FURTHER MATHEMATICS EXAM 2: SOLUTIONS

Core

Question 1

a.

	Country Independant Variable	Totals
No. of Medals Dependant Variable		
Totals		

[A1]

b. Any two of dotplots, barchart or pictograms as the independent variable is categorical (do not accept histograms) **[A1]**

c. For India: 11 Bronze out of a total of 50 medals is 22%. The balance is silver medals (17 silver or 34%). **[A1]**

For South Africa:11 Gold, 12 Silver and 12 Bronze. These approximate to a third of the column each or 33%. **[A1]**

Question 2

a. From *r* value of 0.99

Coefficient of Determination $r^2 = 0.99^2 = 0.98$

Therefore: "We can conclude from this that **98** % of the variation in the total number of medals can be explained by the variation in **number of gold medals**. The other **2**% variation in number of gold medals is due to other factors".

b. For the gradient

$$b = \frac{r \times s_y}{s_x}$$

$$= \frac{+0.99 \times 62.8}{23.9} \quad \textbf{[A1]}$$

$$= 2.601338$$

$$= 2.60$$

For y-intercept

$$a = \bar{y} - b\bar{x}$$

$$= 63.4 - 2.60 \times 22.0 \quad \textbf{[A1]}$$

$$= 6.20$$

c.i

$$z = \frac{score - \bar{x}}{s_x}$$

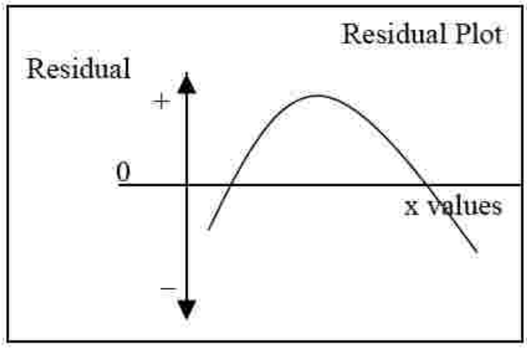
$$= \frac{84-22}{23.9} \quad \textbf{[A1]}$$

$$= 2.59$$

c.ii A z-score of +1 means the actual score is one standard deviation above the mean. **[A1]**

Question 3

a. The general shape expected for the residual plot is



b. Log of Angle is the best transformation as it compresses the *x*-values and has the highest value of coefficient of determination (r^2). **[A1]**

c. y^2 or square of distance as it will stretch the *y* values in the direction of the *y*-axis. **[A1]**

d. For Angle = 37°

$$\text{Distance} = 46.41 \times \log(\text{Angle}^\circ) - 4.55$$

$$= 46.41 \times \log(37^\circ) - 4.55$$

$$= 46.41 \times 1.568 - 4.55$$

$$= 68.2\ metres \quad \textbf{[A1]}$$

For Distance = 71.0 metres

$$\text{Distance} = 46.41 \times \log(\text{Angle}^\circ) - 4.55$$

$$71.0 = 46.41 \times \log(\text{Angle}^\circ) - 4.55 \quad \textbf{[A1]}$$

$$71.0 + 4.55 = 46.41 \times \log(\text{Angle}^\circ)$$

$$\log(\text{Angle}^\circ) = \frac{75.55}{46.41}$$

$$Angle = 10^{1.6279} = 42.5^\circ$$

Module 1: Number patterns

Question 1

a. This is an arithmetic sequence where the common difference is 2600.

Method 1 –Iteration

$$A_1 = 64400$$
$$A_2 = 64400 + 2600 = 67000$$
$$A_3 = 67000 + 2600 = 69600$$

Method 2 – Algebra

$$a = 64400$$
$$d = +2600$$
$$n = 3$$
$$t_n = a + (n-1)d$$
$$= 64400 + (3-1) \times 2600$$
$$= 64400 + 5200$$
$$= 69600$$

On the third morning its expected an attendance of 69600. **[A1]**

b. Using the Arithmetic series formula for

$$a = 64400$$
$$d = +2600$$
$$n = 7$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{7}{2}[2 \times 64400 + (7-1) \times 2600] \quad \textbf{[M1]}$$
$$= 3.5[128800 + 7800]$$
$$= 505400$$

The total for the seven days of the morning session is 505 000 people. **[A1]**

c.i.

