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FURTHER MATHEMATICS

TRIAL EXAMINATION 1

(FACTS, SKILLS AND APPLICATIONS)

2004

Reading Time: 15 minutes
Writing time: 90 minutes

Instructions to students

This exam consists of Section A and Section B.
Section A contains 13 multiple-choice questions from the core, "Data Analysis".
Section A is compulsory and is worth 13 marks.
Section B consists of 5 modules each containing 9 multiple-choice questions. You should choose 3 of these modules and answer every question in each of your chosen modules.
Each of the modules is worth 9 marks.
Section B begins on page 8 of this exam.
There is a total of 40 marks available for this exam.
Students may bring up to two A4 pages of pre-written notes into the exam.
An answer sheet appears on page 35 of this exam.
Formula sheets can be found on pages 33 and 34 of this exam.
Diagrams in this exam are not to scale except where otherwise stated.

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Section A

Core

All questions must be answered.

The following information relates to questions 1 and 2.

The Main Street bus was late to get to its last stop at the depot nine times last month. The number of minutes late on each of these occasions is recorded below.

2.3, 5.1, 6.2, 3.7, 1.8, 7.5, 2.5, 3.2, 4.5.

Question 1

The mean number of minutes late for this bus was closest to

A. 1.8
B. 3.6
C. 3.7
D. 4.1
E. 5.3

Question 2

Which one of the following would **not** be appropriate to use to display the data?

A. a box plot
B. a scatterplot
C. a stemplot
D. a barchart
E. a histogram

The following information relates to questions 3 and 4.

The back-to-back ordered stem plot below shows the distribution of the number of cars sold by a car dealer on each day of trading during May and September.

May						Number of cars sold in a day	September							
			9	9	8	0								
	8	7	2	1	0	1								
9	9	8	6	3	2	2	3	4	5	5	7	8	8	9
	7	7	4	3	1	3	0	0	1	6	7	7	9	
				6	2	4	2	2	3					
						5								

Question 3

For how many days did the car dealer trade in May?

A. 18
B. 21
C. 24
D. 27
E. 31

Question 4

For the data shown it appears that compared to the number of cars sold on each day of trading during May, the number of cars sold on each day of trading during September is

A. on average greater and less variable
B. on average greater and more variable
C. on average less and less variable
D. on average less and more variable
E. negatively skewed

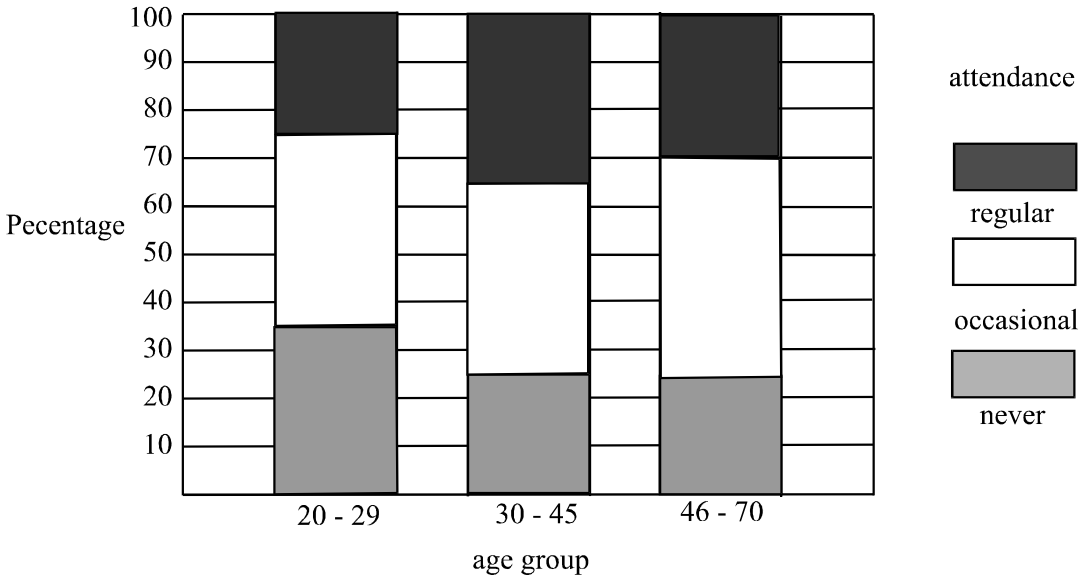
Question 5

The mass of packets of plain copy paper is bell-shaped with a mean of 2500g and a standard deviation of 50g. Out of 10 000 packets of this plain copy paper, we would expect that the number that have a mass greater than 2 550g would be closest to

