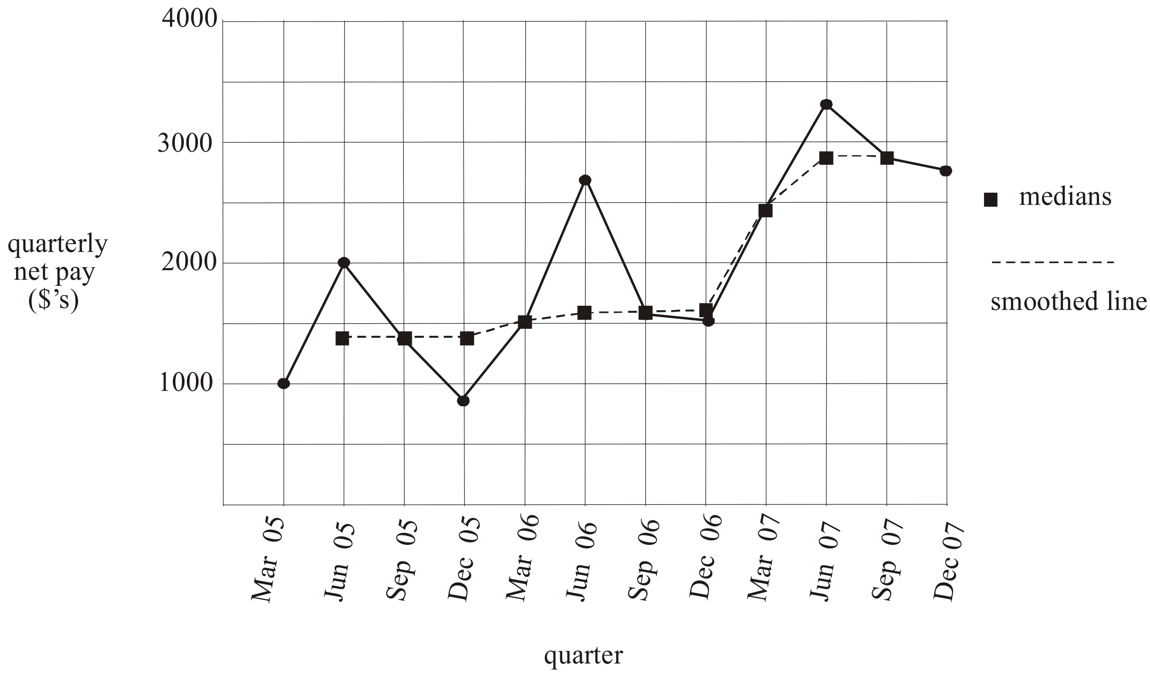
(1 mark)
(only 1 needs to be mentioned)

Question 3

a. The time series plot shows seasonal variation that is trending upwards.

(1 mark)

b.



(1 mark) 5 correct points
(1 mark) 10 correct points
Total 15 marks

SECTION B

Module 1: Number patterns

Question 1

a. At her third session Jane will do $12 + 5 + 5 = 22$ sit - ups .

(1 mark)

b. Use a calculator to count up how many sessions she does to first complete more than fifty sit-ups.
It is at her 9th gym session.

(1 mark)

c. $S_n = a + 5n$
At session 1 Jane does 12 sit-ups.
$12 = a + 5 \times 1$
$12 = a + 5$
$a = 7$

(Check: at session 2 Jane does 17 sit-ups
$17 = a + 5 \times 2$
$17 = a + 10$
$a = 7$)

(1 mark)

d. Use the formula in part **c.**

$$\begin{aligned} S_n &= 7 + 5n \\ 150 &= 7 + 5n \\ 143 &= 5n \\ n &= \frac{143}{5} \\ &= 28 \cdot 6 \end{aligned}$$

During Jane's 29th session she will reach the limit.
At her 28th session she will do

$$\begin{aligned} S_{28} &= 7 + 5 \times 28 \\ &= 147 \end{aligned}$$

so during her 29th session she will reach 150 sit-ups.

(1 mark)

Question 2

a. $r = \frac{110}{100} = \frac{121}{110} = 1 \cdot 1$

(1 mark)

b. We have a geometric sequence so the equation will be of the form

$W_n = a(r)^{n-1}$

$W_n = 100(1 \cdot 1)^{n-1}$

(1 mark)

since *a* represents the first term in the sequence.

c. Method 1 – using a CAS calculator

Solve the equation $200 = 100(1 \cdot 1)^{x-1}$ for x.

So $x = 8 \cdot 272...$

So during his 9th gym session he will first lift 200kg.

(1 mark)

Method 2 – using a graphics calculator

Graph the equation $y = 100(1 \cdot 1)^{x-1}$

Look at the table of values and see the value of *x* where *y* first exceeds 200.

