Year 2005

VCE
Further Mathematics
Trial Examination 1

Suggested Solutions

Question 1 C $38.4 - 34.6 = 3.8$	**Question 2 D** Overall, males earn more than females. Range for males $= 43.2 - 34.2 = 9$ Range for females $= 40.0 - 28.8 = 11.2$ $\therefore$ females have larger range than males 25% of males have a salary less than \$34,600 25% of females have a salary less than \$34,000 and 25% of females have a salary greater than \$37,200, which makes D correct. 50% of males have a salary between \$34,000 and \$38,400
Question 3 E 2.5% are greater than the mean plus 2 times the standard deviation. $77 - 55 = 22$ 2 times the standard deviation $= 22$ standard deviation $= 11$	**Question 4 D** Total number of Liberal voters $= 16 + 83 = 99$ Total number of Labour voters $= 127$ Total number of voters $= 127 + 99 = 226$
Question 5 B Number of people who did not want a republic and voted labour $= 127 - 55 = 72$ Number of people who did not want a republic $= 72 + 83 = 155$ % of people who did not want a republic and voted labour $= \dfrac{72}{155} \times 100 = 46\%$	**Question 6 C** $r^2 = 0.0676$ $r = -0.26$ because x increases as y decreases Hence correlation is weak, negative.
Question 7 B Median of lower 3 points is (65,10) Median of upper 3 points is (85,16) $m = \dfrac{16 - 10}{85 - 65} = \dfrac{6}{20} = 0.3$	**Question 8 A** $b = \dfrac{1}{3}[(y_l + y_m + y_u) - m(x_l + x_m + x_u)]$ $= \dfrac{1}{3}[(10 + 10 + 16) - 0.3(65 + 75 + 85)]$ $= \dfrac{1}{3}[36 - 67.5]$ $= -10.5$

Question 9 C There is an up and down pattern, and the pattern does not repeat itself within a year but over 4 to 5 years. ∴Cyclic trend	**Question 10 A** From the graph it can be seen that there is a negative residual then 2 positive residuals then 2 negative residuals and then a positive residual. Graph B starts with zero residual so not B Graph starts with a positive residual so not C Graph D has negative residual then positive then negative so not D Graph E starts with 2 negative residuals so not E
Question 11 D $1.02 + 0.82 = 1.84$ Summer index + Autumn index $= 4 - 1.84 = 2.16$ Summer index is double Autumn index ∴3 times Autumn index $= 2.16$ Autumn index $= 0.72$ Summer index $= 2 \times 0.72 = 1.44$	**Question 12 A** $\dfrac{3.2 + 2.8 + 4.9}{3} = 3.6$
	Question 13 B Yearly average for 2005 = $\dfrac{7.7 + 5.9 + 7.5 + 9.3}{4} = 7.6$ Yearly average for 2003 = 7.65 Yearly average for 2004 = 7.675 Summer 2003 $5.8 \div 7.65 = 0.7582$ Summer 2004 $5.6 \div 7.675 = 0.7296$ Summer 2005 $5.9 \div 7.6 = 0.7763$ Seasonal index for summer = $\dfrac{0.7582 + 0.7296 + 0.7763}{3} = 0.7547$ Deseasonalised value $= \dfrac{5.9}{0.7547} = 7.8$

Question 1 D	**Question 2 B**
Arithmetic sequence $t_1 = 5$ $t_2 = 8$ $t_3 = 11$ $d = 8 - 5 = 3$	$S_n = \dfrac{n}{2}(a + l)$ $S_n = \dfrac{10}{2}(7 + 37)$ $S_n = 5 \times 44 = 220$
Question 3 D	**Question 4 A**
$S_\infty = \dfrac{a}{1 - r} = \dfrac{3}{1 - \dfrac{1}{2}} = 6$	Total of 8 shares. Sam's share $= \dfrac{1}{8} \times 120,000 = \$15,000$
Question 5 B	**Question 6 C**
$8 + 13 + 18 + 23 = 62$ (four terms) $62 + 28 = 90$ (five terms) $90 + 33 > 100$ $\therefore$ six terms	$t_n = ar^{n-1}$ $t_5 = 3000 \times 0.94^4 = 2,342$

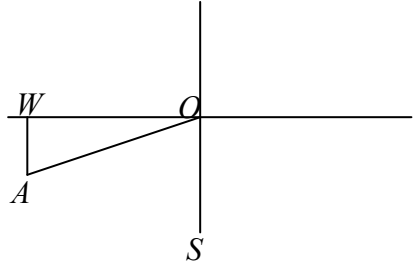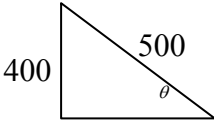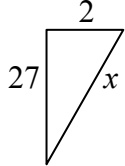| **Question 7 E** Each term is 0.94 times the term that went before. $t_{n+1} = 0.94t_n$ $t_1 = 3000$	**Question 8 B** $S_4 = \dfrac{4}{2}[2a + 3d] = 54$ $S_8 = \dfrac{8}{2}[2a + 7d] = 156$ $4a + 6d = 54$ $8a + 12d = 108$ $8a + 28d = 156$ $16d = 48$ $d = 3$

