Insight Publications

INSIGHT

Trial Exam Paper

2007

FURTHER MATHEMATICS

Written examination 1

MULTIPLE-CHOICE QUESTION BOOK

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A	13	13			13
B	54	27	6	3	27
					Total 40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference that may be annotated (can be typed, handwritten or a textbook), one approved graphics calculator (memory DOES NOT have to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring blank sheets of paper or white out liquid/tape into the examination.

Materials provided
- The question and answer book of 33 pages, with an answer sheet for the multiple-choice questions.
- A separate sheet with miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions
- Write your **name** in the box provided on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- Unless otherwise indicated, diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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Working space

TURN OVER

SECTION A

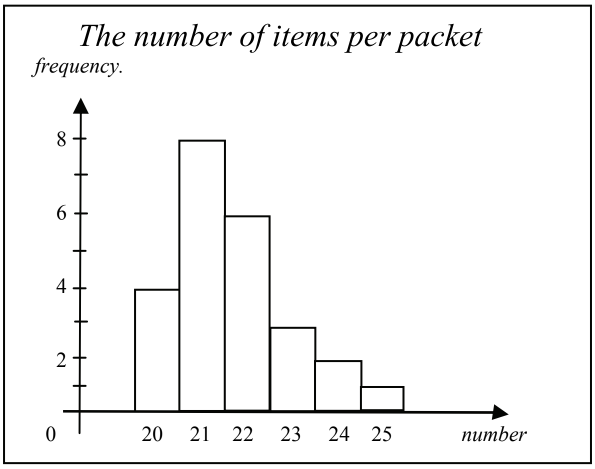
Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
One mark will be awarded for a correct answer; no marks will be awarded for an incorrect answer.
Marks **are not** deducted for incorrect answers.
No marks will be awarded if more than one answer is completed for any question.

Core – Data Analysis

Question 1

At a school carnival, lucky-dips were sold in packets containing a number of items per packet. The distribution is shown in the frequency table below.



The percentage of packets that contained more than 23 items is

A. 3%
B. 6.25%
C. 24%
D. 12.5%
E. 25%

Question 2

Data collected from a test is displayed in a stemplot as shown below.

STEM	LEAF
0	5 6 8 8
1	0 3 3 5
1	5 6 8 8 9 9
2	1 4 4
2	5

The inter-quartile range for this data is

A. 2
B. 5
C. 9
D. 15.5
E. 20

Question 3

Data was collected from a fishing competition on the size of fish caught that day. It was noticed that the data had a bell shaped distribution with a mean of 22.5cm and standard deviation of 2.5cm.

Approximately what percentage of fish were less than 25cm?

A. 16%
B. 95%
C. 50%
D. 68%
E. 84%

Question 4

A student wrote the following numbers to calculate the mean and median.

10, 12, 14, 14, 18, 19, 19

After her calculations, she was informed that her last number 19 was incorrect and in fact was a number **less than** 10.

Compared to her original answers, which of the following is **true**?

A. The actual result had a lower mean and median.
B. The actual result had a lower mean but the same median.
C. The actual result had a lower mean but a higher median.
D. The actual result had the same mean but a lower median.
E. The results remained unchanged.

Question 5

Jon's exam grades for three subjects are shown below. The class average and standard deviation are also shown for each subject.

Subject	***Jon's Mark***	***Class Mean***	***Class Standard deviation***
History	65	70	10
Science	75	75	10
Art	85	90	5

Jon's best performance in relation to the class results was

A. History

B. Science

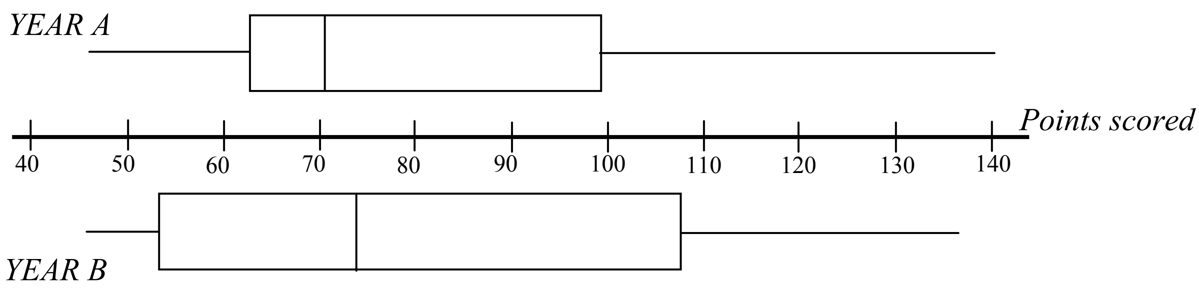
C. Art

D. Art and History were equal

E. Art and Science were equal

Questions 6 and 7 refer to the following information

A junior football team compares the number of points it has scored over the last two years (*A* and *B*) by using the box plots as displayed below.



Question 6

From this box-plot summary, which of the following observations is true?

A. Year A scores were generally higher than Year B

B. 50% of Year A scores were less than 70 points compared to only 25% for Year B

C. If the box-plots were standardised, Year A would have an outlier

D. Both box-plots have a positive skew

E. 25% of Year A scores are more than Year B

SECTION A – continued

Question 7

The variables:

Year (A or B) and **Points scored** are

A. both categorical variables.
B. both numerical variables.
C. categorical and numerical variables respectively.
D. numerical and categorical variables respectively.
E. neither categorical and numerical variables

Question 8

A survey of seven trees noting the age and height of an exotic species gave the following results:

Age (years)	1	4	7	11	12	14	20
Height (metres)	1.8	3.4	5.0	6.2	7.0	6.8	7.4

Further analysis using a least squares regression association would have the **independent variable** and **correlation coefficient** for this data as

A. *Age*, 0.933
B. *Age*, 0.871
C. *Height*, 0.933
D. *Height*, 0.871
E. *Age*, 0.305

Questions 9 and 10 refer to the following information

The population of Victoria from 1990 to 2004 is shown in the table below.

Years from 1990	0	1	3	4	5	7	10	11	13	14
Population in millions	4.40	4.44	4.48	4.50	4.54	4.62	4.77	4.83	4.94	4.99

Table 1

Question 9

Using least squares regression analysis it was found that a linear model to fit this data in **table 1** was

Population in millions = 4.36 + 0.043 × Years from 1990

Using this model, the **residual** for the population in millions for the year 2000 was

A. 4.79
B. 85.59
C. 0.02
D. –0.02
E. Impossible to calculate because of missing data

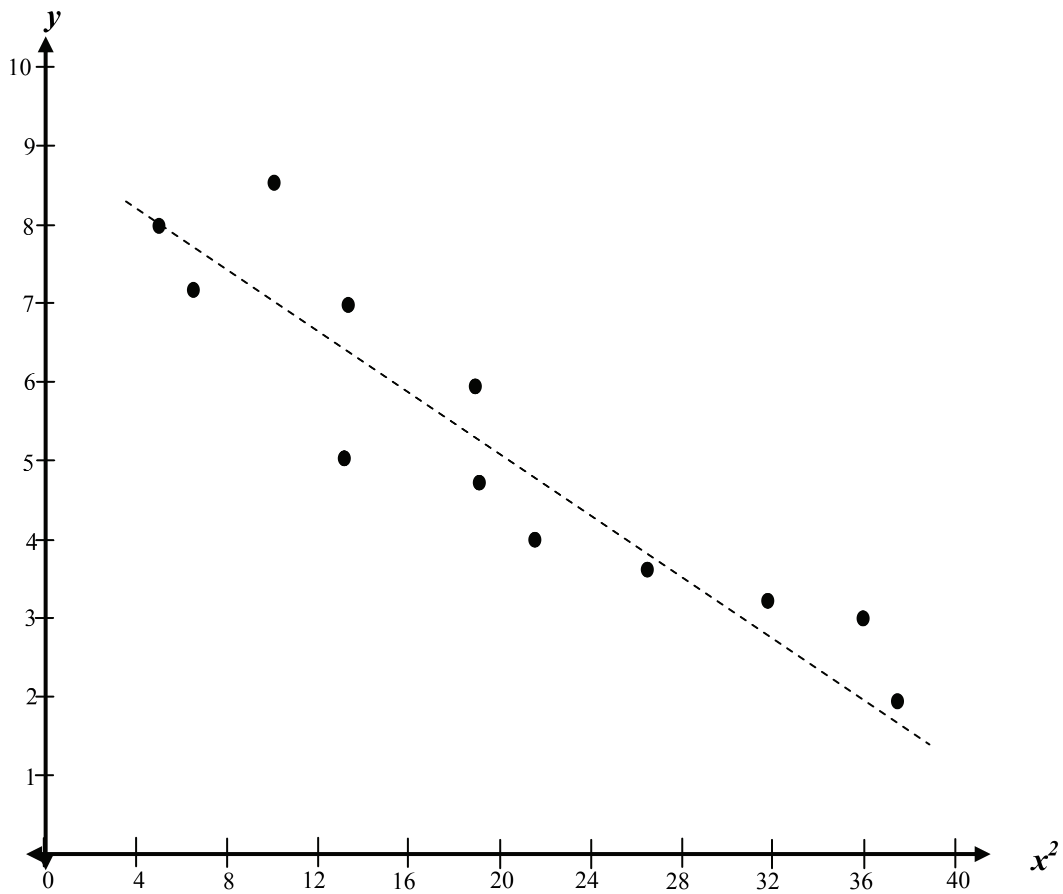
Question 10

When interpreting this linear regression model, which of the following is **true**?

A. The population is increasing at 0.043 million people per year
B. The gradient of 0.043 shows there is are no correlation to predict population
C. The initial population in 1990 is predicted as 0.043 million people
D. The population is decreasing as time increases
E. The population remains steady

Question 11

Data collected has been transformed giving the following resulting graph.



The line of best fit (as indicated by the dash line in the figure) is closest to

A. $y = 8.8 + 0.2x$
B. $y = 8.8 - 0.2x$
C. $y = 0.2 + 8.8x^2$
D. $y = 8.8 - 0.2x^2$
E. $y = 0.2 + 8.8x$

Questions 12 and 13 refer to the following information

The table below shows the seasonal indices of swimwear sales for a particular firm.

Quarter	Summer	Autumn	Winter	Spring
Seasonal Index	1.10	0.95	0.82	

Question 12

The seasonal index for Spring is missing. Once calculated this value shows that sales for spring are

A. typically 3% above average
B. typically 3% below average
C. typically 113% above average
D. typically 13% below average
E. typically 13% above average

Question 13

The deseasonalised figure for swimwear sales in Winter is 88. The **actual** sales for Winter is closest to

A. 107
B. 72
C. 82
D. 99
E. 0

END OF SECTION A
TURN OVER

SECTION B

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected on the answer sheet provided.
Indicate the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.
Choose the response that is **correct** for the question.
One mark will be awarded for a correct answer; no marks will be awarded for an incorrect answer.
Marks **are not** deducted for incorrect answers.
No marks will be awarded if more than one answer is completed for any question.

Module 1: Number Patterns

Before you answer these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions.

Question 1

Which of the following sequences is **not** a geometric sequence?

A. 2, 6, 18, 54, 162 …
B. 2, –6, 18, –54, 162 …
C. 100, 50, 25, 12.5, 6.25 …
D. 1, 3, 6, 12, 36 …
E. 0.1, 1, 10, 100, 1000 …

Question 2

The first four terms of an arithmetic are { –2, 1, 4, 7 … }.

The 25th term for this sequence is

A. 76
B. 73
C. 70
D. 850
E. 45

The following information relates to questions 3 and 4

A large cattle station has a herd of 160 000 cattle at the start of the first year. The cattle population increases by 2% each year. At the end of each year, 4 000 cattle are sold.

Applying this process, at the start of the second year there are now 159 200 cattle.

Question 3

Continuing this process, the number of cattle at the **start** of the third year is

A. 158 384
B. 158 400
C. 155 200
D. 161 552
E. 162 384

Question 4

If A_n is the number of cattle at the start of each year, n, a difference equation that can be used to model the population of the herd over time is

A. $A_{n+1} = 160\,000 - 0.98A_n$ where $A_1 = 4\,000$
B. $A_{n+1} = 160\,000 - 1.02A_n$ where $A_1 = 160\,000$
C. $A_{n+1} = 0.98A_n - 4\,000$ where $A_1 = 159\,200$
D. $A_{n+1} = 1.02A_n - 4\,000$ where $A_1 = 160\,000$
E. $A_{n+1} = 160\,000 - 0.98A_n$ where $A_1 = 160\,000$

Question 5

A stationary car is leaking oil. When first observed the area of the oil patch is 80cm^2. In the first minute it increases by $^1/_{10}$ of its area. In the second and subsequent minutes it increases by $^1/_{10}$ of its **growth** in the previous minute.

The largest possible size the oil slick will grow to is closest to

A. 100cm^2
B. 90cm^2
C. 89cm^2
D. 88cm^2
E. No answer; it will keep on increasing to a very large area

Question 6

For an **arithmetic** sequence, the first term, t_1 is 4 and the sum of twenty terms, S_{20} is 650

For this to be true, the common difference, d and the twentieth term, t_{20} are respectively

A. 3 and 61
B. 4 and 61
C. 3 and 32.5
D. 4 and 64
E. 3 and 64

Question 7

The values of the first four terms are shown in the table below.

Term number	Value of term
1	2
2	5
3	14
4	41

The first order difference equation that best describes this sequence is

A. $t_{n+1} = t_n + 3$
B. $t_{n+1} = (t_n)^2 + 1$
C. $t_{n+1} = 2t_n + 1$
D. $t_{n+1} = t_n + (t_n)^2 - 1$
E. $t_{n+1} = 3t_n - 1$

Question 8

A difference equation is defined by

$$t_{n+1} = -2t_n \quad \text{where} \quad t_1 = -1.$$

The *n*th term, t_n, is given by

A. $1 - 2n$

B. $-(-2)^{n-1}$

C. $(-2)^{n-1}$

D. $(-2)^n$

E. $-1 - 2n$

Question 9

A difference equation is defined by

$$t_n = 2t_{n-1} + t_{n-2} \quad \text{where} \quad t_1 = -1 \quad \text{and} \quad t_2 = 1.$$

The sixth term of this sequence is

A. –7

B. 9

C. 14

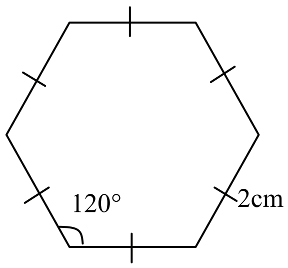
D. 17

E. 41

Module 2: Geometry and trigonometry

Before you answer these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions.