Now, when $x = 8,\ y = 194 \cdot 87$ and when $x = 9$ and $y = 214 \cdot 36$ so during the 9th gym session he will first lift 200kg.

(1 mark)

d. Total of weight lifted for sessions 1 to 9 is given by

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_9 = \frac{100(1 \cdot 1^9 - 1)}{0 \cdot 1}$$

$$= 1357 \cdot 95$$

Total of weight lifted for sessions 1 to 15 is given by

$$S_{15} = \frac{100(1 \cdot 1^{15} - 1)}{0 \cdot 1}$$

$$= 3177 \cdot 25$$

Now, $3177 \cdot 25 - 1357 \cdot 95 = 1819 \cdot 3$.

The total of the weights lifted from the 10th to the 15th session inclusive is given by 1 819kg (to the nearest kg).

(1 mark) making reasonable attempt at finding S_9 and S_{15}

(1 mark) correct answer

Question 3

a. $J_{n+1} = J_n + 2 \quad J_3 = 31$

If $J_3 = 31$ then

$$J_3 = J_2 + 2 = 31$$
$$J_2 = 29$$

So $J_2 = J_1 + 2 = 29$

$$J_1 = 27$$

Jane spends 27 minutes at her first session.

(1 mark)

b. The sequence described is arithmetic because there is a constant difference of 2 between each term.

(1 mark)

c. We have an arithmetic sequence with $a = 27$ and $d = 2$.

$$S_n = \frac{n}{2}[2a + (n-1)d] \text{ (from the formulae sheet)}$$
$$S_{12} = \frac{12}{2}[2 \times 27 + (12-1) \times 2]$$
$$= 6[54 + 11 \times 2]$$
$$= 456$$

Jane spends a total of 456 minutes at her first 12 sessions.

(1 mark)

Question 4

a. For session 1, Jane spends 27 minutes and Michael spends 20 minutes.
So Jane spends 7 minutes more than Michael.

(1 mark)

b. From the graph, the first time that Michael spends more time at the gym during a session is during session number 14.

(1 mark)

c. At session 20, the gap between the points is greatest and hence the difference in the time spent at the gym is the greatest.

(1 mark)
Total 15 marks

Module 2: Geometry and trigonometry

Question 1

a. In ΔABD we use Pythagoras.

$$(AD)^2 = 40^2 - 24^2$$
$$= 1024$$
$$AD = 32\text{m}$$

(1 mark)

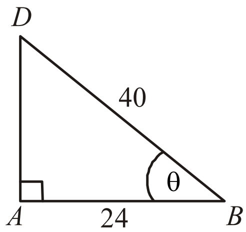
b. Since ΔAFE and ΔABD are similar,

$$\frac{AF}{AB} = \frac{FE}{BD}$$
$$\frac{AF}{24} = \frac{15}{40}$$
$$AF = \frac{15}{40} \times 24$$
$$= 9 \text{ metres}$$

(1 mark)

c. In ΔABD, we have

$$\cos(\theta) = \frac{24}{40}$$
$$\theta = 53^\circ 8'$$



(1 mark)

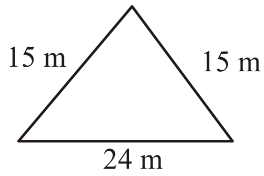
d. ΔAFE and ΔABD are similar. Since the ratio of the sidelengths is $9:24$ or $1:\frac{8}{3}$ then the ratio of the areas will be

$$1:\left(\frac{8}{3}\right)^2$$
$$\text{i.e. } 1:\frac{64}{9}$$
$$\text{or } 9:64$$

(1 mark)

Question 2

a. The area of the triangular end of the triangular prism can be found using Heron's formula



$$A = \sqrt{s(s-a)(s-b)(s-c)} \qquad \text{where } s = \frac{a+b+c}{2} = \frac{15+15+24}{2} = 27$$

So $A = \sqrt{27(27-15)(27-15)(27-24)}$

$= \sqrt{11664}$

$= 108$

(1 mark)

Total area $= 2\times 108 + 2\times 15\times 48 + 2\times 48\times 32 + 2\times 24\times 32$

$= 6264$

Total area $= 6264\text{m}^2$

(1 mark)

b. volume $= 24\times 32\times 48 + 108\times 48$ (from part **a.**)

$= 42048$

volume of building is 42 048 m^3

(1 mark)

c. The model and the actual building are similar shapes.
The ratio of the sidelength of the model to the sidelength of the building is
$2:48$ (the longest sides on the respective bases)

$1:24$

Therefore, the ratio of the volumes of the model to the building is $1:24^3$ or $1:13824$.

(1 mark)

d. Volume of scale model is $\frac{73}{24}\text{m}^3$.