A. 250
B. 500
C. 1 600
D. 3 200
E. 3 400

Question 6

The segmented bar chart below shows the distribution of attendances at AFL football games by women of different age groups for a very large sample of women.

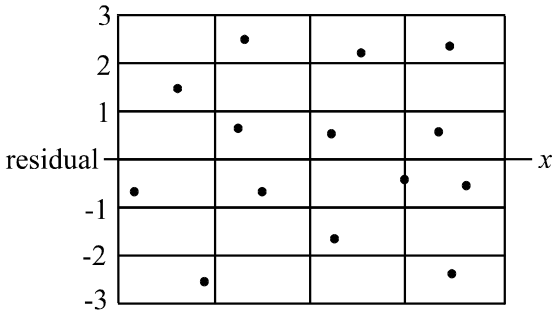


Of the women aged between 30-45, the percentage who regularly attend AFL football games is

A. 25%
B. 30%
C. 35%
D. 40%
E. 65%

Question 7

A regression line is fitted to a set of bivariate data displayed on a scatterplot. A residual plot is made and is shown below.

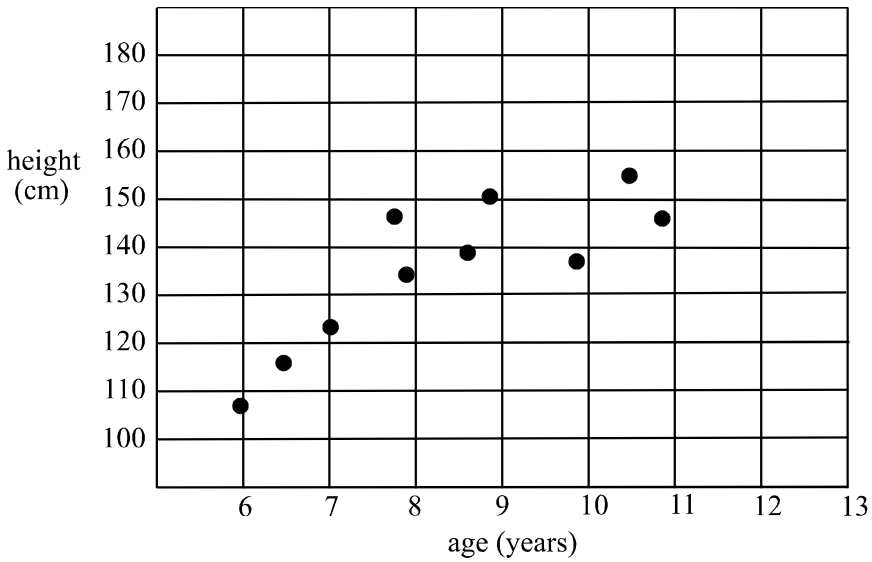


Which one of the following statements is **not true**?

A. There were twenty-eight pieces of data in the set of bivariate data.
B. The relationship between the two variables in the set of bivariate data is probably linear.
C. The residual value of the data when the regression line is fitted is not greater than 3.
D. The quality of fit of the regression line is good.
E. The size of the residuals indicate that the relationship between the two variables in the set of bivariate data is probably linear.

The following information relates to questions 8 to 10.

The scatterplot below displays the height and age of ten primary school children.