For geometric pattern

$$\text{common ratio} = \frac{t_2}{t_1} = \frac{t_3}{t_2}$$
$$= \frac{67620}{64400} = \frac{71001}{67620}$$
$$= 1.05 = 1.05 \quad \textbf{[A1]}$$

There is a common ratio of 1.05

c.ii. This is a geometric sequence where

$$a = 1.05 \quad \text{(common ratio)}$$
$$b = 0 \quad \text{(common difference)}$$
$$c = 64400 \quad \text{(first term)} \quad \textbf{[A1]}$$

d.i. For geometric sequence

$$a = 64400$$
$$r = 1.05$$
$$n = 7 \quad \text{(final evening)}$$
$$t_n = ar^{n-1}$$
$$t_7 = 64400 \times 1.05^{7-1} \quad \textbf{[M1]}$$
$$= 86302.16$$

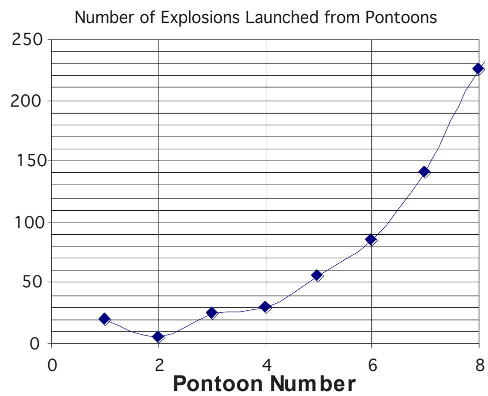
This is below the capacity of the MCG of 88000. **[A1]**

d.ii. For geometric series

$$a = 64400$$
$$r = 1.05$$
$$n = 7 \quad \text{(final evening)}$$
$$S_n = \frac{a(r^n - 1)}{r - 1}$$
$$S_7 = \frac{64400(1.05^7 - 1)}{1.05 - 1} \quad \text{[M1]}$$
$$= \frac{64400(0.407)}{0.05}$$
$$= 524345.344$$

The total attendance to the nearest thousand is expected to be 524000. **[A1]**

Question 2



[A2]

The sequence is

20, 5, 25, 30, 55, 85, 140, 225

Question 3

a. Create a sequence for 8 years of membership:
$a = 10$, $d = 8$
10, 18, 26, 34, 42, 50, 58, 66 **[A1]**

Create a sequence for 8 years of fees:
$a = 88$, $d = 11$
88, 99, 110, 121, 132, 143, 154, 165 **[A1]**

Total collected each year is the product of corresponding terms.

Total $= 10 \times 88 + 18 \times 99 + \ldots 58 \times 154 + 66 \times 165$
$= 42152$
Total fee collected is $42 152. **[A1]**

b. In the 7th year, fees $= 58 \times 154 = 8932$
In the 8th year, fees $= 66 \times 165 = 10890$

Percentage of total $= 100 \times \dfrac{(8932 + 10890)}{42152}$
$= 100 \times 0.470$
$= 47.0\%$ **[A1]**

Module 2: Geometry and trigonometry

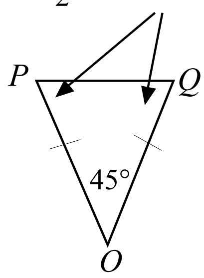
Question 1

a. The figure is a regular octagon and triangle *POQ* is one of eight identical, isosceles triangles with a vertex at the centre of the octagon.
Each angle at the centre has magnitude

$$\frac{360°}{8} = 45°$$ **[A1]**

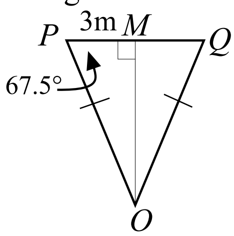
b. Triangle *OPQ* is an isosceles triangle so the angles *OPQ* and *PQO* are equal in size. (67.5°)

$$\frac{180° - 45°}{2} = 67.5°$$



[A1]

c. The triangle *OPM* is a right-angled triangle.



$\tan 67.5° = \dfrac{OM}{3}$ **[M1]**

$OM = 3 \times \tan 67.5°$
$= 7.2426\ldots$
≈ 7.24 metres **[A1]**

d. Triangles *ROS* and *POQ* are similar triangles so RS and PQ are in the same ratio as *ON* and *OM*.

