Neap:

Trial Examination 2007

VCE Further Mathematics Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
A – Core	1	1	15
Section	**Number of modules**	**Number of modules to be answered**	**Number of marks**
B – Modules	6	3	45

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white-out liquid/tape.

Materials supplied

Question booklet of 26 pages with a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.
Please ensure that you write your **name** and your **teacher's name** in the space provided on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2007 VCE Further Mathematics Units 3 & 4 Written Examination 2.

Copyright © 2007 Neap ABN 49 910 906 643 96–106 Pelham St Carlton VIC 3053 Tel: (03) 8341 8341 Fax: (03) 8341 8300 TEVFMU34EX2_QA_07.FM

Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Page

SECTION A – DATA ANALYSIS – CORE MATERIAL

Question 1

This question involves a group of 12 players from within a football club who were used as forwards throughout the season. Table 1 shows how many goal attempts each individual player had made and the number of goals that they scored.

Player	Goal attempts	Goals scored
A	35	19
B	30	12
C	22	8
D	15	11
E	110	76
F	26	15
G	41	27
H	38	30
I	67	39
J	29	18
K	7	3
L	9	5

Table 1

a. Determine the mean and standard deviation of the goals scored. Write your answers correct to two decimal places.

mean = ☐

standard deviation = ☐

2 marks

b. For all forwards in the football league, the mean number of attempts to score a goal during the season was 24.7. For the sample of 12 players in Table 1, determine the percentage of players who made fewer goal attempts than the league mean of 24.7. (Give your answer to the nearest whole number.)

percentage = ☐ %

1 mark

c. Use the data in Table 1 to

i. determine the equation of the least squares regression line that will enable the number of goals scored per forward to be predicted from the number of goal attempts. Write the missing values correct to two decimal places in the spaces provided.

Goals scored = ☐ + ☐ × number of goal attempts

ii. determine the value of Pearson's product–moment correlation coefficient. Write your answer correct to four decimal places.