Question 8

For this data, the variables age and height are respectively

	(age)	(height)
A.	dependent	and numerical
B.	categorical	and discrete
C.	continuous	and dependent
D.	discrete	and continuous
E.	independent	and categorical

Question 9

From the scatterplot, we predict that Peason's product-moment correlation coefficient would be closest to

A. –0.9
B. –0.2
C. 0.1
D. 0.3
E. 0.7

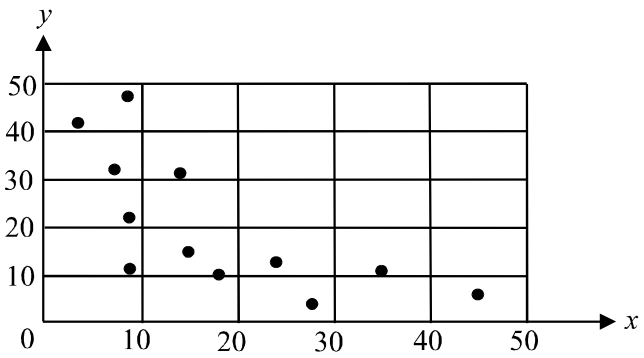
Question 10

A 3-median line is to be fitted to this data. The intercept on the vertical axis of this line is closest to

A. 100
B. 105
C. 110
D. 115
E. 120

Question 11

The scatterplot below displays the relationship between two variables x and y.



To attempt to linearise the data, you would be best advised to use a

A. $\frac{1}{x}$ or a $\log y$ transformation

B. $\log x$ or a y^2 transformation

C. $\log y$ or an x^2 transformation

D. y^2 or a $\frac{1}{y}$ transformation

E. x^2 or $\log x$ transformation

Question 12

The yearly sales of a fertilizer business over eight years are shown below.

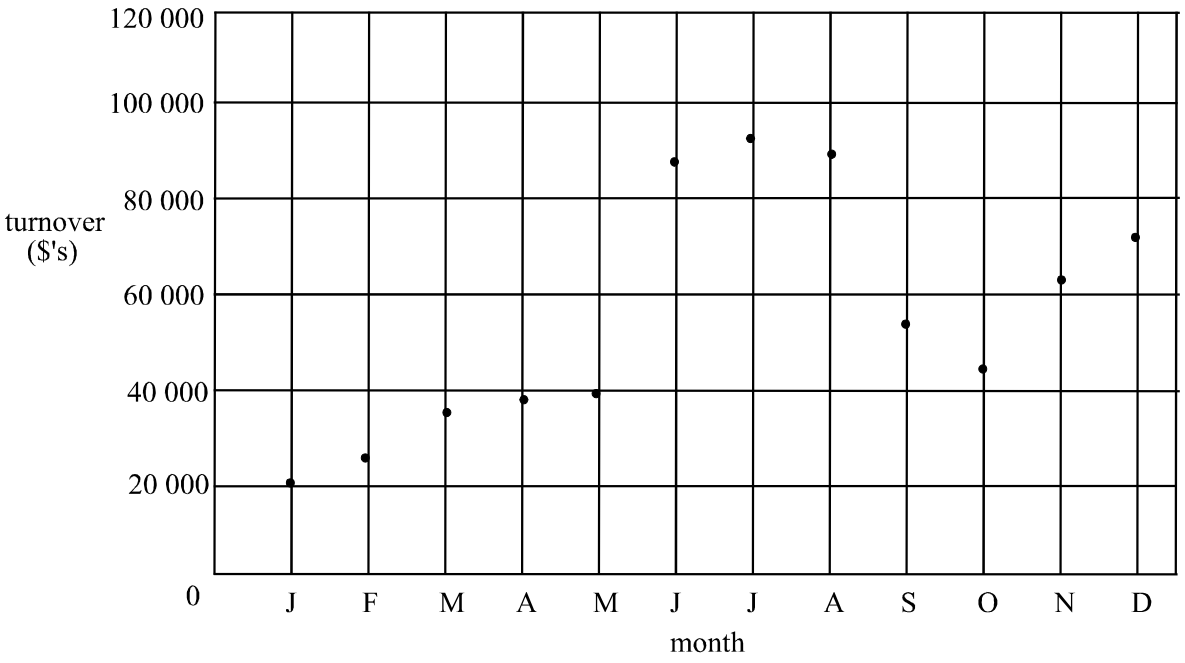
Year	Sales ($1000's)
1	52
2	55
3	57
4	51
5	61
6	59
7	62
8	65