So, volume of actual building

$= \frac{73}{24}\times 24^3$ (from part **c.**)

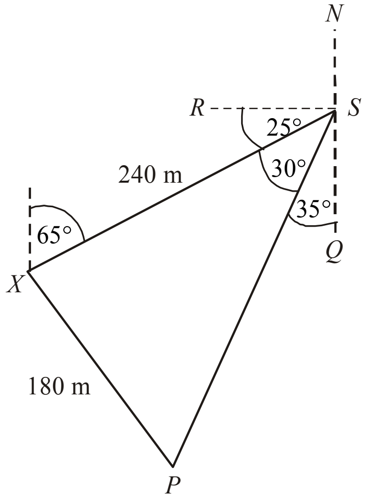
$= 42048\text{m}^3$

(1 mark)

This confirms our answer found in part **b.**

Question 3

a.



Since the bearing of P from S is $215^\circ, \angle QSP = 35^\circ$. Also $\angle NSX = 115^\circ$ (cointerior angles add to give 180°) and so $\angle RSX = 115^\circ - 90^\circ = 25^\circ$.

So $\angle PSX = 90^\circ - 25^\circ - 35^\circ$

$= 30^\circ$

(1 mark)

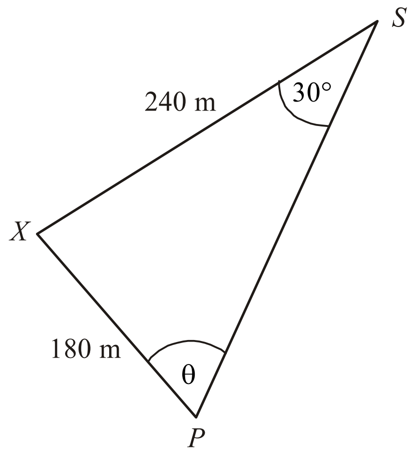
b. In ΔPSX let $\angle SPX = \theta$.

$\frac{\sin\theta}{240} = \frac{\sin(30^\circ)}{180}$

$\sin\theta = \frac{\sin(30^\circ)}{180} \times 240$

$\theta = 41 \cdot 8103^\circ$

$= 41^\circ 49'$ (to the nearest minute)



(1 mark)

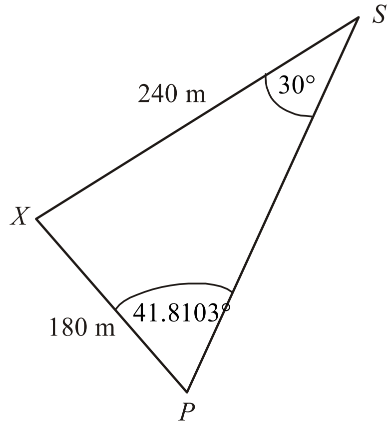
c. In ΔPSX, $\angle PXS = 180^\circ - 30^\circ - 41 \cdot 8103^\circ = 108 \cdot 1897^\circ$. **(1 mark)**

Note: don't use the rounded value from part **b.** and don't round during a question.

$(PS)^2 = 180^2 + 240^2 - 2 \times 180 \times 240 \times \cos(108 \cdot 1897^\circ)$

$PS = 342 \cdot 0102$

$PS = 342\text{m}$ (to the nearest metre)



(1 mark)

d. From part **c.** $\angle PXS = 108 \cdot 1897^\circ$ so the bearing of the post office from X is

$65^\circ + 108 \cdot 1897^\circ = 173 \cdot 1897^\circ$

$= 173^\circ$ (to the nearest degree)

(1 mark)

e. Method 1

Using the formula $\text{Area} = \frac{1}{2}bc\sin A$

$\text{Area} = \frac{1}{2} \times SX \times PS \times \sin(\angle PSX)$

$= \frac{1}{2} \times 240 \times 342 \cdot 0102 \times \sin 30^\circ$ (using your answer from part **c.** Note: don't use your rounded answer)

$= 20520 \cdot 6\text{m}^2$ (to one decimal place)

(1 mark)

Method 2

Using the formula $\text{Area} = \frac{1}{2}bc\sin A$

$\text{Area} = \frac{1}{2} \times SX \times PX \times \sin(\angle PXS)$

$= \frac{1}{2} \times 240 \times 180 \times \sin 108 \cdot 1897^\circ$ (from part **c.**)

$= 20520 \cdot 6\text{m}^2$ (to one decimal place)

(1 mark)

Method 3

Using the formula $\text{Area} = \frac{1}{2}bc\sin A$

$\text{Area} = \frac{1}{2} \times PX \times PS \times \sin(\angle SPX)$

$= \frac{1}{2} \times 180 \times 342 \cdot 0102 \times \sin 41 \cdot 8103^\circ$ (from parts **b.** and **c.**)

$= 20520 \cdot 6\text{m}^2$ (to one decimal place)

(1 mark)

Method 4

Using Heron's formula

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(240 + 180 + 342 \cdot 0102) = 381.0051$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{381 \cdot 0051(381 \cdot 0051 - 240)(381 \cdot 0051 - 180)(381 \cdot 0051 - 342 \cdot 0102)}$$

$= 20520 \cdot 6\text{m}^2$ (to one decimal place)

(1 mark)

Total 15 marks

Module 3: Graphs and relations.