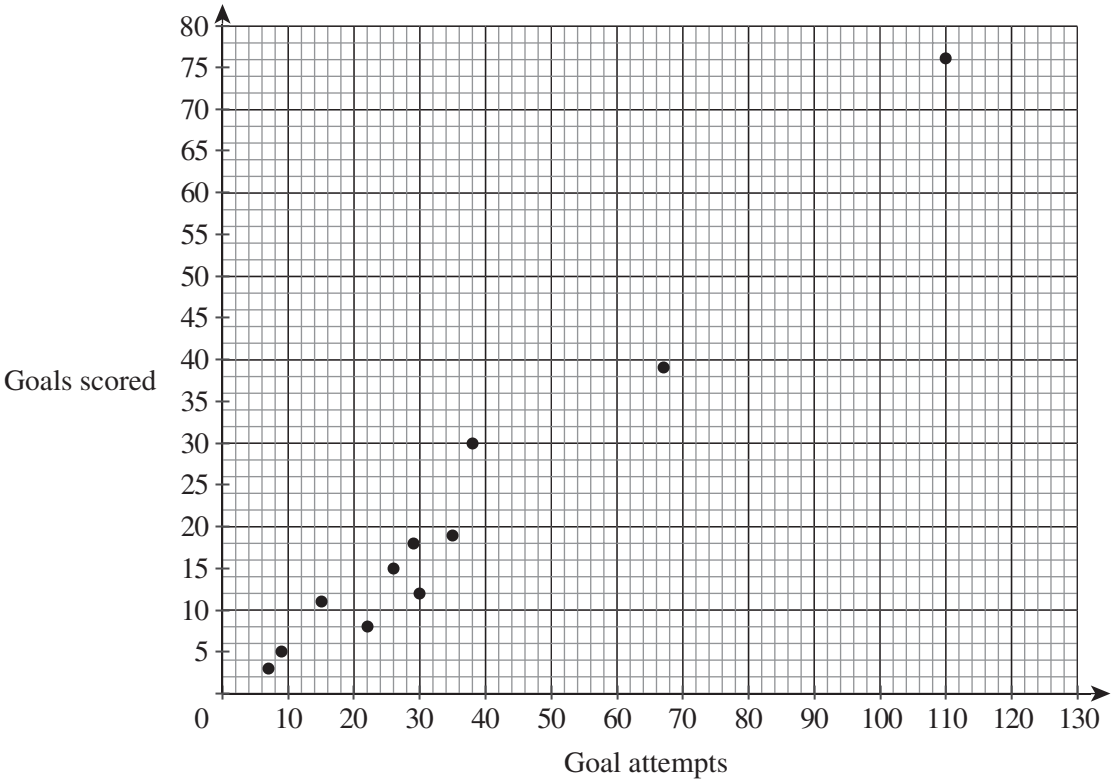
r = ☐

2 + 1 = 3 marks

d. The value of Pearson's product–moment correlation coefficient (calculated in part **c. ii.**) measures the strength and direction of the ☐ relationship between the number of goals scored and the number of goal attempts.

1 mark

e. The scatterplot shown below was constructed from the data displayed in Table 1. The point corresponding to person *G* has not been included. Complete the scatterplot by adding in the data point for person *G*, marking the point with a cross (×). Also on the plot below, draw in the least squares regression line found in **c. i.**



3 marks

f. We wish to predict the number of goals scored based on the number of attempts to score a goal. The dependent variable is

_____

_____

1 mark

g. Complete the following sentences by filling in the spaces.

i. On average, a player's goal tally increases by ☐ goals per goal attempt.

ii. To the nearest whole number, ☐ % of the variation in the goals scored can be explained by the variation in the number of attempts to score a goal.

1 + 1 = 2 marks

h. If we use the least squares regression line obtained in **c. i.** to estimate the number of goals that will be scored by a forward who makes 41 goal attempts, this would leave a residual value (to 2 decimal places) of ☐ goals.

1 mark

i. If we were to use the least squares regression line drawn on Figure 1 to help us predict the number of goal attempts required to score 35 goals, would this be an example of interpolation or extrapolation?

_____

_____

1 mark

Total 15 marks

END OF SECTION A – CORE

SECTION B – MODULES

Module 1: Number patterns

Question 1

The Nail Vision Hardware chain is looking to set up a new wages system to reward loyal employees. They investigate several possible packages.

Package 1 involves each salary increasing by 6% per annum. Penny has a salary of $40 000 in 2007.

a. What would be her salary in 2010 to the nearest dollar?

_____

_____

1 mark

b. How much would she earn over the years 2007 to 2012 (including both 2007 and 2012) to the nearest dollar?

_____

_____

_____

_____

2 marks

Package 2 involves a regular increase of $2500 per annum.

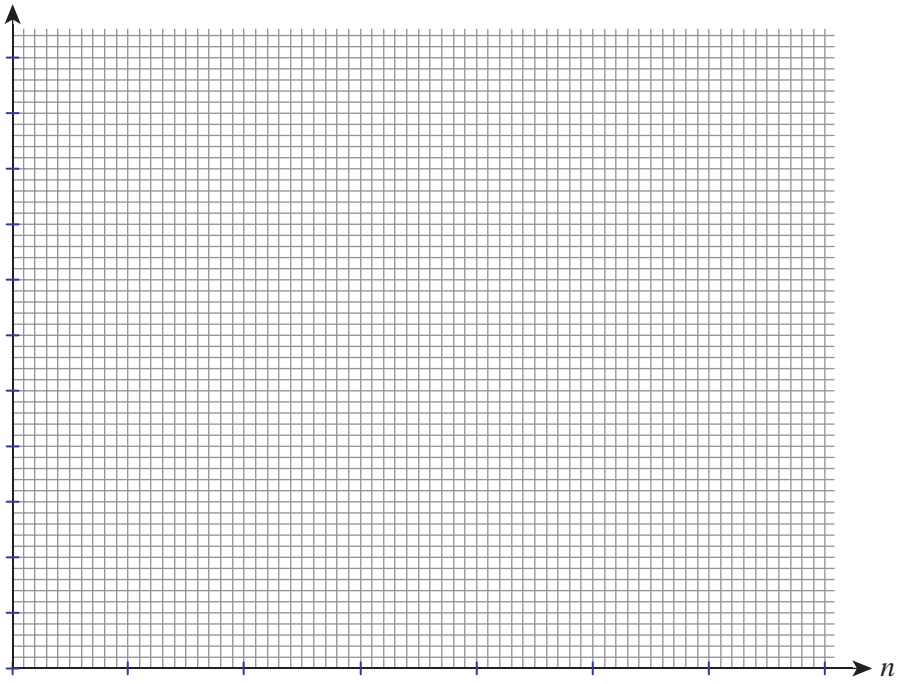
c. What would Penny's salary be in 2010 under this system, to the nearest dollar?