A smoothed value for the sales in Year 4 using the two-term moving mean method with centring is closest to

A. 51
B. 52
C. 53
D. 55
E. 56

Question 13

A time series plot for the monthly turnover in a small business is shown below



A three median trend line was fitted to this data. After this had happened, it was discovered that the monthly turnover for May had been incorrectly recorded as $40 000.
The correct value was $75 000 and a new three median trend line was fitted.
Compared to the previous three median line, this new trend line would

A. have the same slope
B. be very much steeper
C. be a little steeper
D. be a lot less steep
E. be a little less steep.

Section B

Module 1: Number Patterns and Applications

If you choose this module all questions must be answered.

Question 1

For the sequence

8, 15, 22, 29, …

the twentieth term is

A. 134
B. 141
C. 148
D. 159
E. 166

Question 2

For a particular geometric sequence, $t_3 = 9$ and $t_2 = -3$. The values of *a* and *r* respectively of this sequence are

A. $a = -1$ and $r = -3$
B. $a = 1$ and $r = -3$
C. $a = -1$ and $r = -\frac{1}{3}$
D. $a = 1$ and $r = -\frac{1}{3}$
E. $a = \frac{1}{3}$ and $r = -\frac{1}{3}$

Question 3

At a country vet practice the vets see farm animals and domestic animals. The ratio of farm animals compared to domestic animals seen by the vets is 8:5.
In a typical week the vets would see 120 domestic animals.
How many animals in total would the vets typically see in a week?

A. 15
B. 24
C. 156
D. 195
E. 312

Question 4

Muriel began her collection of porcelain dolls when she inherited her grandmother's collection. Each year after that Muriel purchased six to add to her collection. Four years after her collection began she had twenty five porcelain dolls. How many porcelain dolls will Muriel have in her collection thirteen years after beginning her collection?

A. 67
B. 73
C. 75
D. 78
E. 79

The following information relates to Questions 5 and 6.

As part of a school's purchasing policy, the number of books in the school library increases each year by 5%. At the end of each year two hundred old or damaged books are removed from the library. The number of books in the school library in the year prior to the policy being implemented was 10 000.

Question 5

How many books were in the school library after the second year of the policy being implemented?

A. 10 300
B. 10 615
C. 10 825
D. 10 946
E. 11 025

Question 6

A difference equation that describes the number of books in the library from the year prior to the policy being implemented is

A. $t_{n+1} = 0 \cdot 05t_n - 200$ where $t_1 = 10\,000$
B. $t_{n+1} = 0 \cdot 05(t_n - 200)$ where $t_1 = 10\,000$
C. $t_{n+1} = 1 \cdot 05t_n - 200$ where $t_1 = 10\,000$
D. $t_{n+1} = 1 \cdot 05(t_n - 200)$ where $t_1 = 10\,000$
E. $t_{n+1} = 5(t_n - 200)$ where $t_1 = 10\,000$

Question 7

A sequence is defined by the difference equation

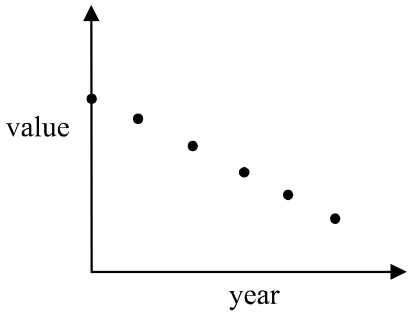
$$t_{n+1} = t_n + 6 \text{ where } t_1 = 5 .$$

The n^{th} term of the sequence is given by t_n where

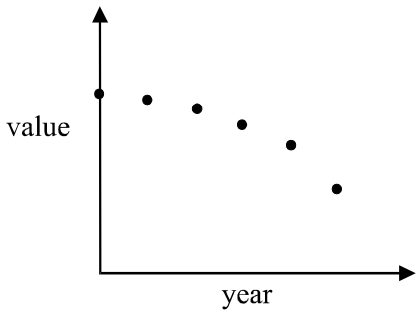
A. $t_n = n + 6$
B. $t_n = n + 5$
C. $t_n = 6n$
D. $t_n = 6n - 1$
E. $t_n = 6n + 4$

Question 8
A garden hose manufacturer reduces the production of its top of the range hose by 15% each year. Which one of the following graphs best reflects the change in the number of top of the range hoses produced by the manufacturer?