$RS : PQ = ON : OM$ **[M1]**

and $\dfrac{RS}{PQ} = \dfrac{ON}{OM}$ and $ON = OM + 2$ **[M1]**

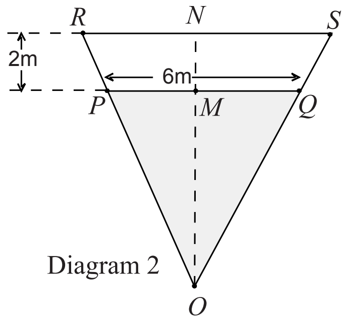


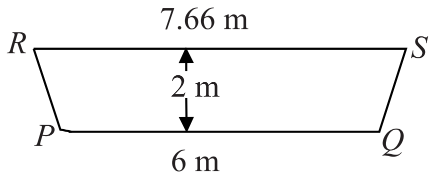
Diagram 2

Substituting:

$$\frac{RS}{6} = \frac{9.243}{7.243}$$

$RS = \dfrac{9.243 \times 6}{7.243} = 7.657 \approx 7.66\text{m}$ **[A1]**

e.



Area of a trapezium = $\frac{1}{2}h(a + b)$ where a and b are the parallel sides and h is the perpendicular height
Area of *PRSQ*

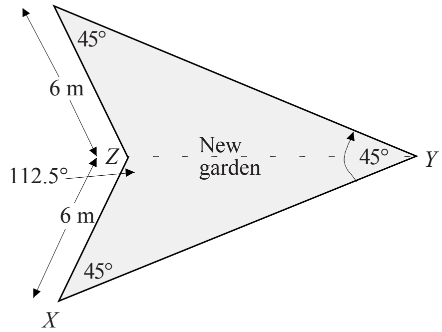
$= \frac{1}{2} \times 2 \times (7.657 + 6)$ **[M1]**

$\approx 13.66 \text{ m}^2$ **[A1]**

f. Area of the path = $8 \times 13.66 \text{ m}^2$
Volume of concrete needed
= area of path × depth **[M1]**
$= 8 \times 13.66 \times 0.15$
$= 16.39 \text{ m}^3$
$\approx 16.4 \text{ m}^3$ **[A1]**

Question 2

a.



On the diagram,
$\angle XZY = 180° - 45° - 22.5°$
$= 112.5°$
Using the sine rule:

$$\frac{XY}{\sin 112.5°} = \frac{6}{\sin 22.5°}$$ **[M1]**

$XY = \dfrac{6 \sin 112.5°}{\sin 22.5°} = 14.485$ metres **[M1]**

$XY = 14.49$ metres correct to two decimal places **[A1]**

b. Area of triangle *ZXY*

$= \frac{1}{2} \times 6 \times 14.485 \times \sin 45°$ **[M1]**

$= 30.728 \text{ m}^2$ **[M1]**

The area of the new garden is $2 \times 30.728 = 61.46 \text{ m}^2$ (correct to two decimal places) **[A1]**

Module 3: Graphs and relations

Question 1

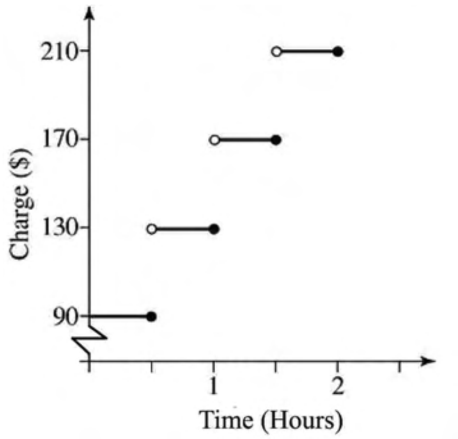
a.

Time	Charge
46 minutes	$50 + 2 × $40 = $130
1 hour 22 min (82min)	$50 + 3 × $40 = $170

[M1]

Total for the two appointments is $300 ($130 + $170) **[A1]**

b.