_____

_____

1 mark

d. Plot both salary sequences for the years 2007 to 2012.



2 marks

Penny is uncertain as to how many years she is likely to stay working with Nail Vision.

e. In what year does package 1 first provide a larger salary than package 2?
Justify and explain your answer.

_____

_____

_____

1 mark

f. If Penny is to finish working at the company at the conclusion of the year in which package 1 is first larger than package 2, is she better off with package 1 or package 2? Determine the difference in the possible total salary that could be received between the two packages over the time that Penny works for the company (in dollars and cents).

_____

_____

_____

_____

_____

3 marks

Question 2

The Nail Vision Hardware chain is looking to expand its operations and open new stores. In 2007 its annual turnover is \$7.0 billion. It intends to open two new stores every year. Each new store opened should bring in \$250 million of turnover in its first year. Nail Vision also hopes that the revenue of each existing store will increase by 4% per annum in future years.

a. Write down a difference equation for turnover, t_n, in billions of dollars.

_____

_____

1 mark

The collapse of a rival hardware chain causes Nail Vision to expand more quickly. This new difference equation applies:

$$t_{n+1} = 1.05t_n + 1, \quad t_1 = 7.0$$

b. Find the year when turnover reaches \$10.0 billion for the first time.

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

2 marks

A third possible difference equation is proposed:

$$T_{n+1} = 1.03T_n + 0.4(T_n - T_{n-1}) \quad , \quad T_1 = 7.0, T_2 = 8.35$$

c. During which years does this model predict a larger turnover than that given by the difference equation in part **a**?

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

2 marks
Total 15 marks

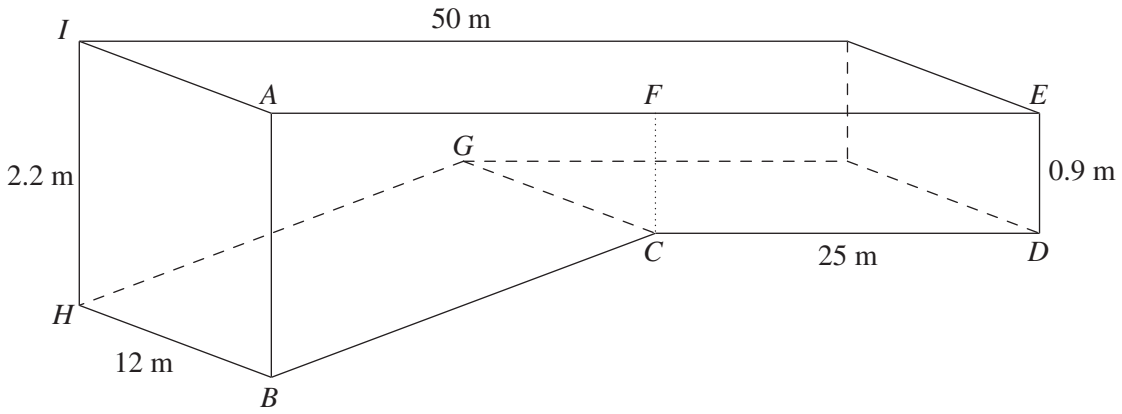
END OF MODULE 1

Module 2: Geometry and trigonometry

Question 1

A proposed swimming pool is to be constructed in the western suburbs of Melbourne. The design of the swimming pool is shown in the diagram below. The pool has two sections: one section has a flat base, while the other section has a sloping base.