Question 1

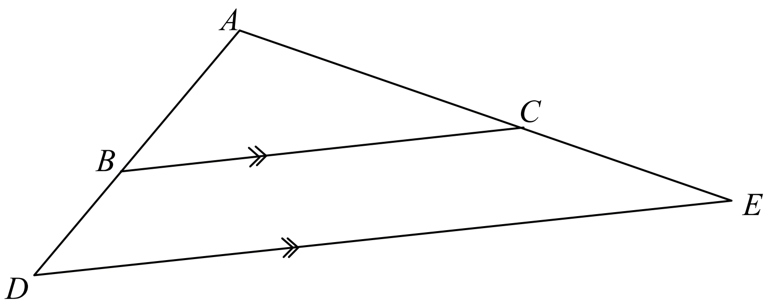


The regular hexagon above of sides 2cm has a total area of approximately

A. 1.73cm^2
B. 3.66cm^2
C. 10.39cm^2
D. 6cm^2
E. 16cm^2

Question 2

The triangles *ABC* and *ADE* are similar figures where $\overline{BC}$ is parallel to $\overline{DE}$.



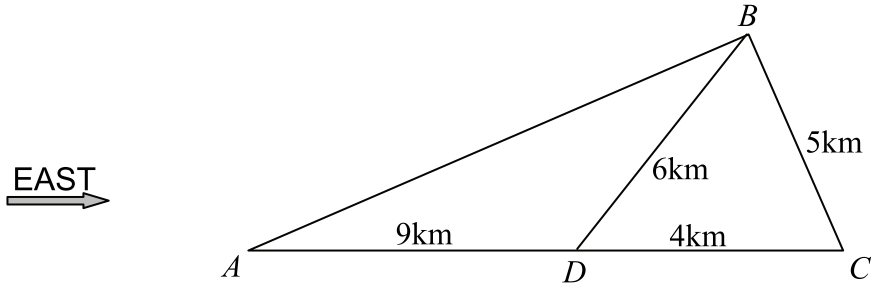
Given that the ratio of *AB*:*AD* is 3:5 and the area of triangle *ADE* is 600cm^2, then the area of triangle *ABC* is

A. 216cm^2
B. 360cm^2
C. 600cm^2
D. $1\ 000\text{cm}^2$
E. $1\ 667\text{cm}^2$

Questions 3 & 4 refer to the following diagram.

Four towns *A*, *B*, *C*, and *D* are connected by a network of straight roads as shown on the map below. Towns *C* and *D* are both due east of town *A*. The distances shown on the map are **horizontal distances**.

(not drawn to scale)



Question 3

The area of the land in triangle *CBD* in square kilometres is closest to

A. 15.0km^2
B. 12.0km^2
C. 11.0km^2
D. 10.0km^2
E. 9.9km^2

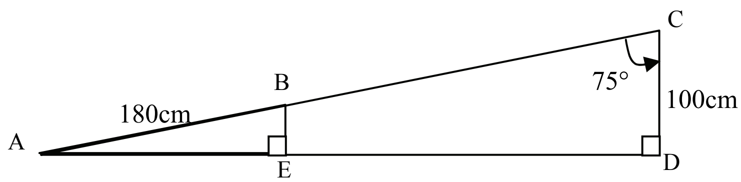
Question 4

The bearing of *B* **from** *D* is approximately

A. 070°T
B. 056°T
C. 048°T
D. 042°T
E. 034°T

Question 5

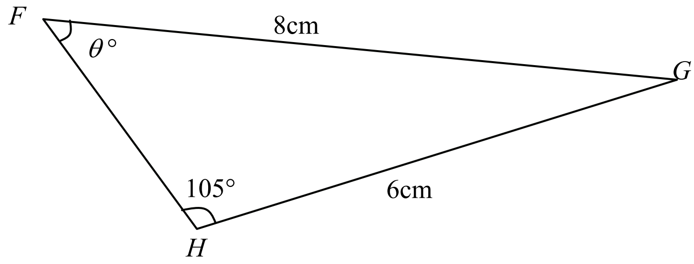
A roofing beam *AC* is supported by two vertical struts *BE* and *CD*.



The $\angle ACD$ is 75° and the lengths of *AB* and *CD* are 180cm and 100cm respectively. The length of *BC* to two decimal places is

A. 26.79cm
B. 103.53cm
C. 180.02cm
D. 206.37cm
E. 386.37cm

Question 6



Using the sine rule, an appropriate form to find θ is

A. $\sin\theta = \dfrac{\sin(105)}{6} \times 8$

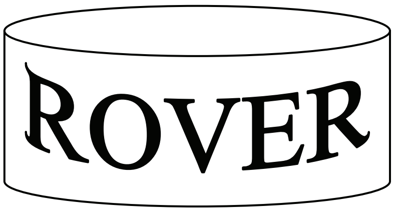
B. $\sin\theta = \dfrac{\sin(105)}{8} \times 6$

C. $\sin\theta = \dfrac{6}{\sin(105)} \times 8$

D. $\sin\theta = \dfrac{8}{\sin(105)} \times 6$

E. $\sin^{-1}\theta = \dfrac{\sin(105)}{6} \times 8$

Question 7

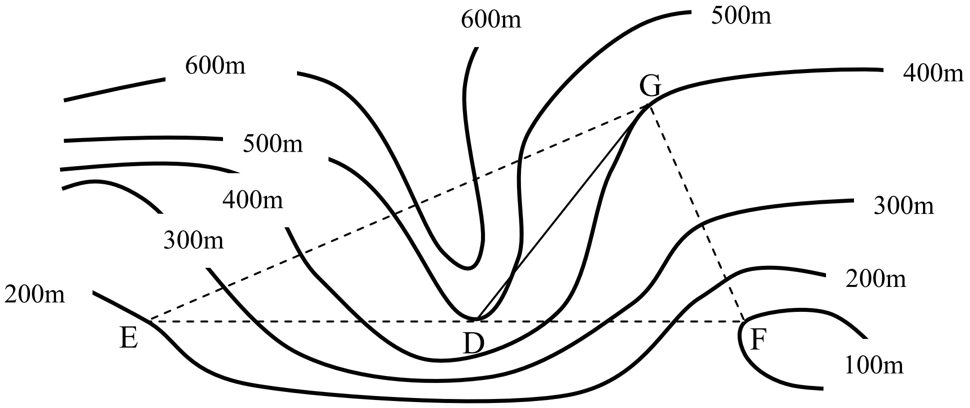


The diameter of Rover's bowl is 20cm. The shape of the dog's bowl is **cylindrical** and it can hold 2200cm^3. The height of the bowl must be at least

A. 5cm
B. 6cm
C. 7cm
D. 8cm
E. 1.8cm

Questions 8 & 9 refer to the following diagram.

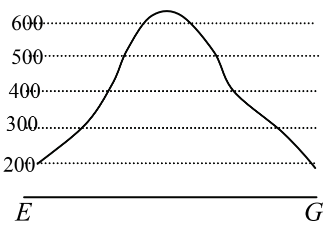
Below is a contour map of roads between four sites *D*, *E*, *F* and *G*.



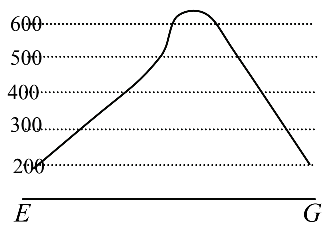
Question 8

The cross-section profile that best describes the contours along the road *EG* is

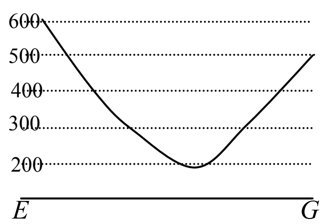
A.



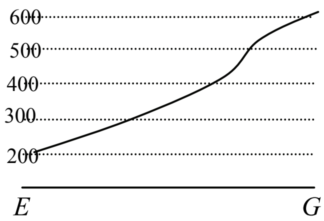
B.



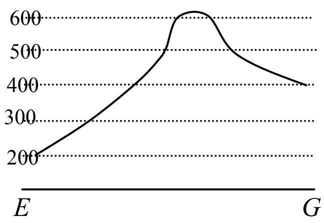
C.



D.



E.



Question 9

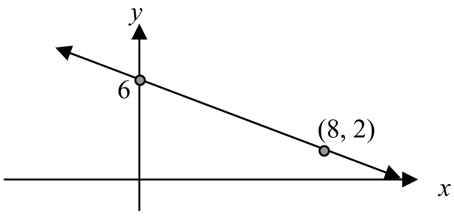
Given that the horizontal distance between *F* and *G* is 2km, the average gradient **from *G* to *F*** is approximately

A. 6.67
B. 0.15
C. 0.20
D. –0.15
E. –6.67

Module 3: Graphs and relations

Before you answer these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions.

Question 1



The equation of the line above is

A. $y + 2x = 6$

B. $y - 2x = 12$

C. $2y + x = 12$

D. $2y - x = 6$

E. $y - x = 6$

Question 2

For the equation

$ay - bx = c$, where a, b and c are constants;

the gradient and y-intercept are respectively

A. $-b$ and c

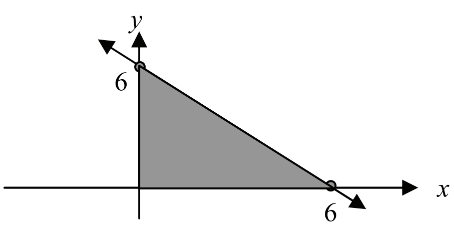
B. b and c/a

C. $-b$ and c/a

D. b/a and c

E. b/a and c/a

Question 3



The feasible region is **shaded** above for an unknown set of constraints.

Which of the following does **not** support this result?

A. $x \geq 0$

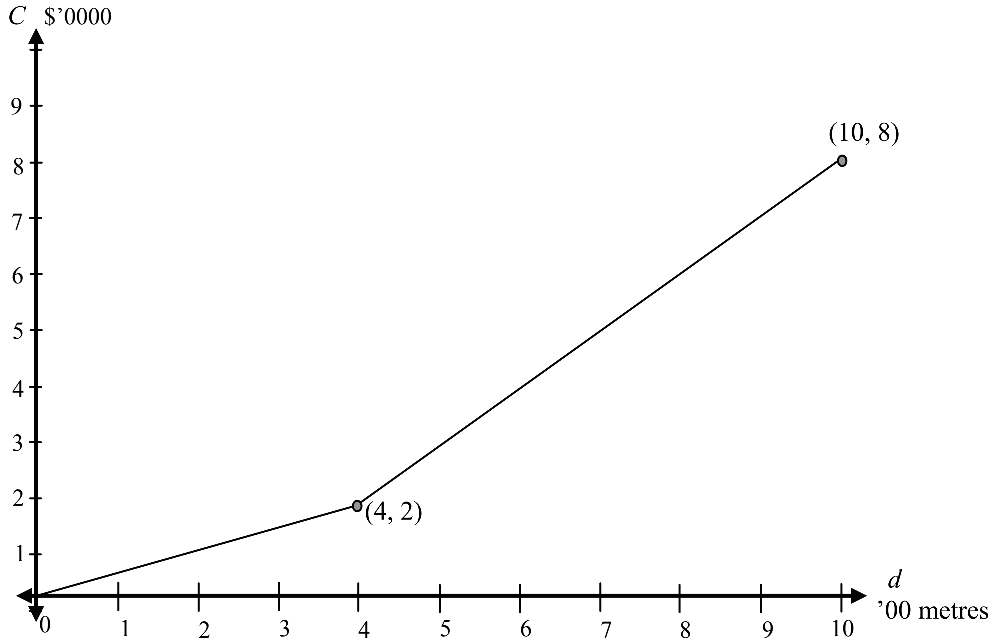
B. $y \geq 0$

C. the point (6, 6) satisfies all constraints

D. $x + y \leq 6$

E. the maximum value of $2x + y$ is at the point (6, 0)

The following graph relates to Questions 4 and 5



The cost, C, in \$'0 000, of laying a pipeline depends on the type of terrain being excavated.

The graph above shows how the cost changes with the distance, d, (in '00 metres). The graph also indicates that at a distance of 400 metres (i.e. $d = 4$), the cost suddenly changes due to the change in terrain being excavated for the pipe-line.

Question 4

Which of the following statements is true?

A. The laying of the pipeline is much quicker after 400 metres.

B. In the first 400 metres, the cost of pipe-line is \$50 per metre.

C. After the first 400 metres, the cost of pipe-line is \$1 per metre.

D. The total cost for 1 kilometre of pipe-line was \$8 000.

E. The change after 400 metres was due to easier excavation.

Question 5

Which of the following rules best describes the graph above?

A. $C = \begin{cases} 0.5d & \text{for } 0 \le d \le 4 \\ d - 2 & \text{for } 4 < d \le 10 \end{cases}$

B. $C = \begin{cases} 0.5d & \text{for } 0 \le d \le 2 \\ d + 2 & \text{for } 2 < d \le 8 \end{cases}$

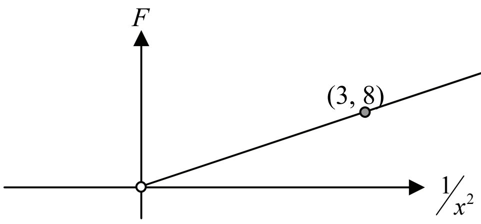
C. $C = \begin{cases} 2d & \text{for } 0 \le d \le 4 \\ d + 2 & \text{for } 4 < d \le 10 \end{cases}$

D. $C = \begin{cases} 0.5d & \text{for } 0 \le d \le 4 \\ 2d - 2 & \text{for } 4 < d \le 10 \end{cases}$

E. $C = \begin{cases} 2d & \text{for } 0 \le d \le 4 \\ d - 2 & \text{for } 4 < d \le 10 \end{cases}$

Question 6

The force of attraction, F, between two bodies decreases the further the distance, x, the bodies are apart. When graphing the force, F, against $1/x^2$, a linear model is obtained as follows.



The rule connecting F and x is

A. $F = 8/3x^2$

B. $F = 3/8x^2$

C. $F = 64/9x^2$

D. $F = 8/3x$

E. $F = 3/8x$

Question 7

The simultaneous equations $2y - 3x = 6$ and $y = x + 2$ have a unique solution of

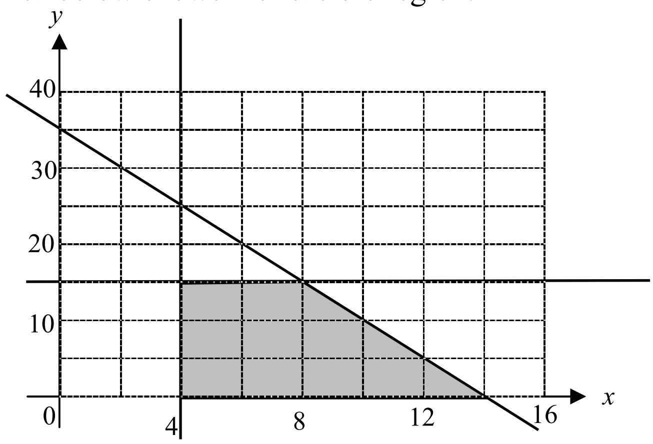
A. (1, 3)
B. (3, 0)
C. (3, 1)
D. (–2, 0)
E. (0, –2)

The following graph relates to Questions 8 and 9

In a linear programming problem involving the number of items a firm can produce in one day subject to a variety of constraints.