Question 1

a. At 1.30pm the temperature is 19°C.

(1 mark)

b. During the first hour the temperature rises most quickly.

(1 mark)

c. **i.** The maximum temperature is 20°C.

(1 mark)

ii. This occurs at 11.30am.

(1 mark)

Question 2

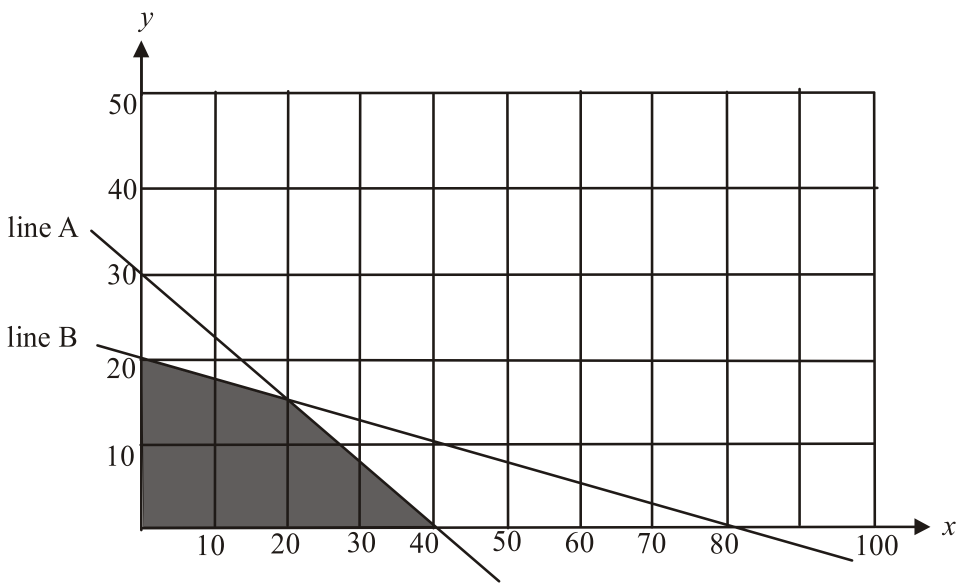
a. $15x + 20y \le 600$

(1 mark)

b. $(20,15)$

(1 mark)

c.



(1 mark)

d. **i.** $P = 15x + 25y$

(1 mark)

ii. At the corner points,

$(0,0), P = 0$

$(20,15), P = 675$

$(40,0), P = 600$

$(0,20), P = 500$

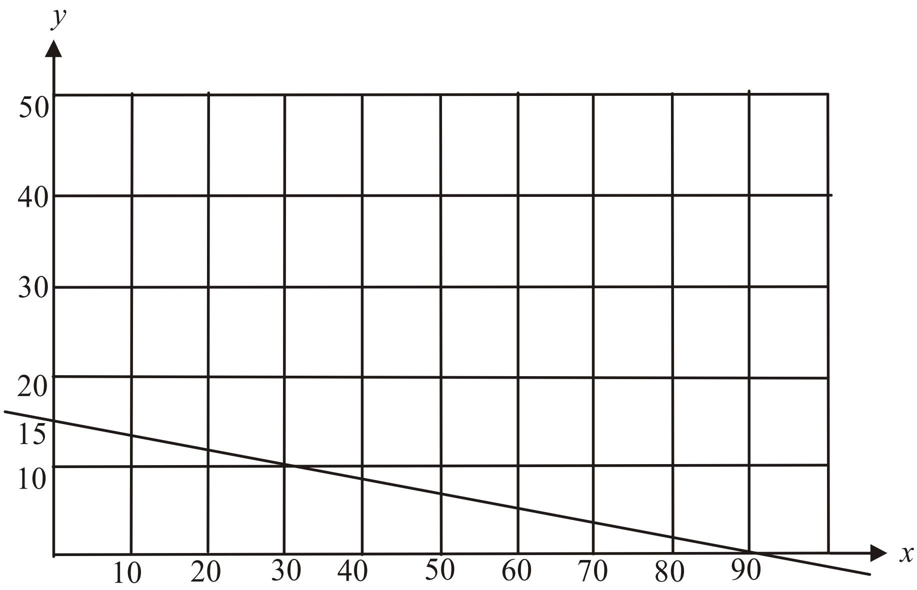
For a maximum profit there should be 20 male clients and 15 female clients.