From the shallow end of the pool, the first 25 metres of the pool has a constant depth of 0.9 metres. Halfway along the length of the pool, the depth begins to increase at a constant rate, reaching a maximum depth of 2.2 metres.



a. Name the shape of the quadrilateral *ABCF*.

_____

_____

1 mark

b. Calculate the distance *EI*. Write your answer in metres, correct to two decimal places.

_____

_____

_____

1 mark

c. Calculate the area of the side of the pool bound by *ABCDEFA*.
Write your answer in square metres, correct to one decimal place.

_____

_____

_____

_____

2 marks

d. Using your answer to part **c.**, find the volume of water required to fill the pool.
Write your answer correct to the nearest cubic metre.

_____

_____

_____

1 mark

e. The sloping section of the base of the pool bound by the rectangle *BCGH* is painted first and that section of the pool is filled before the flat section of the base is painted.
Calculate the volume of water required to fill the section of the pool with the sloping base, up to the level of the flat base.
Write your answer correct to the nearest cubic metre.

_____

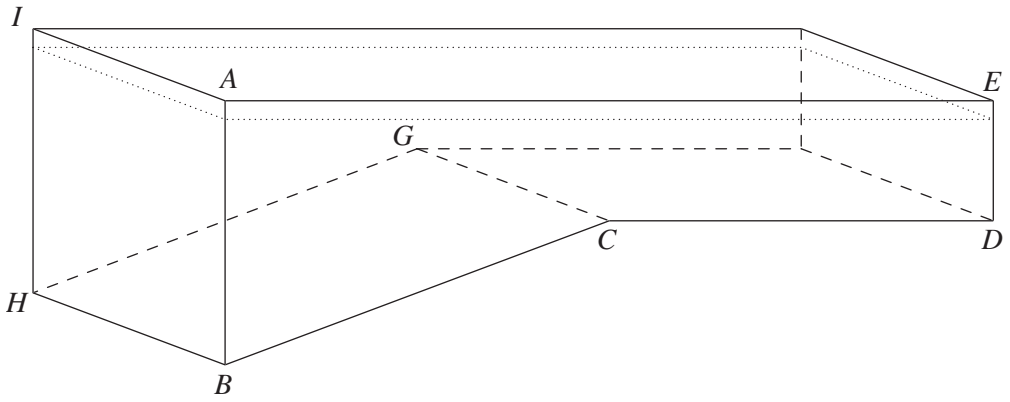
_____

_____

_____

2 marks

f. After the section of the pool with the sloping base is full, the entire pool is filled. Unfortunately, the pump breaks down before the pool is completely full. It is known that the total volume of the water in the pool is 567 cubic metres.



Calculate the distance between the top of the water and the top of the pool.
Write your answer in metres correct to two decimal places.

_____

_____

_____

_____

_____

3 marks

Question 2

It is discovered that the cause of the pump breakdown is the use of a hose that is too small.

The radius of the incorrect hose is 8 cm.

The radius of the correct hose is 12 cm.

By using the properties of similarity, calculate the ratio

volume of water using the incorrect hose : volume of water using the correct hose

_____

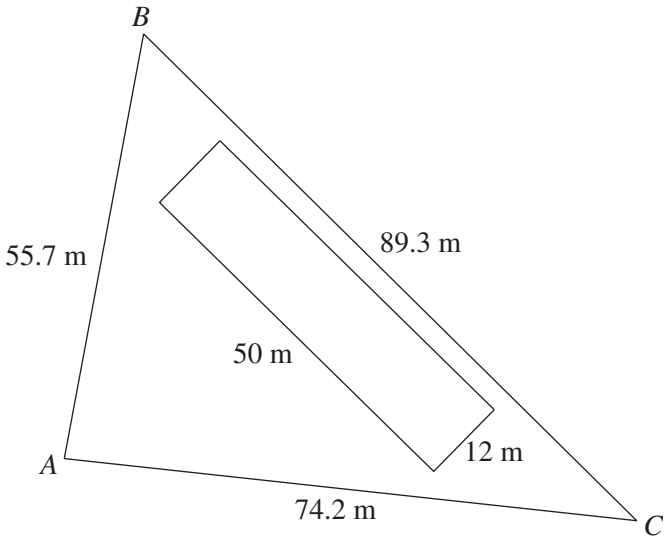
_____

_____

1 mark

Question 3

The block of land on which the pool is to be situated is shown in the diagram below.