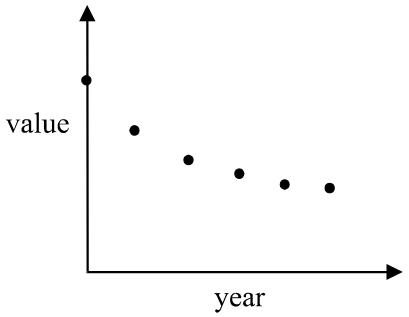
A.



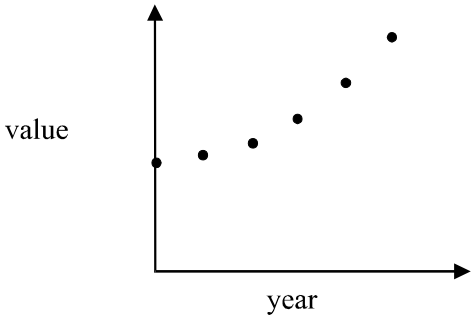
B.



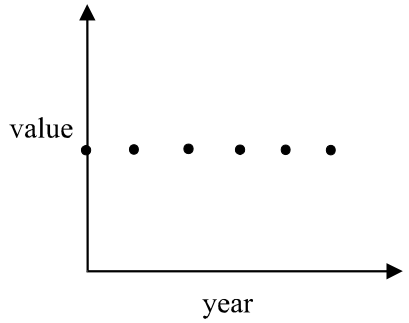
C.



D.



E.



Question 9

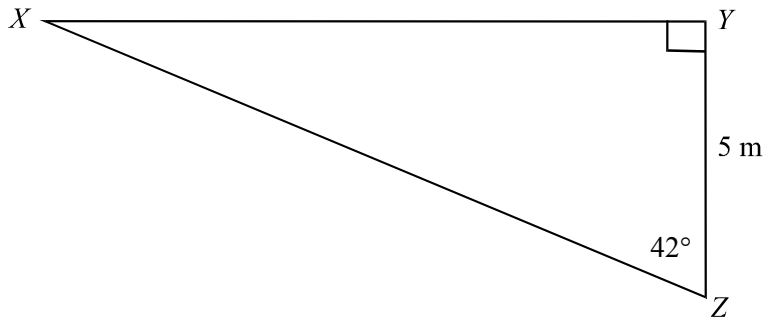
In 2001, the number of cars entering a particular side street was 14 600. The local council has estimated that this will increase on average by 3.5% per annum. According to this estimate the number of cars that will have entered the side street during the years 2001 – 2006 is closest to

A. 15 130
B. 78 292
C. 91 464
D. 95 632
E. 210 801

Module 2: Geometry and Trigonometry

If you choose this module all questions must be answered.