(1 mark)

e. The new inequality is $5x + 30y \le 450$.

(1 mark)

f.



(1 mark)

Question 3

a.

$$\text{rate} = \frac{10}{30} \text{ litres/second}$$

$$= \frac{1}{3} \text{ litre/second}$$

(1 mark)

b. The conditioner is applied between $t = 150$ and $t = 240$.
It takes 90 seconds.

(1 mark)

c. There is $60 - 30 = 30$ litres of water used.

(1 mark)

d. The tap is on between $t = 0$ and $t = 30$, and then between $t = 90$ and $t = 150$ and then between $t = 240$ and $t = 330$.
The total time is $30 + 60 + 90 = 180$ seconds.

(1 mark)
Total 15 marks

Module 4: Business-related mathematics.

Question 1

a.

$$\begin{aligned}\%\text{increase} &= \left(\frac{730000-160000}{160000}\times\frac{100}{1}\right)\% \\ &= 356\cdot 25\%\end{aligned}$$

(1 mark)

b. Marco made a profit of $\$730000-\$160000=\$570000$.
Assuming no costs were incurred, this was all profit so the capital gain was $570 000.

$$\begin{aligned}\text{Now }\ & 46\%\times\$570000 \\ &= \$262200\end{aligned}$$

is the capital gains tax that Marco must pay.

(1 mark)

c. Nick paid $730 000 for the property so he must pay

$$\begin{aligned}&\$2870+6\%\text{ of }(\$730000-\$130000) \\ &=\$2870+6\%\text{ of }\$600000 \\ &=\$38870\end{aligned}$$

(1 mark)

Question 2

a. $7\%\text{ of }\$12000=\$840$
Marco will pay $\$12000-\$840=\$11160$

(1 mark)

b. **i.** Marco will pay $\$1000+12\times\$1050=\$13600$.

(1 mark)

ii.

$$\begin{aligned}\text{Flat interest rate} &= \frac{100\times\text{total interest paid}}{P\times t} \\ &= \frac{100\times1600}{11000\times3} \\ &= 4\cdot85\%\ \text{(to 2 decimal places)}\end{aligned}$$

(Note: the deposit is deducted from the principal.)

(1 mark)

c. Marco is charged interest for the second $6000.

$$\begin{aligned}\text{interest charged} &= \frac{\$6000\times21\times\frac{1}{12}}{100} \\ &= \$105\end{aligned}$$

(1 mark)

Question 3

a. **i.** 12% of $\$12000 = \$1440$.
The kitchen equipment depreciates by \$1 440 each year.

(1 mark)

ii. After 1 year it will be valued at $\$12000 - \$1440 = \$10560$.
After 2 years it will be valued at $\$10560 - \$1440 = \$9120$.
After 3 years it will be valued at $\$9120 - \$1440 = \$7680$.
After 4 years it will be valued at $\$7680 - \$1440 = \$6240$.
After 5 years it will be valued at $\$6240 - \$1440 = \$4800$.
It will take 5 years.

(1 mark)

b. For reducing balance depreciation,

$$\text{value} = P \times \left(1 - \frac{r}{100}\right)^t$$

$$4800 = 12000 \times \left(1 - \frac{12}{100}\right)^t$$

$$\frac{4800}{12000} = 0 \cdot 88^t$$

$$0 \cdot 4 = 0 \cdot 88^t$$

(1 mark)

Method 1 – trial and error

$t = 4 \qquad 0 \cdot 88^4 = 0 \cdot 5996...$

$t = 10 \qquad 0 \cdot 88^{10} = 0 \cdot 2785...$

$t = 7 \qquad 0 \cdot 88^7 = 0 \cdot 4086...$

$t = 8 \qquad 0 \cdot 88^8 = 0 \cdot 3596...$

So during the eighth year the equipment would be valued at \$4 800.

(1 mark)

Method 2 – using equation solver function on calculator
The equation to solve is

$0 = 0 \cdot 88^t - 0 \cdot 4$

$t = 7 \cdot 1678...$

So during the eighth year (i.e. 7 years have elapsed and we are 0.1678 into the eighth year) the equipment would be valued at \$4800.

(1 mark)

Question 4

a. Using *TVM* solver, we have

$$\begin{aligned} N &= 16 \\ I\% &= 9\cdot 5 \\ PV &= 120000 \\ PMT &= ? \\ FV &= 0 \\ P/Y &= 4 \\ C/Y &= 4 \end{aligned}$$

He makes quarterly payments of \$9102.70.