Boundary *AB* is 55.7 metres long, boundary *BC* is 89.3 metres long and boundary *AC* is 74.2 metres long.

a. It is proposed to landscape all of the land outside the pool within the triangle *ABC*.
Calculate the area of land to be landscaped. Write your answer correct to the nearest square metre.

_____

_____

_____

_____

_____

_____

2 marks

b. The bearing of *B* from *A* is 21.7 degrees True. Calculate the bearing of *A* from *C*. Write your answer correct to one decimal place.

_____

_____

_____

_____

_____

_____

2 marks
Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

Question 1

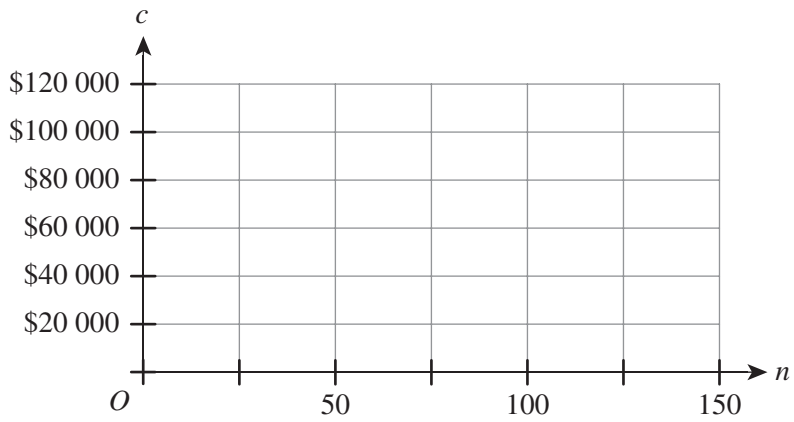
A company, Greenozone, designs and manufactures various models of rainwater tanks. The new *Slenderfit* tank requires $400 worth of materials to make each tank. In addition to this, it costs $15 000 per year to provide the manufacturing facilities, regardless of the number of tanks that are made. It is possible for the facilities to make 150 tanks per year.

a. If n is the number of tanks made, the equation for the total cost of manufacture would be

$c =$ $\qquad\qquad$ $0 \leq n \leq 150$

1 mark

b. Sketch the graph of the equation on the set of axes below:



1 mark

Greenozone are able to sell the tanks to retailers. The first 60 tanks sell for $500 each but the remaining 90 bring in $700 each.

c. Complete the equation for revenue, R, in terms of n.

$$R = \begin{cases} \square & 0 \leq n \leq 60 \\ \square & 60 < n \leq 150 \end{cases}$$

1 mark

d. Sketch this revenue on the above axes.

1 mark

e. What is the least value of n that allows Greenozone to make a profit?

_____

_____

1 mark

Neap:

Trial Examination 2007

VCE Further Mathematics Units 3 & 4

Written Examination 2

Formula Sheet

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Copyright © 2007 Neap ABN 49 910 906 643 96–106 Pelham St Carlton VIC 3053 Tel: (03) 8341 8341 Fax: (03) 8341 8300 TEVFMU34EX2_FS_07.FM

FURTHER MATHEMATICS FORMULAS

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: $\text{seasonal index} = \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n-1)d) = \dfrac{n}{2}[2a + (n-1)d] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, \; r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, \; |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET

Question 2

The process of manufacturing a tank involves two main processes: welding and testing. Two new models are being investigated due to the increased demand for smaller water tanks.