- x represents number of item F that can be produced in one day.
- y represents number of item G that can be produced in one day.

The shaded area below shows the feasible region.



Question 8

The Profit, z, in dollars, for this firm for one day is given by the equation

$$z = 0.5x + 2y$$

Using the feasible region above the maximum profit is

A. $7.00
B. $23.50
C. $28.00
D. $34.00
E. $77.00

Question 9

One of the constraints defining the feasible region indicates that

A. A maximum of 14 items of product F can be made.
B. A minimum of 15 items of product G can be made.
C. The total number of items F and G that can be made is 49.
D. The point (12, 8) satisfies all constraints.
E. There must be at least 4 products of item F produced.

Module 4: Business-related mathematics

Before you answer these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions.

Question 1

The selling price of a DVD is $45 after it was discounted by 20%.

The original marked price was

A. $56.25
B. $54.00
C. $225.00
D. $36.00
E. $60.00

Question 2

In April, Molly received the following statement from her bank showing all the transactions from her savings for the month of March.

Date	Transaction	Debit	Credit	Balance
1 Mar	Balance forward			2 945.18
6 Mar	Withdrawal	234.00		2 711.18
17 Mar	Deposit		400.00	3 111.18
28 Mar	Deposit		255.00	3 366.18

Interest is calculated on the minimum monthly balance. If the rate is 2.4% p.a., the amount of interest Molly will receive for March will be

A. $0.54
B. $65.07
C. $6.73
D. $5.16
E. $5.42

Question 3

Nic wants to buy car at $22 000. On a hire purchase plan, he can purchase the car with a 10% deposit plus repayments of $320 per month for six years.

The total amount of **interest** paid for this car on the hire-purchase plan is

A. $3 040
B. $3 240
C. $5 040
D. $5 240
E. $23 240

Question 4

Nic invests $3 500 in an account that is advertised at 6.2%p.a. with interest compounded quarterly.

The total amount he will have in 4 years is

A. $3 729.43
B. $4 368.00
C. $4 452.11
D. $4 476.59
E. $9 163.48

Question 5

Marion has bought a photocopier for $84 000. For tax purposes she uses reducing balance depreciation. The table below shows the book value of the photocopier after one year.

	Reducing balance Depreciation
Initial	$84 000
At the end of 1 year	$76 440

The book value of the photocopier after the second year is approximately

A. $63 300
B. $68 880
C. $69 560
D. $70 220
E. $71 560

Question 6

A company purchases a delivery van for $29 000. It depreciates at a rate of 25 cents for every kilometre. The van has a scrap value of $5 000.

The Book Value after travelling 20 000 km is

A. $0
B. $5 000
C. $19 000
D. $21 000
E. $24 000

Question 7

Stamp Duty is payable to the state government on a property transaction according to the following rate schedule.

Transfer of Real Property rates	
Range	**Rate**
$0 - $20 000	1.4 per cent of the dutiable value of the property
$20 001 - $115 000	$280 plus 2.4 per cent of the dutiable value in excess of $20 000
$115 001 - $870 000	$2 560 plus 6 per cent of the dutiable value in excess of $115 000
More than $870 000	5.5 per cent of the dutiable value

The stamp duty payable on a property worth $320 000 using the schedule above is

A. $2 560
B. $12 300
C. $14 860
D. $19 200
E. $33 000

Question 8

Joan borrows $120 000 and makes monthly repayments of $1 400.

She uses the annuities formula to calculate how much she owes after X years at a rate of Y% per annum, calculated monthly on the reducing balance.

$$A = 120000(1.0075)^{240} - \frac{1400(1.0075^{240} - 1)}{0.0075}$$

Observing her formula the number of years, X and the rate, Y are

A. 10 years, 7.5%
B. 20 years, 7.5%
C. 20 years, 6.25%
D. 10 years, 9%
E. 20 years, 9%

Question 9

Jordan originally had a home loan of $128 000. The interest rate is 7.2%p.a. compounded monthly and her monthly repayments were $845.

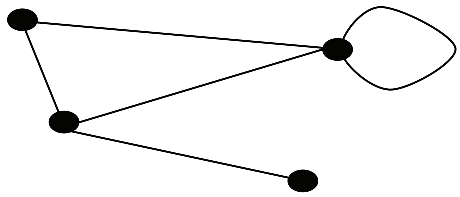
After 5 years she decided to increase her repayments. If the new repayments are $1 000, the **total** still owing after seven years from the original loan (i.e. two years of the new repayment) is

A. $75 300
B. $102 757
C. $115 634
D. $119 622
E. $158 863

Module 5: Networks and decision mathematics

Before you answer these questions you must **shade** the Networks and decisions mathematics box on the answer sheet for multiple-choice questions.

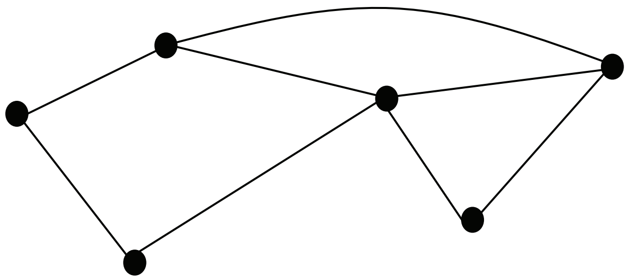
Question 1



For the network above, the sum of degrees is:

A. 5
B. 12
C. 10
D. 8
E. 9

Question 2



Which of the following statements is **false** regarding the network above?

A. Euler's formula for planar graphs applies to the network
B. The sum of degrees is double the number of edges
C. It contains an Euler circuit and Hamiltonian path
D. Many circuits exist
E. It contains an Euler path and Hamiltonian circuit

Question 3

Three students A, B and C are sharing out tasks W, X and Y.

The number of hours to complete each task is shown.

	A	B	C
W	8	9	7
X	13	15	12
Y	15	18	17

If each student must do one task only, how should tasks be allocated to achieve minimum completion time?

A. W → A, X → B, Y → C

B. W → C, X → B, Y → A

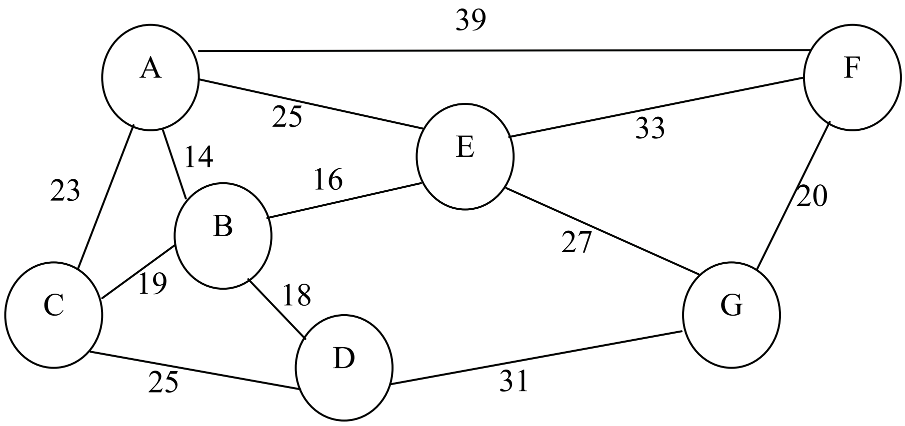
C. W → B, X → A, Y → C

D. W → B, X → C, Y → A

E. W → A, X → C, Y → B

Questions 4 and 5 refer to the following network.

For a shire, the graph below shows the major towns A, B …G connected by the main roads, all distances are in kilometres.



Question 4

The minimum spanning tree for this network is

A. 119 km

B. 114 km

C. 127 km

D. 105 km

E. 96 km

Question 5

A salesperson from town A needs to make deliveries to all towns and return home. If this is done without visiting any other town twice, it would be an example of a

A. Hamiltonian Circuit

B. Complete Graph

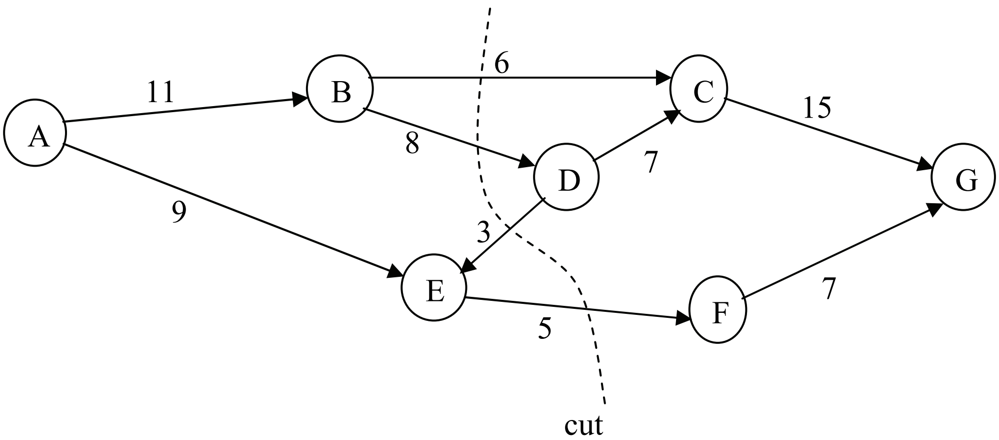
C. Euler Circuit

D. Tree

E. Reachability matrix

Question 6

The following directed graph shows the potential water flow of '000 litres per hour in an irrigation system.



The capacity of the cut in the above network, in '000 litres per hour, is:

A. 19

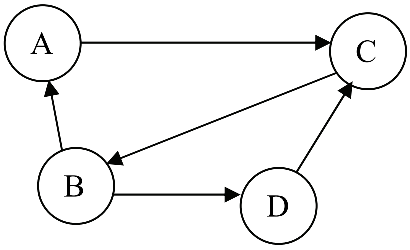
B. 22

C. 25

D. 17

E. 18

Question 7



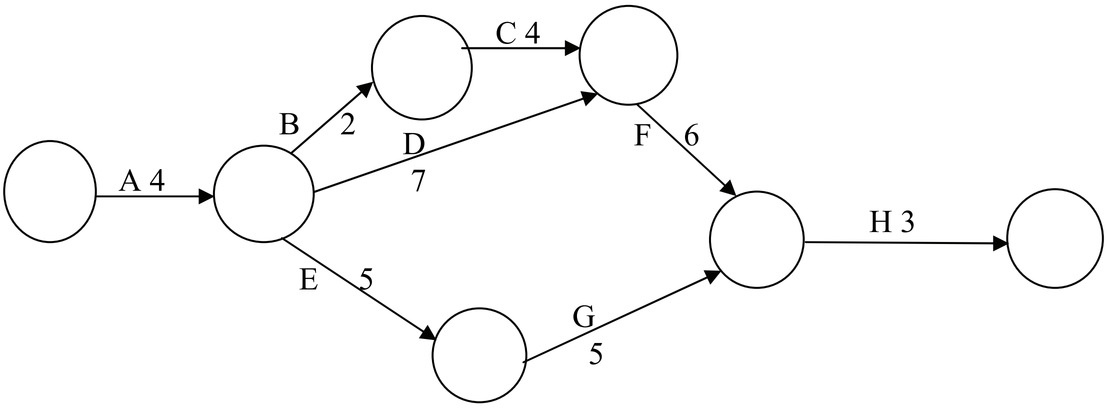
A 2-step reachability matrix for this network is:

A. $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ **B.** $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ **C.** $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ **E.** $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

Questions 8 and 9 refer to the following critical path.

For a particular project there are eight activities to be completed and the time taken to complete each activity is shown in hours.



Question 8

The critical path and completion time for this project is

A. AEGH, 17 hours
B. AEFH, 18 hours
C. ADFH, 20 hours
D. ABCDFH, 20 hours
E. ADCFH, 26 hours

Question 9

Assume that any activity can be **reduced to one** hour by providing extra equipment or labour.

If the project is to be crashed by reducing the completion time of ONE activity only, then this will reduce the completion time of the project by a maximum of

A. 1 hour
B. 2 hours
C. 3 hours
D. 4 hours
E. 5 hours

Module 6: Matrices

Before you answer these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions.

Question 1

Let A = $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$, B = $\begin{bmatrix} 3 & 2 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$

Which of the following operations **can** be performed?

A. $C - B + A$

B. $C - A + B$

C. $AB + C$

D. $BA + C$

E. CBA

Question 2

Let $M = \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}$ and $N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Then M^2N^4 equals

A. $\begin{bmatrix} 8 & 1 \\ 4 & 5 \end{bmatrix}$

B. $\begin{bmatrix} 6 & 1 \\ 2 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 6 & 1 \\ 2 & -1 \end{bmatrix}$

D. $\begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}$

E. Impossible, no solutions

Question 3

Let $R = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ and $T = \begin{bmatrix} 1 & 4 \\ 3 & -3 \end{bmatrix}$

The matrix X such that $RX = T$ will be:

A. $\begin{bmatrix} 1 & 13 \\ 3 & -1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 4 \\ 1 & -1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 12 \\ 3 & -9 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 4 \\ 3 & -6 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 4 \\ 9 & -9 \end{bmatrix}$

Question 4

Two rival companies, KFB (K) and Jocks (J) sell French fries in three sizes: small (S), medium (M) and large (L).

The price of each size of fries, in cents, is listed in a price matrix P, where

$$P = \begin{array}{c} \begin{matrix} S & M & L \end{matrix} \\ \begin{bmatrix} 120 & 180 & 220 \\ 130 & 170 & 210 \end{bmatrix} \end{array} \begin{matrix} \\ K \\ J \end{matrix}$$

Due to competition, KFB reduces the price of all sizes by 10% and Jocks reduces their prices by 8%.

The new price matrix showing the decreased prices can be generated by performing which matrix product?