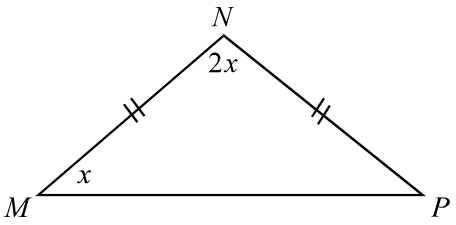
Question 1



In triangle *XYZ*, the length of *XY,* in metres, is closest to

A. 0.15
B. 2.25
C. 3.3
D. 3.7
E. 4.5

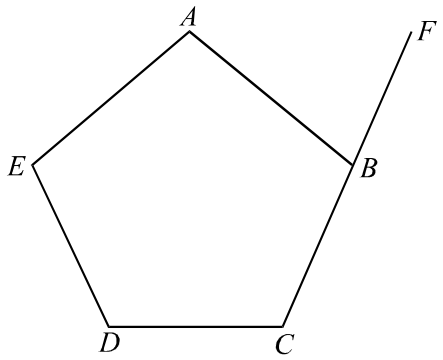
Question 2



In triangle *MNP* the value of *x* is

A. 20°
B. 30°
C. 35°
D. 45°
E. 60°

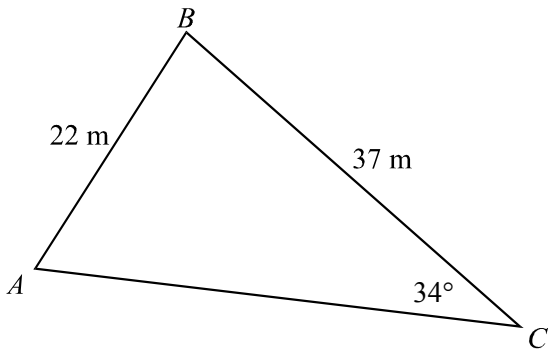
Question 3



ABCDE is a regular pentagon and *B* lies on *CF*. The size of $\angle ABF$ is exactly

A. 60°
B. 72°
C. 84°
D. 100°
E. 108°

Question 4

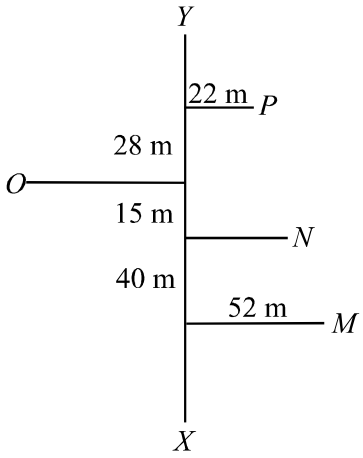


The size of angle *BAC* is closest to

A. 1° 13’
B. 10° 25’
C. 38° 12’
D. 60° 28’
E. 70° 8’

Question 5

The field sketch from a traverse survey of a proposed carpark was conducted with the baseline *XY*.

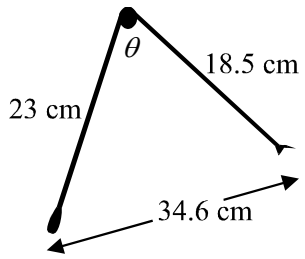


Trees on the site are indicated at the points *M, N, O* and *P*. The straight line distance, in metres, between the trees at *P* and *M* is closest to

A. 77.4
B. 78.9
C. 88.3
D. 94.3
E. 113

Question 6

A pair of broken kitchen tongs has one arm of length 23cm and the other of length 18.5cm. When fully open, the distance between the two ends of the tongs is 34.6 cm.



The angle, θ, which is between the arms of the tongs when they are fully open is closest to

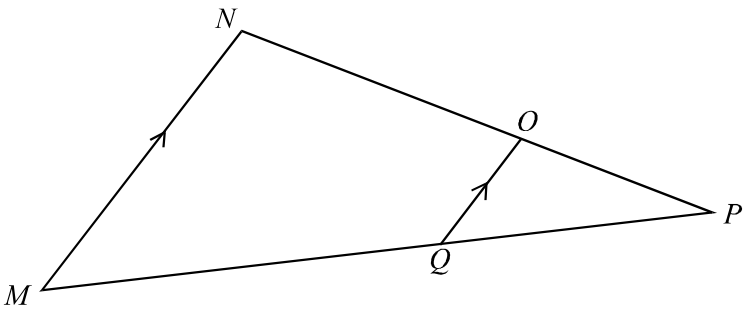
A. 23°
B. 67°
C. 102°
D. 113°
E. 140°

Question 7

A distance of 7 cm on a map corresponds to an actual distance on the ground of 1.75 km. The scale used on the map is