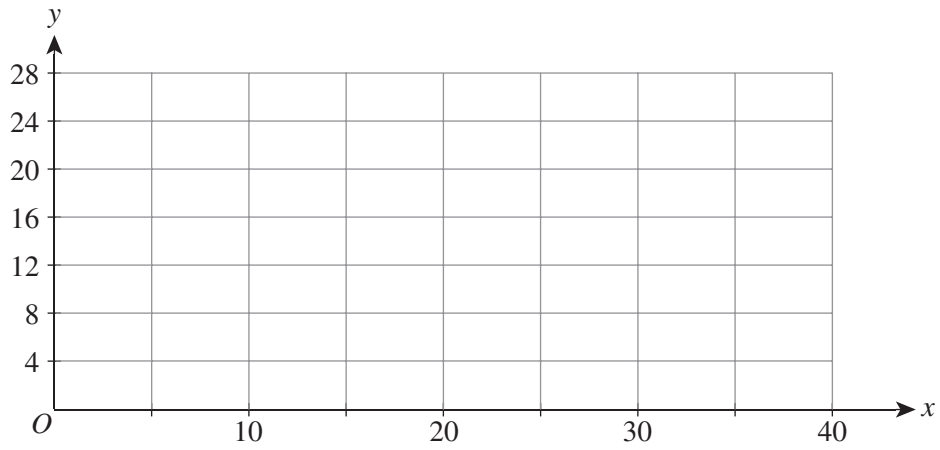
	Domestic	**Garden**	**Time available (hours)**
Welding	2	4	64
Testing	3	4	84

a. If x domestic tanks and y garden tanks are made each week, write down a full set of constraints.

$$x \geq 0 \ , \ y \geq 0$$

2 marks

b. Sketch the corresponding feasible region on the graph below. Clearly identify the coordinates of all corners of the feasible region.



4 marks

c. The company is able to make a profit of $200 on each domestic tank and $310 on each garden tank. Write an expression for the profit, P, in terms of x and y.

_____

_____

1 mark

d. Determine the number of each type of tank (domestic and garden) that should be produced to maximise the company's profit.

2 marks

e. What is the actual maximum profit?

_____

_____

1 mark

Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Question 1

Tracey wants to buy a plasma TV and when she sees one advertised at FJ Hi Fi's monster sale for $4295 she decides to purchase it.

a. If the sale price represents a reduction of 12.5%, calculate the original price of the TV. (Answer correct to the nearest dollar.)

_____

_____

_____

_____

_____

2 marks

Tracey has saved $1500 and is offered a repayment scheme by FJ Hi Fi where she pays the $1500 as a deposit and pays $145 per month for 24 months.

b. How much interest would Tracey pay under this arrangement?

_____

_____

_____

_____

_____

2 marks

c. What is the flat rate of interest per annum that would be paid under this repayment scheme? (Answer correct to two decimal places.)

_____

_____

_____

_____

_____

2 marks

d. Calculate the effective interest rate that Tracey would pay. (Answer correct to two decimal places.)

_____

_____

_____

_____

_____

1 mark

As an alternative to the above repayment scheme Tracey decides to ask the credit union to lend her the $2795 she needs in order to pay cash for the TV. If the credit union agrees to allow her to take out a personal loan at 11.45% per annum, compounding monthly,

e. calculate the monthly repayment required if the loan is to be repaid within two years.

2 marks

f. calculate the amount of interest that Tracey would pay under this arrangement.

1 mark

Question 2

Adrian is 23 years old and has just started work at an engineering company where he earns $30 000 per year. The company must contribute an amount equivalent to 9% of Adrian's salary to his superannuation fund. There is a government tax of 15% imposed upon all such employer contributions.

a. How much is actually added to Adrian's account in the superannuation fund in his first year of work?

1 mark

b. If his salary increases by an average of 3% per year, what will Adrian's salary be in 40 years' time?

2 marks

c. After 40 years, when Adrian is 63, the employer contributions will have accumulated to $1 035 000 in his superannuation account if the fund averages a return of 8.5% during that period.

If Adrian retires at the age of 63 with $1 035 000 in his superannuation fund invested at a more conservative rate of 6.5% per annum, for how many years will his fund be able to pay him an annual pension of $75 000?