A. $\begin{bmatrix} 120 & 180 & 220 \\ 130 & 170 & 210 \end{bmatrix} \begin{bmatrix} 0.1 & 0 \\ 0 & 0.08 \end{bmatrix}$

B. $\begin{bmatrix} 120 & 180 & 220 \\ 130 & 170 & 210 \end{bmatrix} \begin{bmatrix} 0.9 & 0 \\ 0 & 0.92 \end{bmatrix}$

C. $\begin{bmatrix} 0.1 & 0 \\ 0 & 0.08 \end{bmatrix} \begin{bmatrix} 120 & 180 & 220 \\ 130 & 170 & 210 \end{bmatrix}$

D. $\begin{bmatrix} 0.9 & 0 \\ 0 & 0.92 \end{bmatrix} \begin{bmatrix} 120 & 180 & 220 \\ 130 & 170 & 210 \end{bmatrix}$

E. $\begin{bmatrix} 0.9 & 0.1 \\ 0.08 & 0.92 \end{bmatrix} \begin{bmatrix} 120 & 180 & 220 \\ 130 & 170 & 210 \end{bmatrix}$

Question 5

How many of the following four sets of simultaneous linear equations have a **unique** solution?

$2x - y = 3$ $x + y = 1$	$4x = 12$ $2x - y = 4$	$x - 2y = 0$ $-2x + 4y = 10$	$3x - y = 5$ $3x - y = 10$

A. 0
B. 1
C. 2
D. 3
E. 4

Question 6

The order of matrix A is (3×1) and the order of the matrix product AX is (3×4), then the order of matrix X must be

A. (1×4)
B. (3×4)
C. (4×1)
D. (4×3)
E. (3×1)

Question 7

The following system of the linear equations

$$x - 3z = 4$$
$$x + 2y - z = 3$$
$$3y + 2z = 0$$

when written in matrix form is

A. $\begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & 0 \end{bmatrix}\begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & -3 \\ 1 & 2 & -1 \\ 0 & 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & -1 \\ 0 & 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & -3 \\ 1 & 2 & -1 \\ 3 & 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$

E. $\begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & -1 \\ 0 & 3 & 2 \end{bmatrix} + \begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$

Question 8

The solution of the matrix equation

$$\begin{bmatrix} 1 & -2 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \end{bmatrix}$$

is

A. $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$ **B.** $\begin{bmatrix} 2 \\ 10 \end{bmatrix}$ **C.** $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ **E.** $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Question 9

Two locations *A* and *B* are regular popular holiday spots for a large number of people. It is noticed that the 90% of the people that go to location *A* return again the following year and the rest go to location *B*.

Location *B* retains 85% of people each year and the other 15% go to location *A*.

Assuming this pattern of movement is maintained and given that 100 people originally go to location A and 200 go to location *B*, the number that will holiday at each location in the **long term** is

A. 300 holiday at location *A*; 0 at location *B*
B. 200 holiday at location *A*; 100 at location *B*
C. 120 holiday at location *A*; 180 at location *B*
D. 175 holiday at location *A*; 125 at location *B*
E. 180 holiday at location *A*; 120 at location *B*

END OF MULTIPLE-CHOICE QUESTION BOOK

Insight Publications

INSIGHT

Trial Exam Paper

2007

FURTHER MATHEMATICS

Written examination 1

Worked solutions

This book presents:

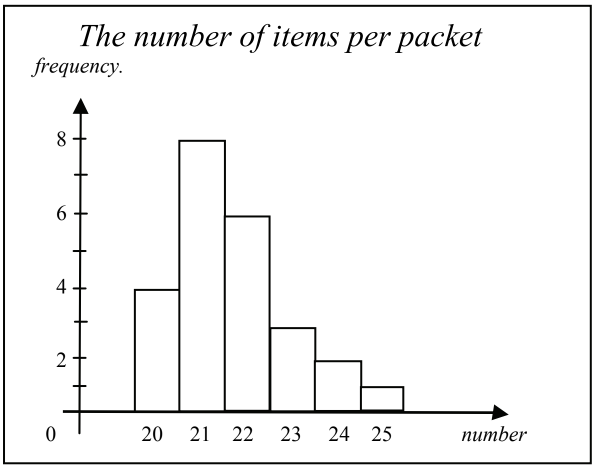
- worked solutions, giving you a series of points to show you how to work through the questions.
- tips and guidelines

SECTION A

Core – Data Analysis

Question 1

At a school carnival, lucky-dips were sold in packets containing a number of items per packet. The distribution is shown in the frequency table below.



The percentage of packets that contained more than 23 items is

A. 3%
B. 6.25%
C. 24%
D. **12.5%**
E. 25%

Answer is D

Worked solution

- More than 23 means 24 or 25 and then you need to use freq to find how many of these occur.

  $\frac{3}{24} \times 100 = 12.5\%$

  (total items)

Question 2

Data collected from a test is displayed in a stemplot as shown below.

STEM	LEAF
0	5 6 8 8
1	0 3 3 5
1	5 6 8 8 9 9
2	1 4 4
2	5

The inter-quartile range for this data is

A. 2

B. 5

C. **9**

D. 15.5

E. 20

Answer is C

Worked solution

- There are 18 values, use the ($n+\frac{1}{2}$ rule) $\therefore$ median = $18+\frac{1}{2} = 9.5$ value.
- This means there are 9 in each half…to find the quartiles look for the median of each half: i.e. the 5th score in each half.
  Q1 = 10, Q3 = 19 $\therefore$ IQR=19-10=9

Question 3

Data was collected from a fishing competition on the size of fish caught that day. It was noticed that the data had a bell shaped distribution with a mean of 22.5cm and standard deviation of 2.5cm.

Approximately what percentage of fish were less than 25cm?

A. 16%

B. 95%

C. 50%

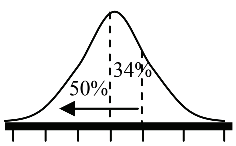
D. 68%

E. **84%**

Answer is E

Worked solution

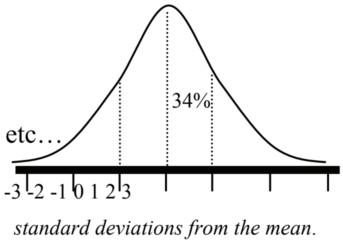
- 25cm is one standard deviation above the mean.



- We need % of fish less than 25cm. 68% lies within $\bar{x} \pm s$, we need only the top half of this i.e. 34%.
- Being symmetrical 50% are below the mean.
  total = 50 + 34 =84%

Tip

- *You should include in your notes a bell shaped curve with appropriate % as follows. It will assist with questions as above but with 'standardising' questions as well (Like Qn5 below).*



Question 4

A student wrote the following numbers to calculate the mean and median.

10, 12, 14, 14, 18, 19, 19

After her calculations, she was informed that her last number 19 was incorrect and in fact was a number **less than** 10.

Compared to her original answers, which of the following is **true**?

A. The actual result had a lower mean and median.

B. **The actual result had a lower mean but the same median.**

C. The actual result had a lower mean but a higher median.

D. The actual result had the same mean but a lower median.

E. The results remained unchanged.

Answer is B

Worked solution

- There are 7 values, the median is the 4^{th} score which is 14 in either cases.
- To find the mean involves adding all values, so a lower value than 19 will decrease the mean.

Question 5

Jon's exam grades for three subjects are shown below. The class average and standard deviation are also shown for each subject.

Subject	***Jon's Mark***	***Class Mean***	***Class Standard deviation***
History	65	70	10
Science	75	75	10
Art	85	90	5

Jon's best performance in relation to the class results was

A. History

B. **Science**

C. Art

D. Art and History were equal

E. Art and Science were equal

Answer is B

Worked solution

- Use $z = \frac{x - \bar{x}}{sd}$ to standardise marks

  History $\frac{65-70}{10} = -0.5$

  Science $\frac{75-75}{10} = 0$ **** best***

  Art $\frac{85-90}{5} = -1$
- These 'z' results show that Science, $z = 0$, is equal to the mean for that subject (50% of class rank) whereas the other subjects are negative z scores meaning they are below class averages.

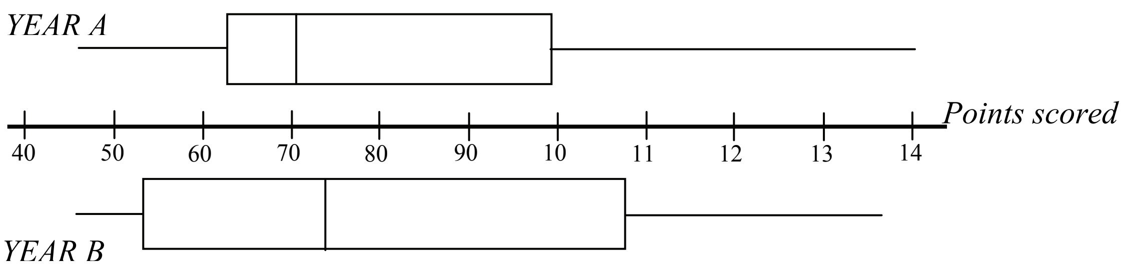
Tip

- *These* z *values are the standard deviations from the class average. The bell shape in Q3 will give you an idea of their 'ranking' compared to the whole class. eg. Something like this table may assist if the question is trying to rate a mark in comparison with the class results.*

Z values	–2	–1	0	1	2
% rank in class	2.5%	16%	50% Class average	84% (top 16%	97.5% (or top 2.5%)

Questions 6 and 7 refer to the following information

A junior football team compares the number of points it has scored over the last two years (*A* and *B*) by using the box plots as displayed below.



Question 6

From this box-plot summary, which of the following observations is true?

A. Year A scores were generally higher than Year B

B. 50% of Year A scores were less than 70 points compared to only 25% for Year B

C. If the box-plots were standardised, Year A would have an outlier

D. **Both box-plots have a positive skew**

E. 25% of Year A scores are more than Year B

Answer is D

Worked solution

- Both are skewed. Observe the range of values from the median, which seem to tail off towards the right.
- All other answers have % errors: eg.
  - Option B: Error being somewhere between 25% & 50% of Year B were less than 70 points
  - Apart from Option C where all values are within 1.5 × IQR, so there are no outliers.

Question 7

The variables:

Year (A or B) and **Points scored** are

A. both categorical variables.
B. both numerical variables.
C. **categorical and numerical variables respectively.**
D. numerical and categorical variables respectively.
E. neither categorical and numerical variables

Answer is C

Worked solution

- **Year** is divided into categories A or B.
- **Points** go from 45 to 140 scored in FOOTBALL being a numerical value.

Question 8

A survey of seven trees noting the age and height of an exotic species gave the following results:

Age (years)	1	4	7	11	12	14	20
Height (metres)	1.8	3.4	5.0	6.2	7.0	6.8	7.4

Further analysis using a least squares regression association would have the **independent variable** and **correlation coefficient** for this data as

A. ***Age*, 0.933**
B. *Age*, 0.871
C. *Height*, 0.933
D. *Height*, 0.871
E. *Age*, 0.305

Answer is A

Worked solution

- Logically the height of a tree generally increases with age. (Although it can be affected by other conditions (nutrients, location, weather …). ∴ Height is DEPENDENT on age. Time (age) is not affected by other factors: independent.
- Need to use calculator to find 'r', the correlation value.

Tip

- *You need to make sure when using your calculator, that the LIST containing your 'x' values are the independent ones i.e. AGE as in the above case when finding regression lines. Which is found by the steps: STAT→CALC→8:LinReg (or 4LinReg)*

Questions 9 and 10 refer to the following information

The population of Victoria from 1990 to 2004 is shown in the table below.

Years from 1990	0	1	3	4	5	7	10	11	13	14
Population in millions	4.40	4.44	4.48	4.50	4.54	4.62	4.77	4.83	4.94	4.99

Table 1

Question 9

Using least squares regression analysis it was found that a linear model to fit this data in **table 1** was

Population in millions = 4.36 + 0.043 × Years from 1990

Using this model, the **residual** for the population in millions for the year 2000 was

A. 4.79
B. 85.59
C. 0.02
D. **–0.02**
E. Impossible to calculate because of missing data

Answer is D

Worked solution

- Predicted value is found using the regression line i.e.:
  Pop.= 4.36 + 0.043 × 10
  = 4.79 million
  (note: year 2000 is 10 from 1990)
  Residual = Actual – Predicted
  = 4.77 – 4.79 = –0.02

Question 10

When interpreting this linear regression model, which of the following is **true**?

A. **The population is increasing at 0.043 million people per year**
B. The gradient of 0.043 shows there is are no correlation to predict population
C. The initial population in 1990 is predicted as 0.043 million people
D. The population is decreasing as time increases
E. The population remains steady

Answer is A

Worked solution

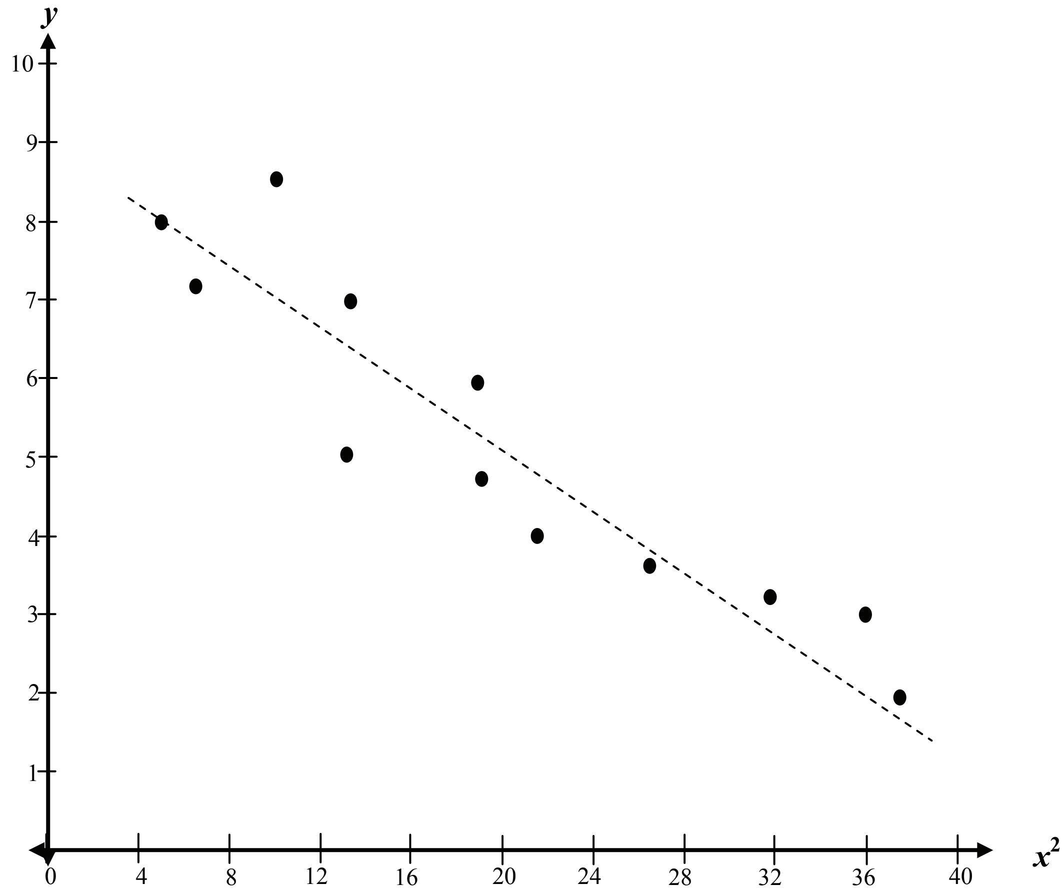
- The regression model $y = a + bx$, the gradient is b which shows the rate of increase (or decrease)
- In this case b = 0.043 show the increase in population (millions) per year.