A. 1:0.25
B. 1:250
C. 1:2 500
D. 1:25 000
E. 1:250 000

Question 8

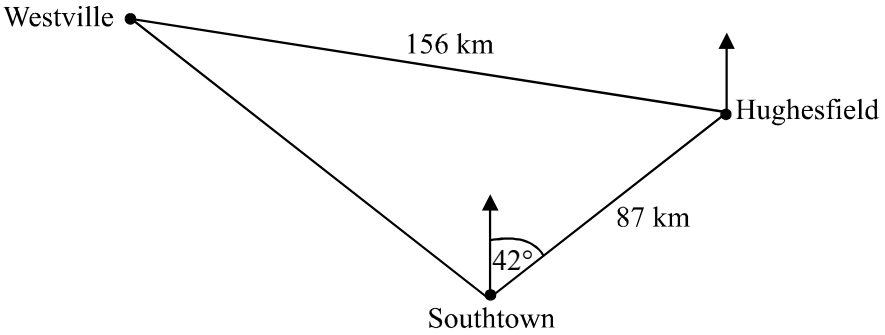


Triangle *MNP* is similar to triangle *QOP*. The area of triangle *MNP* is $108\,\text{cm}^2$ and the area of triangle *QOP* is $12\,\text{cm}^2$.

The ratio *PQ* : *PM* is

A. 1:3
B. 1:4
C. 1:9
D. 2:3
E. 2:5

Question 9



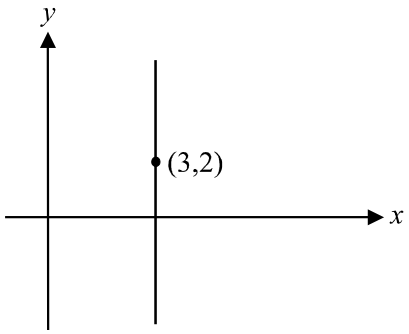
The bearing of Hughesfield from Southtown is 042° and they are located 87 km apart. The bearing of Westville from Hughesfield is 305° and they are located 156 km apart.
How many kilometres (to the nearest km) is Westville north of Southtown?

A. 148
B. 154
C. 186
D. 192
E. 301

Module 3: Graphs and relations

If you choose this module all questions must be answered.

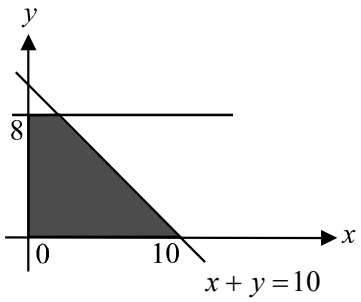
Question 1



The equation of the vertical line shown above which passes through the point (3,2) is

A. $x = 3$
B. $y = 2$
C. $y = \frac{3x}{2}$
D. $y = 3x + 2$
E. $x + y = 5$

Question 2



The region shaded in the diagram above has its boundaries included. The set of inequalities which defines this region is

A. $x \geq 0,\ y \geq 0,\ x \leq 8,\ x + y \leq 10$
B. $x \geq 0,\ y \geq 0,\ x \leq 8,\ x + y \geq 10$
C. $x \geq 0,\ y \geq 0,\ y \leq 8,\ x + y \leq 10$
D. $x \geq 0,\ y \geq 0,\ y \leq 8,\ x + y \geq 10$
E. $x \geq 0,\ y \geq 0,\ y \leq 10,\ x + y \geq 10$