_____

_____

_____

_____

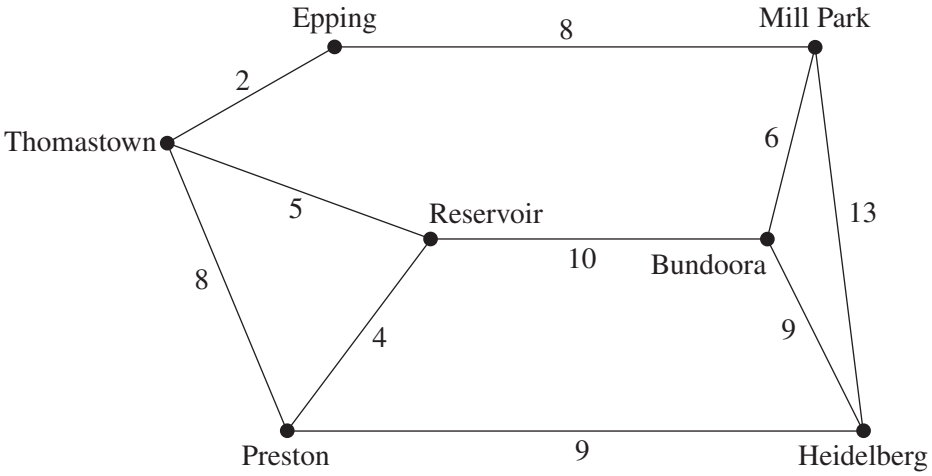
_____

2 marks
Total 15 marks

END OF MODULE 4

Module 5: Networks and decision mathematics

Six students enrolled at a school in Preston attend work experience in each of the suburbs listed in the network diagram below. No student is doing work experience in Preston. The distance between the suburbs, in kilometres, is shown.



The students' teacher, Mandy, begins her day at the school before visiting each of the six work experience sites and then returns to the school in Preston.

Question 1

a. What is the common name for this type of journey?

_____

_____

1 mark

b. List one route that Mandy could take that would minimise the total distance travelled.

_____

_____

1 mark

c. State the total distance that Mandy will travel, following the route listed in part **b.** above.

_____

_____

1 mark

Question 2

Mandy asked two of her colleagues who also work at the school, Doris and Sophia, to visit one of Epping, Mill Park and Bundoora. In other words, between them Mandy, Doris and Sophia need to visit Epping, Mill Park and Bundoora so that each suburb is visited and each teacher makes one visit.

The following table lists the estimated time, in minutes, that it takes for each teacher to travel from her home to each suburb.

	Epping	**Mill Park**	**Bundoora**
Mandy	42	33	36
Doris	38	34	33
Sophia	31	31	27

a. Use the Hungarian algorithm to allocate each teacher to a suburb by finding the minimum time allocation. Use your result to complete the table below.

	Suburb
Mandy	
Doris	
Sophia	

2 marks

b. Calculate the total travel time that allows each suburb to be visited, according to your answer to part **a.**

_____

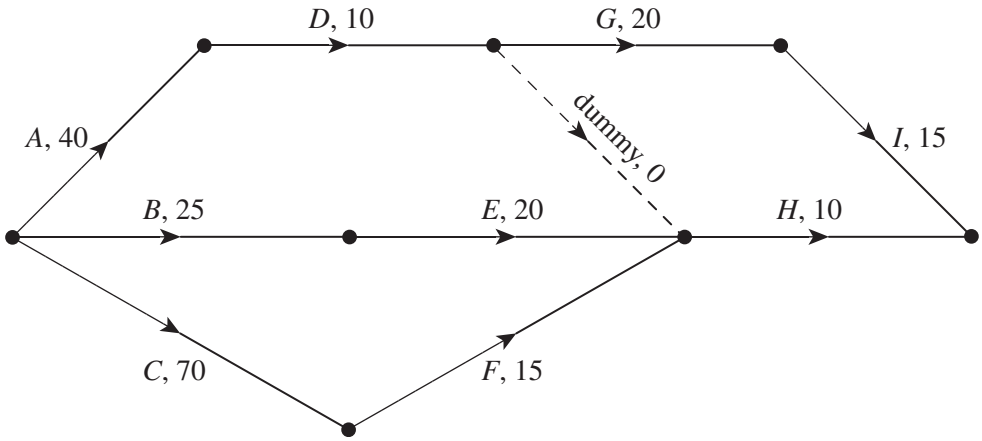
_____

_____

1 mark

Question 3

Carly is a university student studying project management. She is asked to solve the project management problem represented by the network diagram below. The network describes the activities that a horse trainer must complete in order to prepare the horses and their stables before training can commence on any given day.



a. List all the activities that must be completed before activity *H* can begin.