– 1 mark for correct use of open and closed ends

– 1 mark for correct $values (90,130…)

Question 2

a. $P = \$480 + \$320 \times 4$
$= \$1\ 760$ **[A1]**

b. $P = 480 + 320n$ **[A1]**

Question 3

a. Time = 6 years **[A1]**

b. Predicted value (24 years) = $160 000 **[A1]**

Question 4

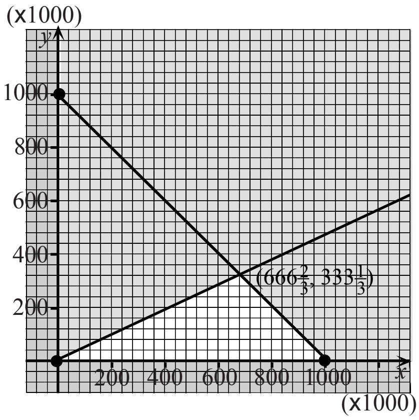
a. Let y = amount invested in shares. **[A1]**

b. The total amount constraint yields
$x + y \le 1\ 000\ 000$ **[A1]**

The investment mix constraint yields
$x \ge 2y$ or **[A1]**
$2y - x \le 0$

c. Given that bonds return 6% the profit on $$x$ worth of bonds is $0.06x$. Similarly the profit on shares is $0.11y$. Therefore the objective function is
$Z = 0.06x + 0.11y$ **[A1]**

d.



Note that the scale is in thousands of dollars.
Determine the vertices of the solution region
x axis: (1 000 000, 0)
y axis: (0, 0)

Intersection of 2 lines
$x + y = 1\ 000\ 000$
$2y - x = 0$
$y = 333\ 333$
$x = 666\ 667$ **[A1]**

To evaluate the maximum of the objective function at vertices.
At (666 667, 333 333)
$Z = 0.06(666\ 667) + 0.11(333\ 333)$
$= 76\ 667$
At (1 000 000, 0)
$Z = 0.06(1\ 000\ 000) + 0.11(0)$ **[M1]**
$= 60\ 000$

Solution is $666 667 in to bonds and $333 333 into shares **[A1]**

Module 4 : Business-related mathematics.

Question 1

a. Monthly payment = $\frac{44 \times 98}{10}$ = \$431.20 **[A1]**

b. A 2.3% increase corresponds to a multiplying factor of

$1 + \frac{2.3}{100} = 1.023$

$98 × 1.023 = $100.254

≈ \$100.25 **[A1]**

c. If *m* is the multiplying factor in the third year then

100.254 × m = 103.06 **[M1]**

$m = \frac{103.06}{100.254}$

= 1.02798…

≈ 1.028

$= 1 + \frac{2.8}{100}$

An increase of 2.8% **[A1]**

Question 2

A perpetuity is the interest earned by the investment in the specified time period (1 month in this case)

Using the simple interest formula:

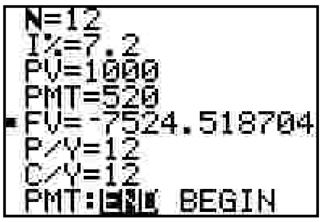
$$I = \frac{\$108000 \times \frac{5.8}{12} \times 1}{100}$$ **[M1]**

= \$522 **[A1]**

Jesse would receive \$522 each month.

Question 3

a. Using the TVM solver on the calculator:



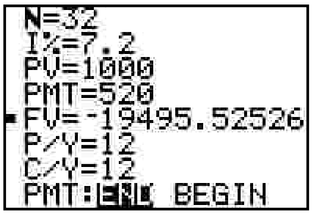
Jesse will owe \$7525 after one year. **[A1]**

Note: Both the present value (PV) and the payment (PMT) are positive because they are both considered 'incoming' amounts.

b. Using the TVM solver:

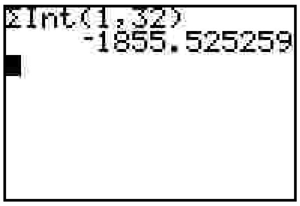
After 33 months the amount he owes will exceed \$20 000. **[A1]**

c. Using the TVM solver:



Jesse will owe \$19 496 after 32 months. **[A1]**

d. Using the calculator:



Or

Interest = 1000 + 520 × 32 – 19 496

= \$1856 **[A1]**

Question 4

a. The \$19 500 that is owed accumulates interest for three months at a rate of 7.2% p.a. compounding monthly.