Tip

- *It seems that the gradient is not well used in practical terms so a sentence like:*
  …the 'y variable' increases by 'b' per 'x variable'….
  could be adapted into your notes.
- *Also note that the value **4.36** in the equation is called the 'y-intercept'. To interpret in practical terms means that in 1990 (i.e. x = 0) the estimated population is 4.36 million.*

Question 11

Data collected has been transformed giving the following resulting graph.



The line of best fit (as indicated by the dash line in the figure) is closest to

A. $y = 8.8 + 0.2x$

B. $y = 8.8 - 0.2x$

C. $y = 0.2 + 8.8x^2$

D. $\mathbf{y = 8.8 - 0.2x^2}$

E. $y = 0.2 + 8.8x$

Answer is D

Worked solution

- Observe the axes, y against x^2. Answer as $y = a + bx^2$, where 'a' is 'y-intercept. If you extend the line, the intercept is $\approx$ 8.8. The line drawn actually passes through 9 on the y axis – but D is the closest answer.
- 'b' is the gradient, choose **any** two points on the line: use Rise/Run and note that it should be negative.
- eg. (5, 8) and (36, 2) are approximately two points along the regression line:

$$\text{Gradient, } b = \frac{2-8}{36-5} \approx -0.2$$

Tip

- *(Please note: eliminating all equations without* x^2 *leaves only two alternatives of which only D has y intercept of 8.8)*
- *Especially with multiple choice questions, sometimes elimination of some responses may provide you more clarity and better chances.*
- *There is also some confusion between* $y = a + bx$*;* $y = ax + b$ *and* $y = mx + c$
- *They are all valid lines so the best thing to remember is that the coefficient of the '*x*' term gives you the gradient.*

Questions 12 and 13 refer to the following information

The table below shows the seasonal indices of swimwear sales for a particular firm.

Quarter	Summer	Autumn	Winter	Spring
Seasonal Index	1.10	0.95	0.82	

Question 12

The seasonal index for Spring is missing. Once calculated this value shows that sales for spring are

A. typically 3% above average
B. typically 3% below average
C. typically 113% above average
D. typically 13% below average
E. **typically 13% above average**

Answer is E

Worked solution

- Being quarterly all indices should add to 4. This gives:
  Spring = 4 – (1.10 + 0.95 + 0.82) = 1.13
- As a percent, 113%, meaning it is 13% above yearly average (100%)

Question 13

The deseasonalised figure for swimwear sales in Winter is 88. The **actual** sales for Winter is closest to

A. 107
B. **72**
C. 82
D. 99
E. 0

Answer is B

Worked solution

- $\text{Deseasonalised Figure} = \dfrac{\text{Actual figure}}{\text{Seasonal Index}}$
- re-arranging this gives
  Actual = Deseasonalised × S.I.
  = 88 × 0.82
  = 72

END OF SECTION A

SECTION B

Module 1: Number Patterns

Question 1

Which of the following sequences is **not** a geometric sequence?

A. 2, 6, 18, 54, 162 …

B. 2, –6, 18, –54, 162 …

C. 100, 50, 25, 12.5, 6.25 …

D. **1, 3, 6, 12, 36 …**

E. 0.1, 1, 10, 100, 1000 …

Answer is D

Worked solution

- Geometric: common ratio must exist.
- For Option D $\frac{t_2}{t_1} \neq \frac{t_3}{t_2}$ i.e. $\frac{3}{1} \neq \frac{6}{3}$

Question 2

The first four terms of an arithmetic are { –2, 1, 4, 7 … }.

The 25th term for this sequence is

A. 76

B. 73

C. **70**

D. 850

E. 45

Answer is C

Worked solution

- It is an Arithmetic Sequence (has a common difference): $a = -2$ & $d = 3$
- Using the general term: $t_n = a + (n-1)d$

$$t_{25} = -2 + (25 - 1)3 = 70$$

The following information relates to questions 3 and 4

A large cattle station has a herd of 160 000 cattle at the start of the first year. The cattle population increases by 2% each year. At the end of each year, 4 000 cattle are sold.

Applying this process, at the start of the second year there are now 159 200 cattle.

Question 3

Continuing this process, the number of cattle at the **start** of the third year is

A. **158 384**
B. 158 400
C. 155 200
D. 161 552
E. 162 384

Answer is A

Worked solution

- So far the Sequence is {160 000, 159 200….
- Next Year: The Growth is 2% of 159 200 = 3 184
- Total = 159 200 + 3 184 – 4 000
  = 158 384

Question 4

If A_n is the number of cattle at the start of each year, n, a difference equation that can be used to model the population of the herd over time is

A. $A_{n+1} = 160\,000 - 0.98A_n$ where $A_1 = 4\,000$
B. $A_{n+1} = 160\,000 - 1.02A_n$ where $A_1 = 160\,000$
C. $A_{n+1} = 0.98A_n - 4\,000$ where $A_1 = 159\,200$
D. $\mathbf{A_{n+1} = 1.02A_n - 4\,000}$ **where** $\mathbf{A_1 = 160\,000}$
E. $A_{n+1} = 160\,000 - 0.98A_n$ where $A_1 = 160\,000$

Answer is D

Worked solution

- An increase of 2% means you have 102% (1.02) of previous amount i.e. $1.02A_n$ then take away 4000 to get next amount, $\therefore 1.02A_n - 4000$

Tip

- *In this case, your answer to Q4 could be used to back-up Q3*

Question 5

A stationary car is leaking oil. When first observed the area of the oil patch is 80cm^2. In the first minute it increases by $^1/_{10}$ of its area. In the second and subsequent minutes it increases by $^1/_{10}$ of its **growth** in the previous minute.

The largest possible size the oil slick will grow to is closest to

A. 100cm^2

B. 90cm^2

C. **89cm2**

D. 88cm^2

E. No answer; it will keep on increasing to a very large area

Answer is C

Worked solution

- The wording "…*will grow to*…" implies no set time period, endless.
- Infinite geometric series where
  r = 0.1 and a = 80
  $$S_\infty = \frac{80}{1-0.1} \approx 89$$
- Answer E is incorrect because the growth becomes practically zero in very short time i.e. 8cm^2 in the first minute, then 0.8cm^2 next minute, then 0.08 ... 0.008 ... 0.0008 etc..

Question 6

For an **arithmetic** sequence, the first term, t_1 is 4 and the sum of twenty terms, S_{20} is 650

For this to be true, the common difference, d and the twentieth term, t_{20} are respectively

A. **3 and 61**

B. 4 and 61

C. 3 and 32.5

D. 4 and 64

E. 3 and 64

Answer is A

Worked solution

- Use $S_{20} = \frac{20}{2}(2a + (n-1)d)$
- $a = 4$, $650 = 10(8 + 19d)$ (Alternatively you can use **solve** on graphics calc. ***solve(10(8 + 19*d*) – 650,* d*, 0)***)
  $65 = 8 + 19d$
  $57 = 19d$
- $\therefore d = 3$ now substitute this into equation for t_{20}.
  $t_{20} = 4 + (19)3 = 61$

Question 7

The values of the first four terms are shown in the table below.

Term number	Value of term
1	2
2	5
3	14
4	41

The first order difference equation that best describes this sequence is

A. $t_{n+1} = t_n + 3$

B. $t_{n+1} = (t_n)^2 + 1$

C. $t_{n+1} = 2t_n + 1$

D. $t_{n+1} = t_n + (t_n)^2 - 1$

E. $\mathbf{t_{n+1} = 3t_n - 1}$

Answer is E

Worked solution

- It is neither arithmetic nor geometric but you are told it is a first order difference equation; so it must have the form : $t_{n+1} = rt_n + d$, : $a = t_1$
  (This eliminates options B & D)
- You will notice that the difference between value of terms in the table is a multiple of 3, meaning $r = 3$, giving us option E left.
- Otherwise you may have noticed the following pattern:
- Triple the previous term and then subtract one will give you the new term.

Question 8

A difference equation is defined by

$$t_{n+1} = -2t_n \quad \text{where} \quad t_1 = -1.$$

The nth term, t_n, is given by

A. $1 - 2n$

B. $\mathbf{-(-2)^{n-1}}$

C. $(-2)^{n-1}$

D. $(-2)^n$

E. $-1 - 2n$

Answer is B

Worked solution

- The sequence generated by this first order difference equation is:
  $\{-1, 2, -4, 8 \ldots\}$ eg. $t_2 = -2t_1 \rightarrow t_2 = -2 \times -1 = 2$ and $t_3 = -2t_2 \rightarrow t_3 = -2 \times 2 = -4$
- Hence it is Geometric where $a = -1$, $r = -2$
- $t_n = ar^{n-1} \therefore t_n = -(-2)^{n-1}$

Question 9

A difference equation is defined by

$$t_n = 2t_{n-1} + t_{n-2} \quad \text{where } t_1 = -1 \quad \text{and} \quad t_2 = 1.$$

The sixth term of this sequence is

A. –7
B. 9
C. 14
D. 17
E. 41

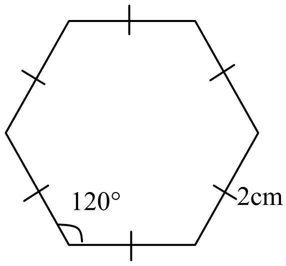
Answer is D

Worked solution

- The sequence for this second order difference equation:
  $t_3 = 2t_2 + t_1 \Rightarrow 2 \times 1 + -1 = 1$
  $t_4 = 2t_3 + t_2 \Rightarrow 2 \times 1 + 1 = 3$
  $t_5 = 2t_4 + t_3 \Rightarrow 2 \times 3 + 1 = 7$
  $t_6 = 2t_5 + t_4 \Rightarrow 2 \times 7 + 3 = 17$

Module 2: Geometry and trigonometry

Question 1



The regular hexagon above of sides 2cm has a total area of approximately

A. 1.73cm^2

B. 3.66cm^2

C. **10.39cm^2**

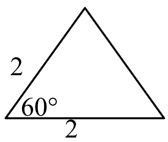
D. 6cm^2

E. 16cm^2

Answer is C

Worked solution

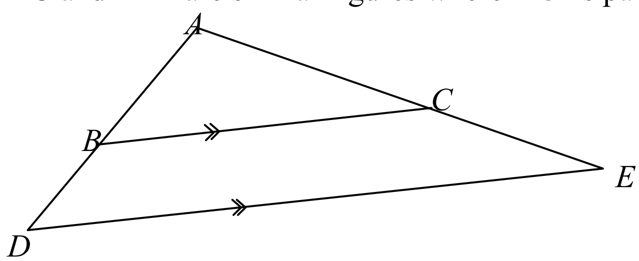
- A hexagon has 6 equilateral Δ′s with all sides 2cm with angle of 60°.



- Area of one triangle is
  $\frac{1}{2} \times 2 \times 2 \times \sin 60 \approx 1.73$
  $\therefore$ Total area = $6 \times 1.73 \approx 10.39$

Question 2

The triangles *ABC* and *ADE* are similar figures where $\overline{BC}$ is parallel to $\overline{DE}$.



Given that the ratio of *AB*:*AD* is 3:5 and the area of triangle *ADE* is 600cm^2, then the area of triangle *ABC* is

A. **216cm^2**

B. 360cm^2

C. 600cm^2

D. $1\ 000\text{cm}^2$

E. $1\ 667\text{cm}^2$

Answer is A

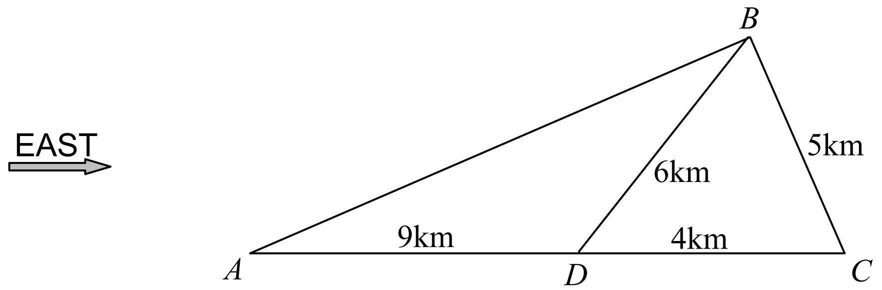
Worked solution

- Length ratio is 3:5 →Area ratio is $3^2 : 5^2$ giving an area scale factor of $9/25$
  (not $25/9$ since area *ABC* is less than area of *ADE*)
- $\therefore$ Area of *ABC* = 600 × $9/25$
  = 216

Questions 3 & 4 refer to the following diagram.

Four towns *A*, *B*, *C*, and *D* are connected by a network of straight roads as shown on the map below. Towns *C* and *D* are both due east of town *A*. The distances shown on the map are **horizontal distances**.

(not drawn to scale)



Question 3

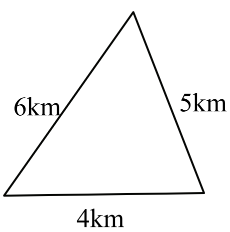
The area of the land in triangle *CBD* in square kilometres is closest to

A. 15.0km^2
B. 12.0km^2
C. 11.0km^2
D. **10.0km^2**
E. 9.9km^2

Answer is D

Worked solution

- Use Heron's Formula: Area $= \sqrt{s(s-a)(s-b)(s-c)}$
  where $s = \dfrac{a+b+c}{2}$



$$s = \frac{6+4+5}{2} = 7.5$$

- Area $= \sqrt{7.5(7.5-6)(7.5-4)(7.5-5)}$
  ≈ 9.92

Question 4

The bearing of *B* **from** *D* is approximately

A. 070°T

B. 056°T

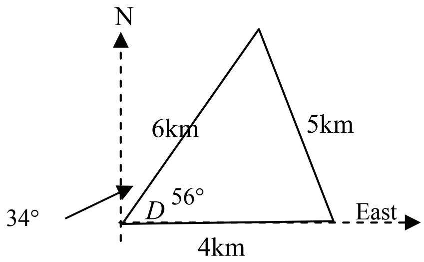
C. 048°T

D. 042°T

E. **034°T**

Answer is E

Worked solution



- Use cosine rule to find angle opposite side 5

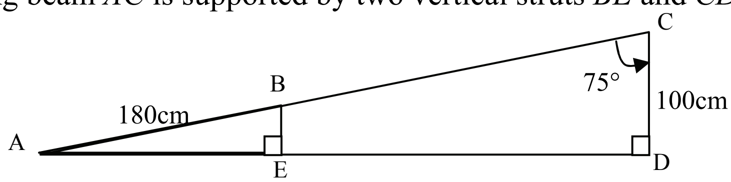
  $\cos D = \dfrac{6^2 + 4^2 - 5^2}{2 \times 6 \times 4}$

  $D = \cos^{-1}(0.5625) = 56°$ from East

  $\therefore$ 034°T

Question 5

A roofing beam *AC* is supported by two vertical struts *BE* and *CD*.