_____

_____

1 mark

Some of the information in the network diagram above is summarised in the following table.

b. Complete the missing times in the table below.

Activity	Earliest starting time (minutes)	Latest starting time (minutes)
A	0	10
B	0	40
C	0	0
D	40	
E	25	65
F	70	70
G	50	60
H		85
I	70	80

2 marks

c. Some of the activities can be delayed without delaying the entire project. List all such activities.

_____

_____

1 mark

d. Calculate the slack (float time) of activity *G*.

_____

_____

1 mark

e. Which activity (or activities) can be delayed for the longest time without delaying the entire project? Justify your answer.

_____

_____

_____

_____

_____

2 marks

Question 4

Due to delays caused by a drought, Carly must help the horse trainer choose between delaying activity *D* by 20 minutes or delaying activity *F* by 20 minutes.

a. State which activity (activity *D* or activity *F*) should be chosen in order to minimise the completion time of the entire project.

_____

_____

1 mark

b. Using your answer to part **a.**, by how many minutes would the entire project now be delayed?

_____

_____

1 mark

Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

The migration pattern of Mongolian storks is under investigation. Three winter nesting places have been identified, areas *A*, *B* and *C*.

- Every winter, 80% of storks who nested at area *A* in the previous year return to area *A*.
- Equal numbers of the storks which previously nested at area *A* now nest at area *B* and area *C*.
- Of the storks which previously nested at area *B*, 70% return the next year and 20% go to area *A*.
- Of those previously nesting at *C*, 60% return the next year while 20% nest at area *A* the next year.

It may be assumed that all storks nest at one of these three locations every year.

Letting $S_0 = \begin{bmatrix} 0.4 \\ 0.3 \\ 0.3 \end{bmatrix}$ gives the proportions of storks located at areas *A*, *B* and *C*, respectively, for 2007.

a. Write down a transition matrix that could be used to predict the nesting numbers at the three locations based on the previous year's figures.

1 mark

b. Determine the proportions of storks at each location for 2008 and 2009.

_____

_____

_____

_____

_____

Thus complete the following:

In 2008, _____% nest at area *A*, _____% nest at area *B* and _____% nest at area *C*.

In 2009, _____% nest at area *A*, _____% nest at area *B* and _____% nest at area *C*.

3 marks

c. If these patterns were to continue, what can be said about the long-term stork populations at each location? If populations stabilise, at what proportions does this occur?

_____

_____

_____

_____

2 marks

A new industrial complex is being planned for area *C*. Over the next few years, the birds who attempt to settle at area *C* are taken to area *A* and area *B* in equal numbers.

d. Alter your existing transition matrix to reflect this change.

2 marks

e. Find the new long-term proportion of the stork populations at area *A* and area *B*.

_____

_____

Hence complete the following sentence.

The long-term proportion of the stork population at area *A* is _____% and at area *B* is _____%.

1 mark

Question 2

The breeding patterns of the storks are also under observation. Adult females produce an average of 1.9 eggs each per year. Immature females (those between 1 and 2 years of age) produce a mean of 0.8 eggs each. In total, 289 eggs per year are produced.

It is also known that there is a total of 210 adult females and immature females in the area.

a. Write down a matrix equation, the solution of which is the number of each of the adult females and immature females.

2 marks

b. Solve the above equation and state the numbers of adult females and immature females.

1 mark

It is hoped that a breeding program can be introduced that will have the effect of increasing the average number of eggs produced by adult females to 2.5 eggs per annum, while also increasing the average number of eggs produced by immature females to 1.0 eggs per annum.

c. Use matrix methods to determine the total number of eggs that would result from this.

_____

_____

1 mark

d. Determine the required numbers of eggs per adult female and eggs per immature female so that 500 eggs are produced per year. It may be assumed that each adult female will continue to produce an average of 1.5 more eggs than an immature female does each year. Give your answer to 2 decimal places.

_____

_____

2 marks
Total 15 marks

END OF QUESTION AND ANSWER BOOKLET