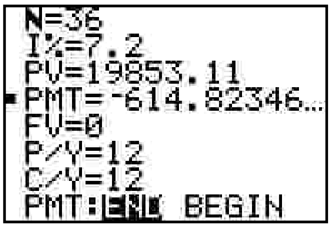
$$7.2\% \text{ p.a.} = \frac{7.2\%}{12} = 0.6\% \text{ per month}$$ **[M1**

Using the compound interest formula: $A = PR^n$ where

$$R = 1 + \frac{r}{100} = 1 + \frac{0.6}{100} = 1.006$$

The missing figure is 1.006 **[A1]**

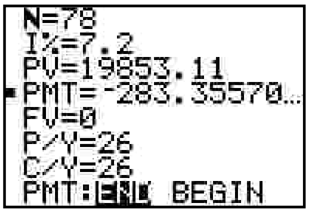
b. Using the TVM solver :



Jesse will need to repay $614.82 per month for 36 months to totally repay the loan. **[A1]**

c. Using the calculator.
Monthly payments: Interest paid over the 36 months is $2280.53

Fortnightly payments: Interest paid over the 78 fortnights is $2248.63 **[M1]**



Jesse will save $31.90 ($32 to the nearest dollar) in interest if he makes fortnightly payments rather than monthly payments.
Alternative method for calculating interest

Monthly :
Interest = $614.82 × 36 – $19853.11
= $2280.41

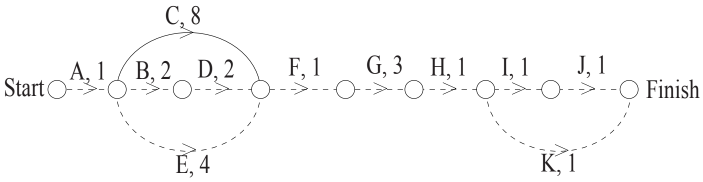
Fortnightly:
Interest = $283.36 × 78 – $19853.11
= $2248.97
A saving of $31.44; $31 to the nearest dollar.

Either answer is acceptable. **[A1]**

Module 5 : Networks and decision mathematics

Question 1

a. Activity C has activity A as an immediate predecessor and is an immediate predecessor to activity F.



[A1

b. Activities that cannot be delayed without delaying the whole project are activities on the critical path: A, C, F, G, H, I, J **[A1]**

c. The earliest completion time of the whole project is the sum of the durations of the activities on the critical path. i.e 1 + 8 + 1 + 3 + 1 + 1 + 1 = 16 days **[A1]**

d. The slack time of an activity is the difference between its latest start time and its earliest start time. For activity E this is 5 – 1 = 4 days. **[A1]**

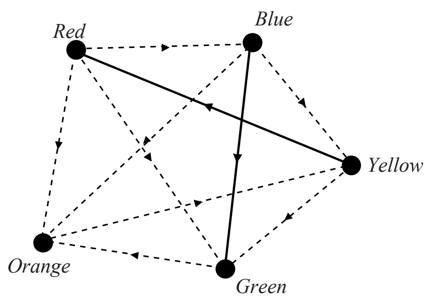
e. The latest **finish** time for activity B is 7 days. **[A1]**
It is the sum of the latest start time for activity B (5 days) and the duration of B (2 days). **[M1]**

Question 2

a. The number of games played

$$= \frac{n(n-1)}{2} = \frac{5 \times 4}{2} = 10$$

b. The missing results are:
Blue defeated ***Green*** **[A1]**
and ***Yellow*** defeated ***Red*** **[A1]**



c.

	R	B	Y	G	O
Red	0	1	0	1	1
Blue	0	0	1	1	1
Yellow	1	0	0	1	0
Green	0	0	0	0	1
Orange	0	0	1	0	0

Blue defeated Green so this is a '1'

Yellow did not defeat Blue so this is a '0'

[A1][A1]

d. The '1' in the matrix M^2 gives the second-level defeat of Blue over Red **[A1]**
Blue defeated Yellow and Yellow defeated Red. **[A1]**

e.i. The matrix $M + M^2$ is found by adding the corresponding elements in the two matrices.

$$M + M^2 = \begin{bmatrix} 0 & 1 & 2 & 2 & 3 \\ 1 & 0 & 2 & 2 & 2 \\ 1 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

[H1]

ii. Adding the elements in each row gives the dominance vector ie. the sum of the level one and level two wins in the competition.