The $\angle ACD$ is 75° and the lengths of *AB* and *CD* are 180cm and 100cm respectively. The length of *BC* to two decimal places is

A. 26.79cm

B. 103.53cm

C. 180.02cm

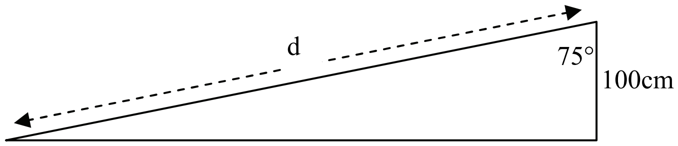
D. **206.37cm**

E. 386.37cm

Answer is D

Worked solution

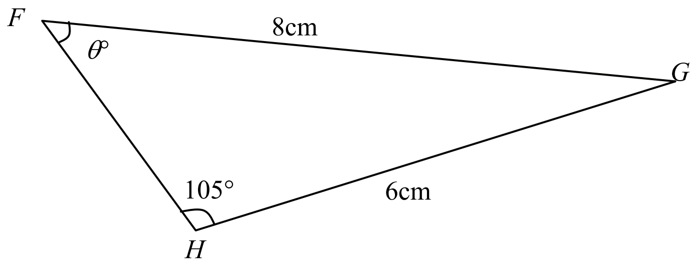
- Being a right angled triangle



- Use SOHCAHTOA

$d(AC) = 100/\cos(75)$
$= 386.37$
$\therefore BC = 386.37 - 180$
$= 206.37\text{cm}$

Question 6



Using the sine rule, an appropriate form to find θ is

A. $\sin\theta = \dfrac{\sin(105)}{6} \times 8$

B. $\boldsymbol{\sin\theta = \dfrac{\sin(105)}{8} \times 6}$

C. $\sin\theta = \dfrac{6}{\sin(105)} \times 8$

D. $\sin\theta = \dfrac{8}{\sin(105)} \times 6$

E. $\sin^{-1}\theta = \dfrac{\sin(105)}{6} \times 8$

Answer is B

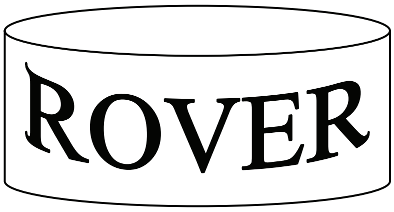
Worked solution

- $\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)}$ OR $\dfrac{\sin(A)}{a} = \dfrac{\sin(B)}{b}$

$\therefore \dfrac{\sin(\theta)}{6} = \dfrac{\sin(105)}{8}$

$\equiv \sin(\theta) = \dfrac{\sin(105)}{8} \times 6$

Question 7



The diameter of Rover's bowl is 20cm. The shape of the dog's bowl is **cylindrical** and it can hold 2200cm^3. The height of the bowl must be at least

A. 5cm

B. 6cm

C. **7cm**

D. 8cm

E. 1.8cm

Answer is C

Worked solution

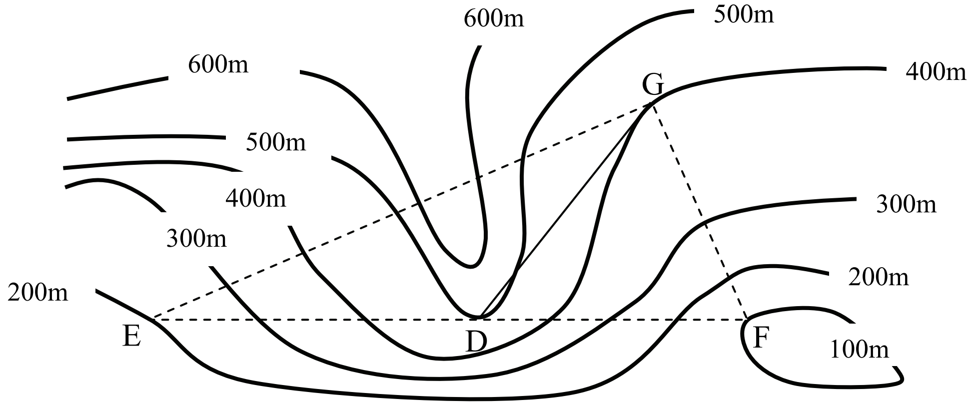
- $V = \pi r^2 h$
- It is important to note that $r = 10$ not 20 since the radius, r is half the diameter, d
  $\therefore\ r = d/2 = 20/2 = 10$
- $2200 = \pi 10^2 h$
  $h = \dfrac{2200}{(\pi 100)} \approx 7.00$

Tip

- *Calculator use. When dividing by more than one term, it is recommended to use brackets (as in the previous answer) OR calculate numerator and denominator separately before you divide.*

Questions 8 & 9 refer to the following diagram.

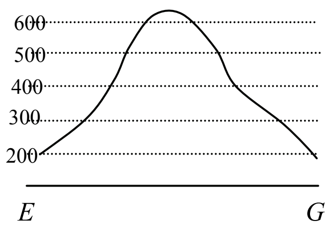
Below is a contour map of roads between four sites *D*, *E*, *F* and *G*.



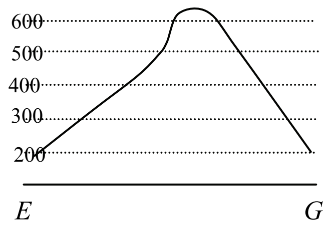
Question 8

The cross-section profile that best describes the contours along the road *EG* is

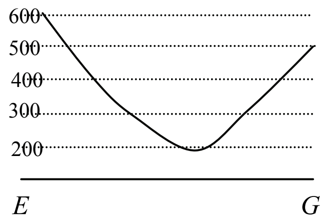
A.



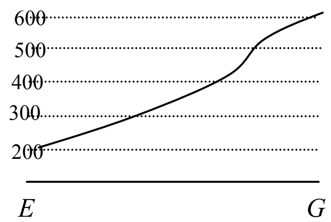
B.



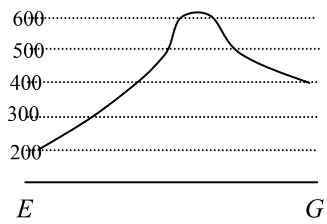
C.



D.



E.



Answer is E

Worked solution

- Need to examine contours on this grid. In this case the road *EG* rises from 200m at *E*, reaches a peak height of 600m before a decline to a height of 400m at *G*.
- Option E is the only one with town *G* at a height of 400m

Question 9

Given that the horizontal distance between *F* and *G* is 2km, the average gradient **from *G* to *F*** is approximately

A. 6.67

B. 0.15

C. 0.20

D. **–0.15**

E. –6.67

Answer is D

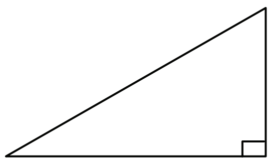
Worked solution

- Gradient = $\dfrac{height(contours)}{run(horizontal)}$

$$= \frac{100-400}{2000} = -0.15$$

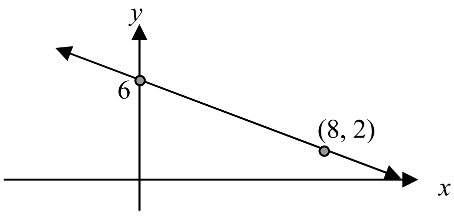
Tip

- *Generally there could be a number of questions based on this triangle when applying contours, such as elevation (depression), average gradient (rate of slope), distance between points …*



Module 3: Graphs and relations

Question 1



The equation of the line above is

A. $y + 2x = 6$

B. $y - 2x = 12$

C. $\mathbf{2y + x = 12}$

D. $2y - x = 6$

E. $y - x = 6$

Answer is C

Worked solution

- $y = mx + c \longrightarrow$ $c = 6$, $m = -4/8$ or $-\frac{1}{2}$
  $y = -\frac{1}{2}x + 6$
  $2y = -x + 12$ (× 2 both sides)
  $2y + x = 12$ (add x both sides)

- (Note: The line above shows a negative gradient but in this case x & y variables are both on the LHS of equation)

Question 2

For the equation

$ay - bx = c$, where a, b and c are constants;

the gradient and y-intercept are respectively

A. $-b$ and c

B. b and c/a

C. $-b$ and c/a

D. b/a and c

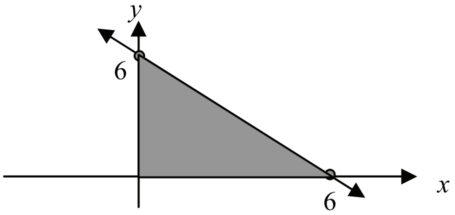
E. $\mathbf{b/a}$ **and** $\mathbf{c/a}$

Answer is E

Worked solution

- Must get y by itself
  $ay - bx = c$
  $ay = bx + c$ (add bx to both sides)
  $y = \frac{b}{a}x + \frac{c}{a}$ (÷ a both sides)
  ↑ gradient ↑ y-intercept

Question 3



The feasible region is **shaded** above for an unknown set of constraints.

Which of the following does **not** support this result?

A. $x \geq 0$

B. $y \geq 0$

C. **the point (6, 6) satisfies all constraints**

D. $x + y \leq 6$

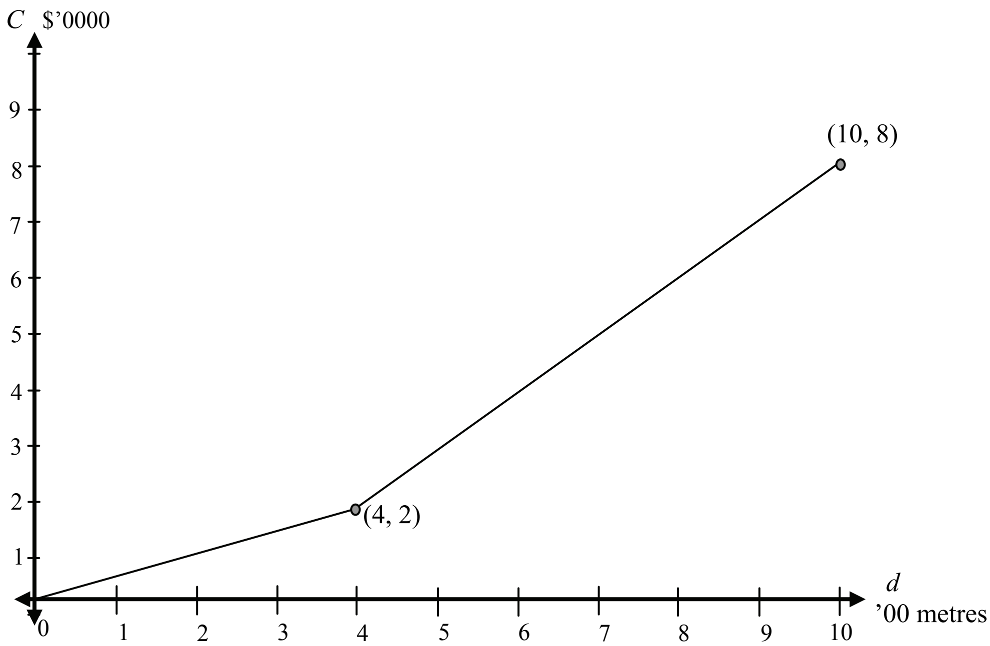
E. the maximum value of $2x + y$ is at the point (6, 0)

Answer is C

Worked solution

- The point (6, 6) is not within shaded (feasible) region.

The following graph relates to Questions 4 and 5



The cost, C, in $'0 000, of laying a pipeline depends on the type of terrain being excavated.

The graph above shows how the cost changes with the distance, d, (in '00 metres). The graph also indicates that at a distance of 400 metres (i.e. $d = 4$), the cost suddenly changes due to the change in terrain being excavated for the pipe-line.

Question 4

Which of the following statements is true?

A. The laying of the pipeline is much quicker after 400 metres.

B. In the first 400 metres, the cost of pipe-line is $50 per metre.

C. After the first 400 metres, the cost of pipe-line is $1 per metre.

D. The total cost for 1 kilometre of pipe-line was $8 000.

E. The change after 400 metres was due to easier excavation.

Answer is B

Worked solution

- The first stage, $0 \le d \le 4$, ends at point (4, 2) and being a uniform gradient (hence constant rate) means in practical terms, the cost of $20 000 for 400 metres which is a rate of $50 per metre.
- Other options: A is incorrect, it is not a time series, the steeper line in fact means it is dearer to lay pipe.
- Option C meant that units were not taken into account.
- Option D should be $80 000
- Option E because of increased cost rate, maybe the excavation was harder not easier.

Question 5

Which of the following rules best describes the graph above?

A. $\boldsymbol{C = \begin{cases} 0.5d & \text{for } 0 \le d \le 4 \\ d - 2 & \text{for } 4 < d \le 10 \end{cases}}$

B. $C = \begin{cases} 0.5d & \text{for } 0 \le d \le 2 \\ d + 2 & \text{for } 2 < d \le 8 \end{cases}$

C. $C = \begin{cases} 2d & \text{for } 0 \le d \le 4 \\ d + 2 & \text{for } 4 < d \le 10 \end{cases}$

D. $C = \begin{cases} 0.5d & \text{for } 0 \le d \le 4 \\ 2d - 2 & \text{for } 4 < d \le 10 \end{cases}$

E. $C = \begin{cases} 2d & \text{for } 0 \le d \le 4 \\ d - 2 & \text{for } 4 < d \le 10 \end{cases}$

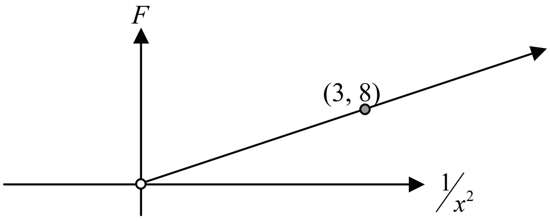
Answer is A

Worked solution

- 1st Stage
  $y = mx + c \quad c = 0, m = ½$
  $*C = ½d$
- 2nd Stage
  $m = (8 - 2)/(10 - 4) = 1$
  $C = 1d + c$
- Sub in the point (4, 2) to find c
  $2 = 4 + c \Rightarrow c = -2$
  $C = d - 2$

Question 6

The force of attraction, F, between two bodies decreases the further the distance, x, the bodies are apart. When graphing the force, F, against $1/x^2$, a linear model is obtained as follows.