(1 mark)

b. $\text{Marco's total payments} = 16 \times \$9102\cdot 70$

$= \$145643\cdot 20$

Now, $\$145643\cdot 20 - \$120000$

$= \$25643.20$

So Marco pays a total of \$25643.20 in interest.

(1 mark)

c. If Marco paid the loan off in 2 years his quarterly instalments would be found using *TVM* solver:

$$\begin{aligned} N &= 8 \\ I\% &= 9\cdot 5 \\ PV &= 120000 \\ PMT &= ? \\ FV &= 0 \\ P/Y &= 4 \\ C/Y &= 4 \end{aligned}$$

Quarterly instalments would be \$16647.00. **(1 mark)**

Over 2 years he would pay in total $\$16\,647\cdot 00 \times 8 = \$133176$.

This means he pays $\$133176 - \$120000 = \$13176$ in interest.

So paying it off in 2 years rather than 4 saves Marco

$\$25643.20 - \$13176$

$= \$12467.20$ in interest.

(1 mark)

Total 15 marks

Module 5: Networks and decision mathematics

Question 1

a. The degree of vertex *F* is 5.

(1 mark)

b. Some examples of a Hamiltonian circuit are

A G F E D C B A
A G E D C F B A
A B C D E F G A

Other routes are possible.
The route must start and finish at *A* and pass through each vertex just once.

(1 mark)

c. An Euler circuit does not exist for this graph because all the vertices do not have an even degree.

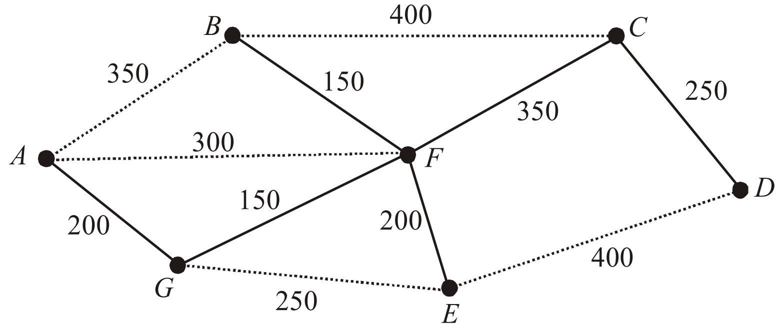
(1 mark)

d. Using trial and error, the shortest route from *A* to *D* is *A G E D*.

(1 mark)

Question 2

a.



(1 mark)

b. Total length of paths to be resurfaced is $200+150+150+350+250+200=1300\text{m}$.

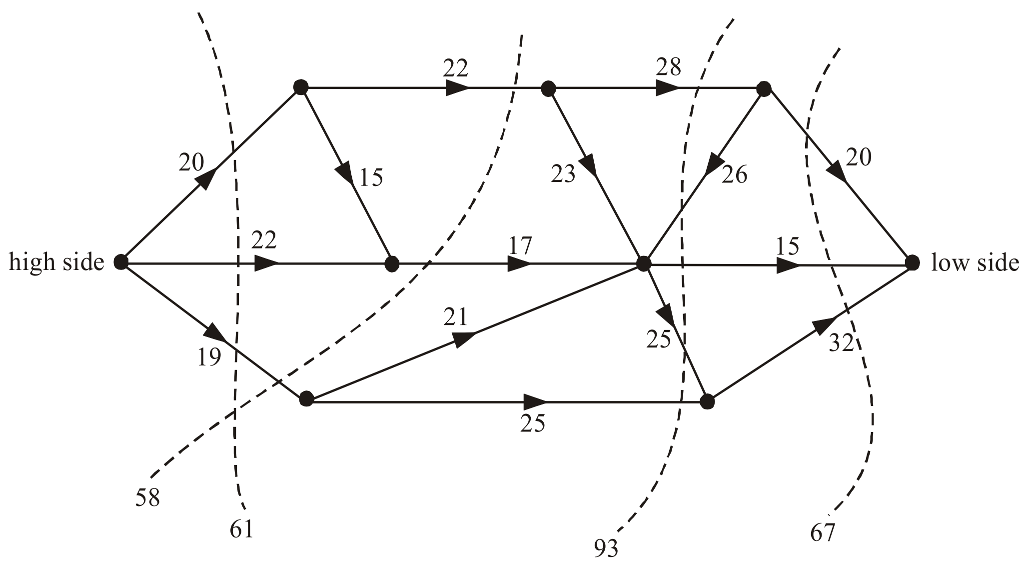
(1 mark)

Question 3

a. The capacity of the cut shown is $28+15+25+25=93$ litres/sec.
Note that since the edge with capacity 26 is flowing in the opposite direction (low to high) it is not counted.