$$\begin{bmatrix} 8 \\ 7 \\ 6 \\ 2 \\ 3 \end{bmatrix}$$

[H1]

f. The ranking of the teams (most wins to least number of wins) is:
Red, Blue, Yellow, Orange and Green **[A1]**

Module 6 Matrices

Question 1
(a)

Matrix	**Order of the Matrix** m × n	Name the element that has the value of **1** e.g. $a_{2,3}$
$A = \begin{bmatrix} 2 & 0 & 3 \\ 5 & -2 & 1 \end{bmatrix}$	2 × 3	$a_{2,3}$
$B = \begin{bmatrix} -4 & -1 \\ 2 & 1 \\ 3 & 0 \end{bmatrix}$	3 × 2	$b_{2,2}$
$C = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$	2 × 2	$C_{1,1}$
	[A1]	**[A1]**

(b)

Possible Products	*Order of Matrix*	Shown Workings
A.B	2 × 2	2 × **3** × **3** × 2
A.C	Does not exist	2 × 3 × 2 × 2
B.A	3 × 3	3 × **2** × **2** × 3
B.C	3 × 2	3 × **2** × **2** × 2
C.A	2 × 3	2 × **2** × **2** × 3
C.B	Does not exist	2 × 2 × 3 × 2

[A1] correctly identifying those that do not exist.
[A1] for the remaining four possible products.

c. Using a graphics calculator, set up the two matrices ***A* and *B***.

```
[A]
        [[2 0  3]
         [5 -2 1]]
[B]
         [[-4  -1]
          [2   1 ]
          [3   0 ]]
[A][B]
        [[1    -2]
         [-21  -7]]
```

$$\begin{bmatrix} 2 & 0 & 3 \\ 5 & -2 & 1 \end{bmatrix} \times \begin{bmatrix} -4 & -1 \\ 2 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -21 & -7 \end{bmatrix}$$

Question 2

a.

$$\begin{bmatrix}15 & 105\\20 & 190\end{bmatrix}\begin{bmatrix}a\\d\end{bmatrix}=\begin{bmatrix}-195\\-410\end{bmatrix}$$ **[A1]**

b. Determinant = 15 × 190 – 20 × 105
= 750 **[A1]**

c.

$$\begin{bmatrix}a\\d\end{bmatrix}=\frac{1}{750}\begin{bmatrix}190 & -105\\-20 & 15\end{bmatrix}\begin{bmatrix}-195\\-410\end{bmatrix}$$

$$=\begin{bmatrix}8\\-3\end{bmatrix}$$ **[M1]**

The first term, *a* is 8 and the common difference, *d* is –3. **[A1]**

Question 3

a. Probability or Transition Matrix

Current Year

$$T=\begin{bmatrix}0.6 & 0.3 & 0.01\\0.15 & 0.5 & 0.01\\0.25 & 0.2 & 0.98\end{bmatrix}$$

Columns: 12, 3, *Non* (Current Year); rows: 12, 3, *Non* (*Next Year*)

For columns totaling to 1.0 **[M1] [A1]**

b. Initial State Matrix

$$M_{2006}=\begin{bmatrix}200\\100\\0\end{bmatrix}\begin{matrix}12-month\\3-month\\People\ Telemarketed\end{matrix}$$ **[A1]**

c. **i & ii**

$$M_{2007}=TM_{2006}$$

$$M_{2007}=\begin{bmatrix}0.6 & 0.3 & 0.01\\0.15 & 0.5 & 0.01\\0.25 & 0.2 & 0.98\end{bmatrix}\begin{bmatrix}200\\100\\0\end{bmatrix}$$ **[A1]**

$$=\begin{bmatrix}150\\80\\70\end{bmatrix}$$ **[A1]**

d. For long term use T^n where n is large eg $n = 50$

$$M_{longterm}=T^nM_{2006}$$

$$M_{long\ term}=\begin{bmatrix}0.6 & 0.3 & 0.01\\0.15 & 0.5 & 0.01\\0.25 & 0.2 & 0.98\end{bmatrix}^{50}\begin{bmatrix}200\\100\\3500\end{bmatrix}$$ In three

$$=\begin{bmatrix}180.415\\124.035\\3495.549\end{bmatrix}$$ **[M1]**

Total membership = 180.415 + 124.035
= 304.45

That is approximately 304 members. **[A1]**

END OF QUESTION AND ANSWER BOOK