The rule connecting F and x is

A. $\boldsymbol{F = 8/3x^2}$

B. $F = 3/8x^2$

C. $F = 64/9x^2$

D. $F = 8/3x$

E. $F = 3/8x$

Answer is A

Worked solution

- Linear: $F = k\dfrac{1}{x^2}$
- value of k is 8/3 being the gradient of the line
- $\therefore\ F = \dfrac{8}{3} \times \dfrac{1}{x^2} = \dfrac{8}{3x^2}$

Question 7

The simultaneous equations $2y - 3x = 6$ and $y = x + 2$ have a unique solution of

A. (1, 3)

B. (3, 0)

C. (3, 1)

D. **(–2, 0)**

E. (0, –2)

Answer is D

Worked solution

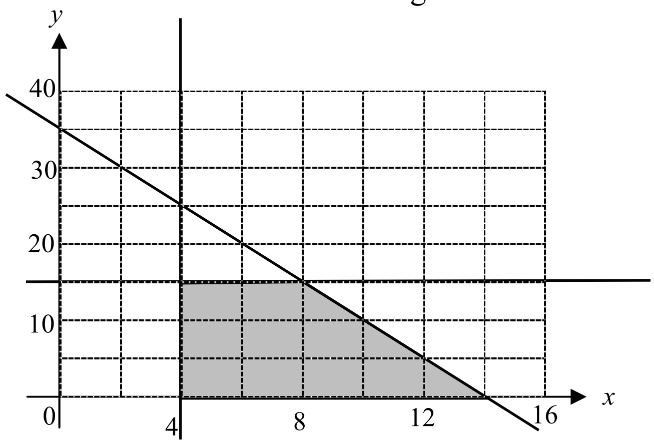
- Using substitution method
  Where $2y - 3x = 6$ (A) and $y = x + 2$ (B)
- substituting B into A gives:
  $2(x + 2) - 3x = 6$ (expand bracket)
  $2x + 4 - 3x = 6$ (collect like terms)
  $-x + 4 = 6$ (subtract 4 from both sides)
  $-x = 2$ ($\div -1$ both sides)
  $x = -2$
- sub this into B gives
  $y = -2 + 2$
  $= 0$
- Answer is $(x, y) = (-2, 0)$

The following graph relates to Questions 8 and 9

In a linear programming problem involving the number of items a firm can produce in one day subject to a variety of constraints.

- x represents number of item F that can be produced in one day.
- y represents number of item G that can be produced in one day.

The shaded area below shows the feasible region.



Question 8

The Profit, z, in dollars, for this firm for one day is given by the equation

$$z = 0.5x + 2y$$

Using the feasible region above the maximum profit is

A. $7.00
B. $23.50
C. $28.00
D. **$34.00**
E. $77.00

Answer is D

Worked solution

Feasible end points	Value of $z = 0.5x + 2y$
(4, 0)	$0.5 \times 4 + 0 = 2$
(14,0)	$0.5 \times 14 + 0 = 7$
(8, 15)*	$0.5 \times 8 + 2 \times 15 = 34$*
(4, 15)	$0.5 \times 4 + 2 \times 15 = 32$

* maximum

Question 9

One of the constraints defining the feasible region indicates that

A. A maximum of 14 items of product F can be made.

B. A minimum of 15 items of product G can be made.

C. The total number of items F and G that can be made is 49.

D. The point (12, 8) satisfies all constraints.

E. **There must be at least 4 products of item F produced.**

Answer is E

Worked solution

- Graphically one of the constraints is
  $x \geq 4$, which supports option E
- Option A is incorrect because there are no upper constraints for x alone
- Option B should say a **maximum** of 15
- Option C incorrect this refers to $x + y \leq 49$, no constraint supports these points
- Option D is outside feasible region.

Module 4: Business-related mathematics

Question 1

The selling price of a DVD is $45 after it was discounted by 20%.

The original marked price was

A. **$56.25**
B. $54.00
C. $225.00
D. $36.00
E. $60.00

Answer is A

Worked solution

- 80% of x is 45
  $0.80x = 45$
  $x = 45/0.8$
  $= 56.25$

Tip

- *Need to key words like selling price, marked price …*
- *The most common mistake is to find 80% of 45 is* x

Question 2

In April, Molly received the following statement from her bank showing all the transactions from her savings for the month of March.

Date	Transaction	Debit	Credit	Balance
1 Mar	Balance forward			2 945.18
6 Mar	Withdrawal	234.00		2 711.18
17 Mar	Deposit		400.00	3 111.18
28 Mar	Deposit		255.00	3 366.18

Interest is calculated on the minimum monthly balance. If the rate is 2.4% p.a., the amount of interest Molly will receive for March will be

A. $0.54
B. $65.07
C. $6.73
D. $5.16
E. **$5.42**

Answer is E

Worked solution

- Minimum for March is $2 711.18
- Rate is 2.4%p.a. = 0.2% per month
- Interest, I = $2 711.18 × 0.2/100
  = $5.42

Question 3

Nic wants to buy car at $22 000. On a hire purchase plan, he can purchase the car with a 10% deposit plus repayments of $320 per month for six years.

The total amount of **interest** paid for this car on the hire-purchase plan is

A. $3 040

B. **$3 240**

C. $5 040

D. $5 240

E. $23 240

Answer is B

Worked solution

- Deposit 10% of 22 000 = 2 200
- Total payment = 2 200 + 320 × 12 × 6
  = 25 240

Amount of interest = 25 240 – 22 000
  = 3 240

Question 4

Nic invests $3 500 in an account that is advertised at 6.2%p.a. with interest compounded quarterly.

The total amount he will have in 4 years is

A. $3 729.43

B. $4 368.00

C. $4 452.11

D. **$4 476.59**

E. $9 163.48

Answer is D

Worked solution

- Compound Interest
  Rate = 6.2%p.a. ÷ 4 = 1.55% per qtr
- The number of terms, $n = 4 \times 4 = 16$
  $\therefore A = 3500(1.0155)^{16}$
  = $4 476.59
- **Alternatively** use TVM solver
  N= 16 (quarterly over 4 yrs)
  I%= 6.2
  PV= 3500
  PMT= 0 (no regular repayments)
  *FV= 4476.59 (alpha solve)
  P/Y= 4 (quarterly)
  C/Y=4

Tip

- *Watch for key words* ***compound*** *and periods of interest i.e.* ***monthly, quarterly****, etc…; it indicates to use the compound interest formula and the rate and number of terms may need attention.*

Question 5

Marion has bought a photocopier for \$84 000. For tax purposes she uses reducing balance depreciation. The table below shows the book value of the photocopier after one year.

	Reducing balance Depreciation
Initial	\$84 000
At the end of 1 year	\$76 440

The book value of the photocopier after the second year is approximately

A. \$63 300
B. \$68 880
C. **\$69 560**
D. \$70 220
E. \$71 560

Answer is C

Worked solution

- As stated, need to use reducing balance depreciation formula:
  Book Value, $BV = P(1 - \frac{r}{100})^n$
  but firstly we need to find 'r'
- r, depreciation rate
  $= \frac{84000 - 76440}{84000} \times 100\%$
  = 9%
- 2nd year : Using formula
  Book value = $84000(1 - 0.09)^2$
  = \$69 560.40

Question 6

A company purchases a delivery van for \$29 000. It depreciates at a rate of 25 cents for every kilometre. The van has a scrap value of \$5 000.

The Book Value after travelling 20 000 km is

A. \$0
B. \$5 000
C. \$19 000
D. \$21 000
E. **\$24 000**

Answer is E

Worked solution

- Amount of Depreciation
  = 0.25 × 20000
  = 5000
- Book Value = Original Price – Depreciation
  Book Value = 29000 – 5000
  = \$24 000

Question 7

Stamp Duty is payable to the state government on a property transaction according to the following rate schedule.

Transfer of Real Property rates	
Range	**Rate**
$0 - $20 000	1.4 per cent of the dutiable value of the property
$20 001 - $115 000	$280 plus 2.4 per cent of the dutiable value in excess of $20 000
$115,001 - $870 000	$2 560 plus 6 per cent of the dutiable value in excess of $115 000
More than $870 000	5.5 per cent of the dutiable value

The stamp duty payable on a property worth $320 000 using the schedule above is

A. $2 560
B. $12 300
C. **$14 860**
D. $19 200
E. $33 000

Answer is C

Worked solution

- $320 000 fits into the third category ($115 001 - $870 000)
- Stamp Duty Payable rate is
  2560 plus 6% of value in <u>excess of $115000</u>
- = 2560 + 0.06 × (320000 – 115000)
- =$14 860

Question 8

Joan borrows $120 000 and makes monthly repayments of $1 400.

She uses the annuities formula to calculate how much she owes after X years at a rate of Y% per annum, calculated monthly on the reducing balance.

$$A = 120000(1.0075)^{240} - \frac{1400(1.0075^{240} - 1)}{0.0075}$$

Observing her formula the number of years, X and the rate, Y are

A. 10 years, 7.5%
B. 20 years, 7.5%
C. 20 years, 6.25%
D. 10 years, 9%
E. **20 years, 9%**

Answer is E

Worked solution

- 240 months ⟶ $Y = 240/12 = 20$ years
- $R = 1.0075$ but $R = 1 + r/100$
- $r = 0.0075 \times 100$
  $= 0.75\%$ per month
  $= 9\%$ p.a.

Question 9

Jordan originally had a home loan of $128 000. The interest rate is 7.2%p.a. compounded monthly and her monthly repayments were $845.

After 5 years she decided to increase her repayments. If the new repayments are $1 000, the **total** still owing after seven years from the original loan (i.e. two years of the new repayment) is

A. $75 300

B. $102 757

C. **$115 634**

D. $119 622

E. $158 863

Answer is C

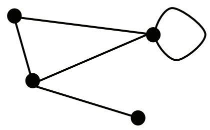
Worked solution

1st five years	Next 2 years
N= 60	N= 24
I%= 7.2	I%= 7.2
PV= 128000	PV= 122459
PMT= -845	PMT= -1000
FV=(–122459)	FV=(–115634)
P/Y= 12	P/Y= 12
C/Y=12	C/Y= 12

- Using the amount still owing after the first five years ($122 459), this will become the principal for the next stage. Think of it as starting a new loan on what you owe because the original conditions have changed, in this case repayments are now $1 000.

Module 5: Networks and decision mathematics

Question 1



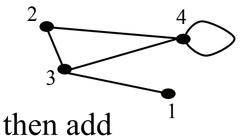
For the network above, the sum of degrees is:

A. 5
B. 12
C. **10**
D. 8
E. 9

Answer is C

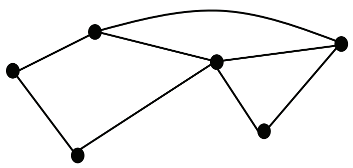
Worked solution

- Every edge has 2 degrees.
  5 edges gives 5 × 2
  = 10 degrees
- Otherwise you can label the degree of each vertex



- then add
  2 + 3 + 4 + 1
  = 10

Question 2



Which of the following statements is **false** regarding the network above?

A. Euler's formula for planar graphs applies to the network
B. The sum of degrees is double the number of edges
C. **It contains an Euler circuit and Hamiltonian path**
D. Many circuits exist
E. It contains an Euler path and Hamiltonian circuit

Answer is C

Worked solution

- To have an Euler circuit all degrees must be even. This network has two odd degrees.

Tip

- *Students need to be able to distinguish between a circuit, Euler Circuit and Hamiltonian Circuit as well as their respective paths. Also Euler's formula for planar graphs:* V + F – E = *2 must be connected and have no edges overlapping.*

Question 3

Three students A, B and C are sharing out tasks W, X and Y.

The number of hours to complete each task is shown.

	A	B	C
W	8	9	7
X	13	15	12
Y	15	18	17

If each student must do one task only, how should tasks be allocated to achieve minimum completion time?

A. W → A, X → B, Y → C

B. W → C, X → B, Y → A

C. W → B, X → A, Y → C

D. **W → B, X → C, Y → A**

E. W → A, X → C, Y → B

Answer is D

Worked solution

- Trial and error possible although the Hungarian algorithm can be used.

	A	B	C
W	8	9	7
X	13	15	12
Y	15	18	17

row reduction

1	2	0
1	3	0
0	3	2

column reduction

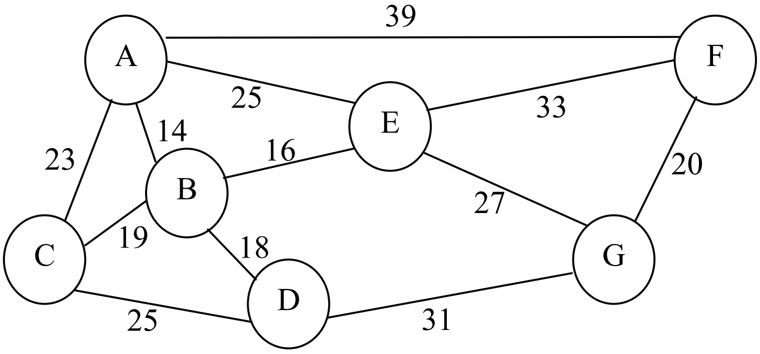
1	0	0
1	1	0
0	1	2

(zeroes in shade give answer)

- W → B
X → C
Y → A
- time of 36 hrs

Questions 4 and 5 refer to the following network.

For a shire, the graph below shows the major towns A, B …G connected by the main roads, all distances are in kilometres.