Question 9   E

$s_1v_1 = s_2v_2$

$20 \times v_1 = 15 \times 1$

$v_1 = \dfrac{15}{20} = \dfrac{3}{4} = 0.75$

Question 1 C	**Question 2 C**
$\cos\theta = \dfrac{8}{14}$ $\theta = \cos^{-1}(\dfrac{8}{14}) = 55^0$	 Angle *WAO* and angle *SOA* are equal (alternate angles) $\therefore$ bearing is $180 + 55 = 235^0$T
Question 3 D $\sin\theta = \dfrac{400}{500}$ $\theta = \sin^{-1}(\dfrac{400}{500}) = 53^0$	**Question 4 D** 1 : 64 1000 : 64000 Ratio of sides = ratio of cube root of volumes Ratio of sides =10 : 40 Ratio of area of face = square of sides Ratio of area of face = 100 : 1600 Area of face of cube B = 1600
Question 5 E Triangles AOB and DOC are similar (AAA) $\therefore$ corresponding sides are in the same ratio $\dfrac{2.5}{1} = \dfrac{OC}{4}$ $OC = 10$	**Question 6 C** $x^2 = 2^2 + 27^2$ $x = \sqrt{2^2 + 27^2}$ $x = 27.1$

Question 7   D
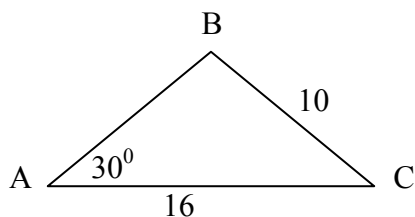
$$\frac{10}{\sin 30} = \frac{16}{\sin B}$$

$$\sin B = \frac{16 \sin 30}{10}$$

$$B = \sin^{-1}\left(\frac{16 \sin 30}{10}\right)$$

$$B = 53^0 \text{ or } 180 - 53 = 127^0$$

Question 8   D

By Pythagoras, $AC = 13$

Using the cosine rule on triangle ACD

$$13^2 = 8^2 + 6^2 - 2 \times 8 \times 6 \cos \theta$$

$$69 = -96 \cos \theta$$

$$\cos \theta = -0.71875$$

$$\theta = 180 - 44 = 136^0$$

Question 9   D

Area of 2 triangles + area of two slanting rectangles + area of base rectangle

Slant side of triangle $= 5$ (Pythag)

$$A = 2 \times \frac{1}{2} \times 6 \times 4 + 2 \times 12 \times 5 + 6 \times 12$$

$$A = 24 + 120 + 72$$

$$A = 216$$

Question 1   A

x intercept occurs when $y = 0$

$3x - 6 = 0$

$x = 2$

Question 2   D

$2x - 3y = -7$ (1)

$3x + 8y = 2$ (2)

$(1) \times 3 \rightarrow 6x - 9y = -21$ (1a)

$(2) \times 2 \rightarrow 6x + 16y = 4$ (2a)

$(2a) - (1a) \rightarrow -25y = -25$

$y = 1$

When $y = 1$,

$3x = 2 - 8 = -6$

$x = -2$

Question 3   B

30 cents = $0.3 for one minute of calls

= $0.3n$ for n minutes of calls, plus the $35 basic charge.

$C = 0.3n + 35$

Question 4   E

Gradient = 2 since parallel to line with gradient of 2

$y = 2x + c$

When $x = -1, y = -4$

$-4 = -2 + c$

$c = -2$

$y = 2x - 2$

$\therefore y - 2x + 2 = 0$

Question 5   A

The line $x + y = 3$ cuts the axes at $(3,0)$ and $(0,3)$

$\therefore$ not E

The line $2y - 3x + 4 = 0$ cuts the y axis when $x = 0$

$2y + 4 = 0$

$y = -2$

The line $2y - 3x + 4 = 0$ cuts the x axis

when $y = 0$

$-3x + 4 = 0$

$x = \dfrac{4}{3} = 1\dfrac{1}{3}$

$\therefore$ not C or D

We want the area under the line $x + y = 3$

and over the line $2y - 3x + 4 = 0 \therefore$ A

Question 6   C

Profit / book = Revenue / book − Cost / book

$P = \dfrac{25x}{x} - \dfrac{18x - 1200}{x}$

$P = \dfrac{25x - 18x + 1200}{x}$

$P = \dfrac{7x + 1200}{x}$

Question 7 A

The equation of the line joining the points (3,7) and (5,11) is of the form

$y = mx + c$

$m = \dfrac{11 - 7}{5 - 3} = \dfrac{4}{2} = 2$

$y = 2x + c$

When $x = 3, y = 7$

$7 = 6 + c$

$c = 1$

Equation of this line is $y = 2x + 1$

$\therefore$ not C, D, E

The open circles are not included.