(1 mark)

b.

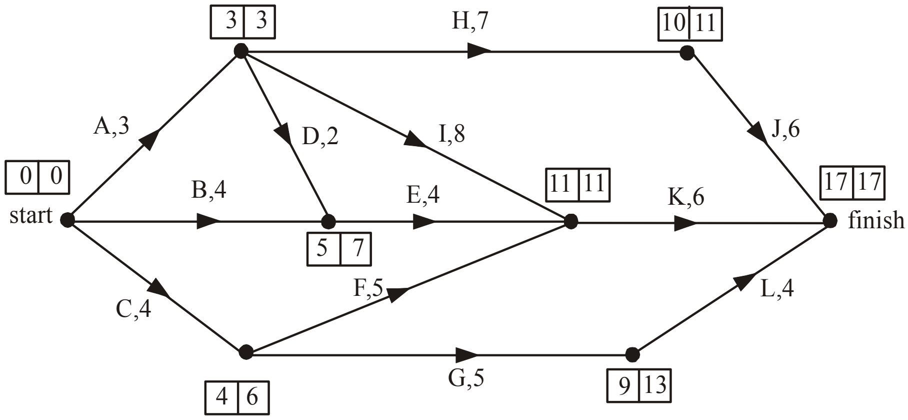


The minimum cut is 58 so the maximum capacity is 58 litres/sec.

(1 mark)

Question 4

a.



The earliest start times are indicated in the left hand box for each activity and the latest start times are indicated in the right hand box. The earliest start time for activity *K* is 11 weeks.

(1 mark)

b. The latest start time for activity *D* is 7 – 2 = 5 weeks.

(1 mark)

c. The sum of the duration of the activities along the critical paths is 17 weeks, so the minimum time is 17 weeks.

(1 mark)

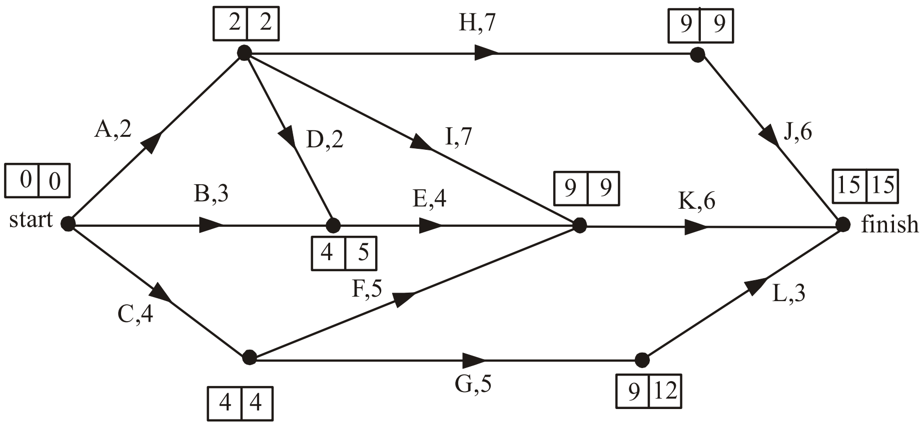
d. The float time for activity *F* is $6-4=2$ weeks.

(1 mark)

e. The critical path is *A* ,*I* and *K*.
Since *A* and *I* are on the critical path, individually, they will reduce the overall completion time.

(1 mark)

f.



The minimum completion time is now 15 weeks.

(1 mark)

g. Those activities that are now critical to the project being completed by this reduced time are *A*, *H*, *J*, *I*, *K*, *C* and *F*.

(1 mark)
Total 15 marks

Module 6: Matrices

Question 1

a. **i.**

$$M = \begin{bmatrix} 5 & 4 \end{bmatrix} \quad N = \begin{bmatrix} 4 & 5 & 6 & 7 \\ 3 & 6 & 2 & 5 \end{bmatrix}$$

$$\begin{aligned} MN &= \begin{bmatrix} 5 & 4 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 & 7 \\ 3 & 6 & 2 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 32 & 49 & 38 & 55 \end{bmatrix} \end{aligned}$$

(1 mark)

ii. N is a 2×4 matrix
M is a 1×2 matrix
Since the number of columns of N (4 columns) does not equal the number of rows of M (1 row), NM does not exist.

(1 mark)

iii. P is a 4×3 matrix
N is a 2×4 matrix
The order of NP will be 2×3.

(1 mark)

b. Using the matrix MN which gives the maximum number of pages for the two assessment tasks for the two subjects in the four homerooms, we see that for 12C the number is 38.