Question 4

The minimum spanning tree for this network is

A. 119 km

B. **114 km**

C. 127 km

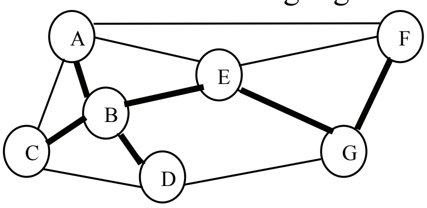
D. 105 km

E. 96 km

Answer is B

Worked solution

- Using Prim's algorithm (i.e. start with lowest edge, AB with 14, **from** either of these vertices select the lowest edge, BE with 16 etc… taking care not to form any circuits):
- the minimum tree is highlighted below



- Total = 14 + 16 + 18 + 19 + 27 + 20 = 114

Question 5

A salesperson from town A needs to make deliveries to all towns and return home. If this is done without visiting any other town twice, it would be an example of a

A. **Hamiltonian Circuit**

B. Complete Graph

C. Euler Circuit

D. Tree

E. Reachability matrix

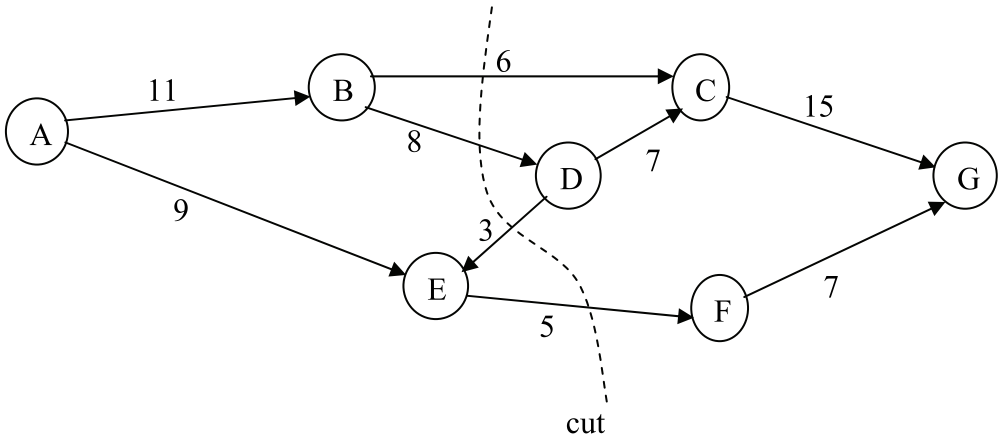
Answer is A

Worked solution

- Hamiltonian circuit starts and finishes at the same vertex (town) and must go through all other vertices once.
- (Note: An Euler circuit goes along all **edges** once not vertices.)

Question 6

The following directed graph shows the potential water flow of '000 litres per hour in an irrigation system.



The capacity of the cut in the above network, in '000 litres per hour, is:

A. **19**

B. 22

C. 25

D. 17

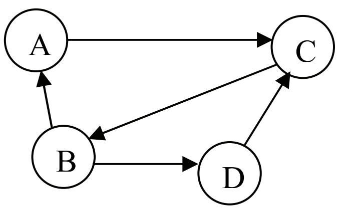
E. 18

Answer is A

Worked solution

- A cut is where all **necessary** edges are removed to stop 'flow' from source to sink (i.e. A to G above).
- In this case the cut DE is not needed.
- Capacity = 6 + 8 + 5 = 19

Question 7



A 2-step reachability matrix for this network is:

A. $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

B. $\begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}$

C. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

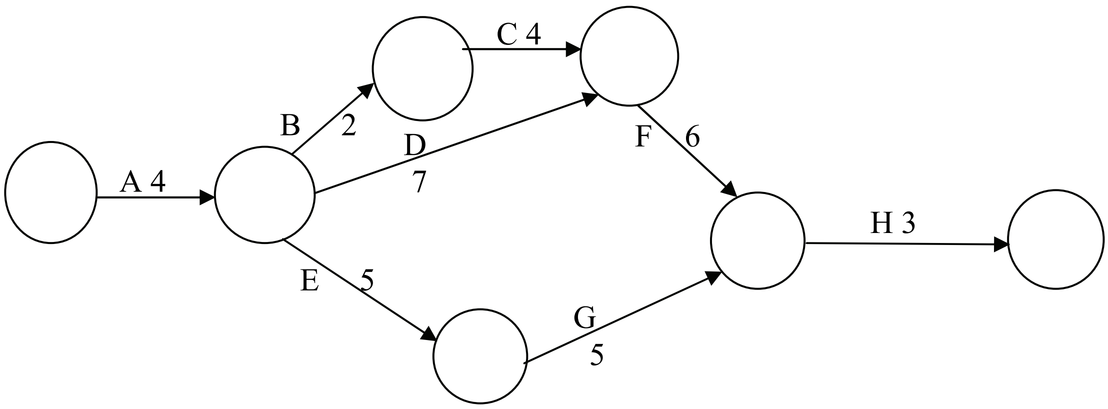
Answer is B

Worked solution

- Reachability:
  diagonal all 0
  A can reach B (ACB)
  B can reach C twice (BAC)&(BDC)
  C can reach A (CBA)
  C can reach D (CBD)
  D can reach B (DCB)
- Only matrix B gives this.

Questions 8 and 9 refer to the following critical path.

For a particular project there are eight activities to be completed and the time taken to complete each activity is shown in hours.



Question 8

The critical path and completion time for this project is

A. AEGH, 17 hours

B. AEFH, 18 hours

C. **ADFH, 20 hours**

D. ABCDFH, 20 hours

E. ADCFH, 26 hours

Answer is C

Worked solution

- The critical path is the 'longest' path because all activities must be completed before you start the next one.
- ADFH
  4 + 7 + 6 + 3
  = 20 hrs

Question 9

Assume that any activity can be **reduced to one** hour by providing extra equipment or labour.

If the project is to be crashed by reducing the completion time of ONE activity only, then this will reduce the completion time of the project by a maximum of

A. 1 hour

B. 2 hours

C. **3 hours**

D. 4 hours

E. 5 hours

Answer is C

Worked solution

- Need to look along all activities on the critical path.
- If Activity A can be reduced from 4 to 1 hour this would decrease the project by 3 hours.*
- Activity D can only be reduced by 1 otherwise ABCFH becomes the critical path.
- Activity F can also be reduced by 3, afterwards AEGH becomes the critical path. *
- Activity H can only be reduced by 2.
- Therefore the best is 3 hours.

Module 6: Matrices

Question 1

Let $A = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$

Which of the following operations **can** be performed?

A. $C - B + A$

B. $C - A + B$

C. $\boldsymbol{AB + C}$

D. $BA + C$

E. CBA

Answer is C

Worked solution

- When adding or subtracting matrices, their order must be identical. This eliminates choices A & B.
- When multiplying, the number of columns on the 1st matrix must equal the number of rows on the 2nd matrix. This eliminates E
- Option C satisfies all.
- The operation AB + C has order AB $(2 \times 1)(1 \times 2)$ giving a (2×2) matrix which can be added to matrix C, also (2×2)

Question 2

Let $M = \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}$ and $N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Then M^2N^4 equals

A. $\begin{bmatrix} 8 & 1 \\ 4 & 5 \end{bmatrix}$

B. $\begin{bmatrix} \mathbf{6} & \mathbf{1} \\ \mathbf{2} & \mathbf{3} \end{bmatrix}$

C. $\begin{bmatrix} 6 & 1 \\ 2 & -1 \end{bmatrix}$

D. $\begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}$

E. Impossible, no solutions

Answer is B

Worked solution

- N is the identity matrix which means $N^4 = N$
  So $M^2 N^4 = M^2$
- Use of Graphics calculator: MATRIX where $[A] = \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}$ then perform $[A]$^2
- Otherwise by hand

  $$M^2 = \begin{bmatrix} 2 \times 2 + 1 \times 2 & 2 \times 1 + 1 \times -1 \\ 2 \times 2 + -1 \times 2 & 2 \times 1 + -1 \times -1 \end{bmatrix}$$

- Either way will give answer B

Question 3

Let $R = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ and $T = \begin{bmatrix} 1 & 4 \\ 3 & -3 \end{bmatrix}$

The matrix X such that $RX = T$ will be:

A. $\begin{bmatrix} 1 & 13 \\ 3 & -1 \end{bmatrix}$ **B.** $\begin{bmatrix} \mathbf{1} & \mathbf{4} \\ \mathbf{1} & \mathbf{-1} \end{bmatrix}$ **C.** $\begin{bmatrix} 1 & 12 \\ 3 & -9 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 4 \\ 3 & -6 \end{bmatrix}$ **E.** $\begin{bmatrix} 1 & 4 \\ 9 & -9 \end{bmatrix}$

Answer is B

Worked solution

- $RX = T$
  $R^{-1}RX = R^{-1}T$
  $X = R^{-1}T$
- (**note** order other answers are due to wrong order or not finding the inverse)
- Use Graphics calculator
  MATRIX
  2 × 2
- $[A] = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ $[B] = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$

  then $[A]^{-1}[B] =$

Question 4

Two rival companies, KFB (K) and Jocks (J) sell French fries in three sizes: small (S), medium (M) and large (L).

The price of each size of fries, in cents, is listed in a price matrix P, where

$$P = \begin{array}{c} \\ \end{array}\begin{array}{c} \begin{matrix} S & M & L \end{matrix} \\ \begin{bmatrix} 120 & 180 & 220 \\ 130 & 170 & 210 \end{bmatrix} \end{array} \begin{matrix} K \\ J \end{matrix}$$

Due to competition, KFB reduces the price of all sizes by 10% and Jocks reduces their prices by 8%.

The new price matrix showing the decreased prices can be generated by performing which matrix product?

A. $\begin{bmatrix} 120 & 180 & 220 \\ 130 & 170 & 210 \end{bmatrix} \begin{bmatrix} 0.1 & 0 \\ 0 & 0.08 \end{bmatrix}$

B. $\begin{bmatrix} 120 & 180 & 220 \\ 130 & 170 & 210 \end{bmatrix} \begin{bmatrix} 0.9 & 0 \\ 0 & 0.92 \end{bmatrix}$

C. $\begin{bmatrix} 0.1 & 0 \\ 0 & 0.08 \end{bmatrix} \begin{bmatrix} 120 & 180 & 220 \\ 130 & 170 & 210 \end{bmatrix}$

D. $\begin{bmatrix} \mathbf{0.9} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.92} \end{bmatrix} \begin{bmatrix} \mathbf{120} & \mathbf{180} & \mathbf{220} \\ \mathbf{130} & \mathbf{170} & \mathbf{210} \end{bmatrix}$

E. $\begin{bmatrix} 0.9 & 0.1 \\ 0.08 & 0.92 \end{bmatrix} \begin{bmatrix} 120 & 180 & 220 \\ 130 & 170 & 210 \end{bmatrix}$

Answer is D

Worked solution

- Options A & B can't be multiplied.
- Reducing by 10% means 90% (0.9) remains and reducing by 8% has 92% (0.92) remaining.
- Option C will only give the reductions.
- D is the only logical answer remaining.

Question 5

How many of the following four sets of simultaneous linear equations have a **unique** solution?

$2x - y = 3$	$4x = 12$	$x - 2y = 0$	$3x - y = 5$
$x + y = 1$	$2x - y = 4$	$-2x + 4y = 10$	$3x - y = 10$

A. 0
B. 1
C. **2**
D. 3
E. 4

Answer is C

Worked solution

- When the determinant is zero, there are no unique solutions. This means that the two equations are either parallel (no solution) OR they are collinear (infinite solutions)
- In this question the 3rd & 4th pair have determinant of zero.
- Only 2 sets (the first two) will have a unique solution.

Question 6

The order of matrix A is (3 × 1) and the order of the matrix product AX is (3 × 4), then the order of matrix X must be

A. **(1 × 4)**
B. (3 × 4)
C. (4 × 1)
D. (4 × 3)
E. (3 × 1)

Answer is A

Worked solution

- When multiplying, the number of columns on the 1st matrix must equal the number of rows on the 2nd matrix.

(3 × 1) (a × b) gives (3 × 4)

match

$\therefore$ a = 1 & b = 4 : order (1 × 4)

Question 7

The following system of the linear equations

$$x - 3z = 4$$
$$x + 2y - z = 3$$
$$3y + 2z = 0$$

when written in matrix form is

A. $\begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{-3} \\ \mathbf{1} & \mathbf{2} & \mathbf{-1} \\ \mathbf{0} & \mathbf{3} & \mathbf{2} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{bmatrix} = \begin{bmatrix} \mathbf{4} \\ \mathbf{3} \\ \mathbf{0} \end{bmatrix}$

C. $\begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & -1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & -3 \\ 1 & 2 & -1 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$

E. $\begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & -1 \\ 0 & 3 & 2 \end{bmatrix} + \begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$

Answer is B

Worked solution

- Easier to re-write equations in columns with co-efficients as:
  $1x + 0y - 3z = 4$
  $1x + 2y - 1z = 3$
  $0x + 3y + 2z = 0$
- The first matrix in the product is the coefficients of x, y & z.

  $\begin{bmatrix} 1 & 0 & -3 \\ 1 & 2 & -1 \\ 0 & 3 & 2 \end{bmatrix}$
- Only option B has this. (Option E cannot be added and is illogical.)

Question 8

The solution of the matrix equation

$$\begin{bmatrix} 1 & -2 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \end{bmatrix}$$

is

A. $\begin{bmatrix} \mathbf{4} \\ \mathbf{-1} \end{bmatrix}$ **B.** $\begin{bmatrix} 2 \\ 10 \end{bmatrix}$ **C.** $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ **E.** $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Answer is A

Worked solution

- Calculator easiest way to solve although it can be done by hand:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 14 \end{bmatrix} \text{ where } \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -4 & 1 \end{bmatrix} \text{is the inverse of } \begin{bmatrix} 1 & -2 \\ 4 & 2 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 2 \times 6 + 2 \times 14 \\ -4 \times 6 + 1 \times 14 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

Question 9

Two locations *A* and *B* are regular popular holiday spots for a large number of people. It is noticed that the 90% of the people that go to location *A* return again the following year and the rest go to location *B*.

Location *B* retains 85% of people each year and the other 15% go to location *A*.

Assuming this pattern of movement is maintained and given that 100 people originally go to location *A* and 200 go to location *B*, the number that will holiday at each location in the **long term** is

A. 300 holiday at location *A*; 0 at location *B*

B. 200 holiday at location *A*; 100 at location *B*

C. 120 holiday at location *A*; 180 at location *B*

D. 175 holiday at location *A*; 125 at location *B*

E. **180 holiday at location *A*;120 at location *B***

Answer is E

Worked solution

- The transition matrix is

  $$T = \begin{bmatrix} 0.90 & 0.15 \\ 0.10 & 0.85 \end{bmatrix}$$

- Initial state matrix is
- $S_0 = \begin{bmatrix} 100 \\ 200 \end{bmatrix}$
- Need to find the steady-state solution, i.e. T^n S_0 for a large n
  (Using n = 20, 30, 40…gave)
- $\begin{bmatrix} 180 \\ 120 \end{bmatrix}$ which is option E

END OF WORKED SOLUTIONS