The closed circles are included.

$\therefore$ A

Question 8 C

The height of the water increases rapidly at first, and then less rapidly, showing that the vase is narrow at the bottom but getting wider nearer the top.

Question 9 E

From the labels on the axes, it can be seen that

$y = kx^2$

When $x^2 = 16, y = 80$

$80 = k \times 16$

$k = 5$

$\therefore y = 5x^2$

Module 4 Business-related mathematics. Suggested solutions.

Question 1 B Amt. Repaid $= 530 \times 4 = 2120$ Interest $= 2120 - 2000 = \$120$	**Question 2 E** On Jan.1st Luke has a balance of \$4,400 On Jan.12th Luke has a balance of \$5,000 On Jan.17th Luke has a balance of \$4,700 On Jan.28th Luke deposited \$200 Total deposits for Jan.=$200 + 400 + 200 = \$800$
Question 3 D On Jan.1st Luke has a balance of \$4,400 On Jan.12th Luke has a balance of \$5,000 On Jan.17th Luke has a balance of \$4,700 On Jan.28th Luke has a balance of \$4,900 Minimum balance for the month = \$4,400	**Question 4 D** 10% of $1,800 = 180$ $180 + 1,800 = \$1,980$
Question 5 C Change in population from 2000 to 2005 $= 720 - 480 = 240$ Decrease in 5 years = 240 Decrease /year $= 240 \div 5 = 48$ Decrease in 8 years $= 48 \times 8 = 384$ Number rabbits after 8 years $= 720 - 384 = 336$	**Question 6 C** $A = PR^n = \dfrac{Q(R^n - 1)}{R - 1}$ $P = 10,000$ $R = 1 + \dfrac{8}{12 \times 100} = 1.0067$ $n = 9$ $A = 10,000(1.0067)^9 - \dfrac{600(1.0067^9 - 1)}{0.0067}$ $A = \$5072.4$

Module 4 Business-related mathematics. Suggested solutions.

Question 7 D	**Question 8 D**
$A = PR^n$	When $t = 0, N = 1500$
$P = 30,000$	Double $N = 3000$
$R = 1 + \dfrac{5.5}{2 \times 100} = 1.0275$	$3000 = 1500(1.08)^t$
	$2 = 1.08^t$
$n = 10$	$\log_{10}(2) = t \log_{10}(1.08)$
$A = 30,000 \times (1.0275)^{10} = \$39,349.5$	$t = \dfrac{\log_{10}(2)}{\log_{10}(1.08)}$
	$t = 9.006$

Question 9 E

$A = PR^n - \dfrac{Q(R^n - 1)}{R - 1}$

$0 = 200,000(1.015)^{100} - \dfrac{Q(1.015^{100} - 1)}{0.015}$

$Q(1.015^{100} - 1) = 13296.13695$

$Q = \$3874.11$

Module 5 Networks and decision mathematics. Suggested solutions.

Question 1 C	**Question 2 D**
V + F – E = 2	A = 2 E = 2
V = 10	B = 6 F = 1
F – E = –8	C = 2 G = 3
E – F = 8	D = 2
E = F + 8	
Minimum value for F = 1	Sum = 2 + 6 + 2 + 2 + 2 + 1 + 3 = 18
∴ Minimum value for E = 1 + 8 = 9	

Question 3 C	**Question 4 C**
A Hamiltonian circuit passes through each vertex once, beginning and ending at the same vertex.	A simple graph has no loops and no multiple edges between two vertices.

Question 5 D	**Question 6 E**
A to A is 0 (top left hand corner) ∴ must be D	Minimum spanning tree = 3 + 1 + 5 + 3 + 2 + 6 = 20

Module 5 Networks and decision mathematics. Suggested solutions.

Question 7 B	**Question 8 E**
A critical path is one where any delay on this path will cause a delay in completing the project.	Andrew plays 2 sports and Ben plays 3 sports. ∴ not A One person plays tennis and one person plaus squash. ∴ not B Dave and Carla both participate in one sport each. ∴ not C Number of people playing both cricket and tennis = 1 Number of people skiing = 3 ∴ not D Number of people playing squash, netball or tennis = 3 ∴ E

Question 9 A
Start with A
ABD = 14, AD = 5, ACD = 11 ∴ ABD
ABDE = 21, ABE = 20 ∴ ABDE
ABDEG = 29, ABDG = 22, ABDFG = 24
∴ ABDEG is the critical path.

End of suggested solutions
2005 Further Mathematics Trial Examination 1