(1 mark)

c.

$$\begin{bmatrix} 4 & 3 & 6 & 8 \\ 5 & 6 & 8 & 3 \\ 6 & 2 & 7 & 4 \\ 7 & 5 & 4 & 2 \end{bmatrix} \begin{bmatrix} f \\ a \\ b \\ p \end{bmatrix} = \begin{bmatrix} 530 \\ 550 \\ 425 \\ 460 \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} f \\ a \\ b \\ p \end{bmatrix} &= \begin{bmatrix} 4 & 3 & 6 & 8 \\ 5 & 6 & 8 & 3 \\ 6 & 2 & 7 & 4 \\ 7 & 5 & 4 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 530 \\ 550 \\ 425 \\ 460 \end{bmatrix} \\ &= \begin{bmatrix} 20 \\ 40 \\ 15 \\ 30 \end{bmatrix} \end{aligned}$$

So $f = 20,\ a = 40, b = 15, p = 30$

(1 mark) for attempting to find or use the inverse matrix
(1 mark) for the correct answer

Question 2

a. There were $865 + 345 + 26 = 1236$ people.

(1 mark)

b. Since the columns of a transition matrix must add to give 1, $a = 0 \cdot 6$.

(1 mark)

c. From the transition matrix we see that 0.7 or 70% of people studying this year are predicted to be working next year.

(1 mark)

d. **i.**

$$\begin{aligned} S_1 &= TS_0 \\ &= \begin{bmatrix} 0\cdot2 & 0\cdot1 & 0\cdot1 \\ 0\cdot7 & 0\cdot8 & 0\cdot3 \\ 0\cdot1 & 0\cdot1 & 0\cdot6 \end{bmatrix} \begin{bmatrix} 865 \\ 345 \\ 26 \end{bmatrix} \\ &= \begin{bmatrix} 210\cdot1 \\ 889\cdot3 \\ 136\cdot6 \end{bmatrix} \end{aligned}$$

(1 mark)

ii. 0.3 or 30% of people move from other activities to full-time work each year.
In the first year that number is $0 \cdot 3 \times 26 = 7 \cdot 8$.
So 8 (to the nearest whole number) moved to full-time work.

(1 mark)

e.

$$\begin{aligned} S_5 &= T^5 S_0 \\ &= \begin{bmatrix} 137\cdot341 \\ 858\cdot372 \\ 240\cdot288 \end{bmatrix} \begin{matrix} s \\ w \\ o \end{matrix} \end{aligned}$$

To the nearest whole number there are expected to be 858 people working after five years.

(1 mark)

f.

$$S_5 = T^5 S_0$$
$$= \begin{bmatrix} 137\cdot 341 \\ 858\cdot 372 \\ 240\cdot 288 \end{bmatrix} \begin{matrix} s \\ w \\ o \end{matrix}$$

$$S_{10} = T^{10} S_0$$
$$= \begin{bmatrix} 137\cdot 333 \\ 851\cdot 683 \\ 246\cdot 984 \end{bmatrix} \begin{matrix} s \\ w \\ 0 \end{matrix}$$

$$S_{20} = T^{20} S_0$$
$$= \begin{bmatrix} 137\cdot 333 \\ 851\cdot 467 \\ 247\cdot 2 \end{bmatrix} \begin{matrix} s \\ w \\ 0 \end{matrix}$$

$$S_{30} = T^{30} S_0$$
$$= \begin{bmatrix} 137\cdot 333 \\ 851\cdot 467 \\ 247\cdot 2 \end{bmatrix} \begin{matrix} s \\ w \\ 0 \end{matrix}$$

A steady state has been reached.

(1 mark)

g. $P_n = TR_{n-1} + TS_{n-2} \quad n \geq 2$

When $n = 2$, we have

$$P_2 = TR_1 + TS_0$$
$$= T^2 R_0 + TS_0 \qquad \text{(Note } TR_1 = T \times (TR_0)\text{)}$$
$$= \begin{bmatrix} 0\cdot 2 & 0\cdot 1 & 0\cdot 1 \\ 0\cdot 7 & 0\cdot 8 & 0\cdot 3 \\ 0\cdot 1 & 0\cdot 1 & 0\cdot 6 \end{bmatrix}^2 \begin{bmatrix} 943 \\ 257 \\ 17 \end{bmatrix} + \begin{bmatrix} 210\cdot 1 \\ 889\cdot 3 \\ 136\cdot 6 \end{bmatrix} \qquad \text{from part } \textbf{d. i.}$$
$$= \begin{bmatrix} 353\cdot 4 \\ 1776\cdot 2 \\ 323\cdot 4 \end{bmatrix} \begin{matrix} s \\ w \\ o \end{matrix}$$

(1 mark)

h. There are 1776 people from both research projects who are in full time work two years after the first of the research projects began.
(Note that this is at the end of the first year for the second research project.)

(1 mark)
Total 15 marks