Question 3

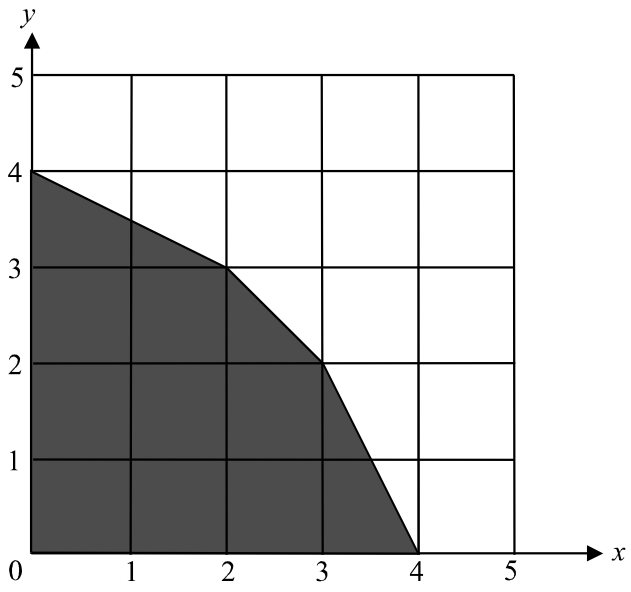
The pair of simultaneous equations

$$2x + 1 = -y$$
$$\text{and} \quad 3x + 2y = -3$$

has the solution

A. $x = -3, \ y = -1$
B. $x = -3, \ y = 1$
C. $x = -1, \ y = 1$
D. $x = 1 \ \ y = -3$
E. $x = 1, \ y = 3$

Question 4



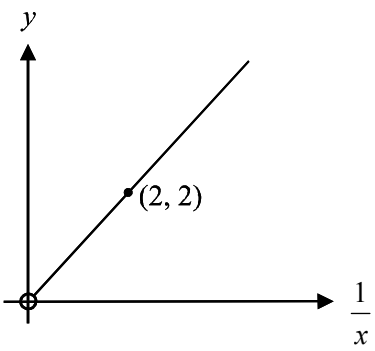
The feasible region for a linear programming problem is indicated by the shaded region on the diagram above. The objective function is $P = 4x + y$.
The maximum value of P is

A. 4
B. 11
C. 14
D. 16
E. 17

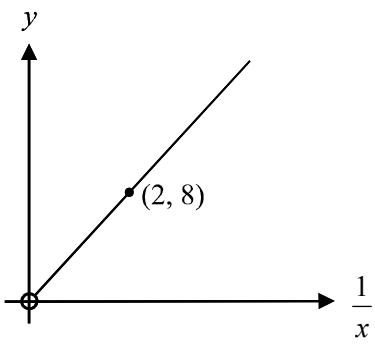
Question 5

The graph of the relation $y = \frac{2}{x}$, for $x > 0$, could be

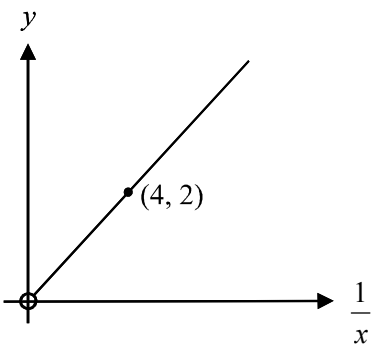
A.



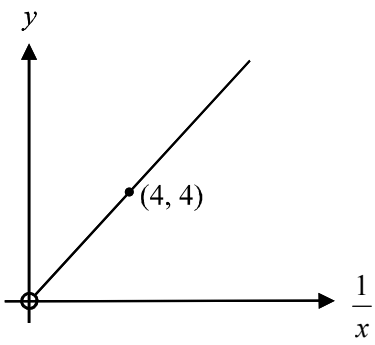
B.



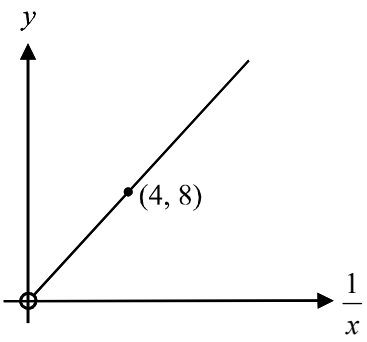
C.



D.



E.



Question 6

Paul delivers advertising brochures and local papers to houses in his area each week.

Let x equal the number of advertising brochures he delivers in a week.
Let y equal the number of local papers he delivers in a week.

The weight of an advertising brochure is 100 grams and the weight of a local paper is 180 grams.
The size of his trolley and the number of trips he can make in a week mean that Paul can deliver up to 90 000 gram (90kg) of material per week.
Also, in his contract he must deliver a minimum of 200 local newspapers in a week.
The constraints, that describe the number of advertising brochures and local papers Paul can deliver in a week, are given by

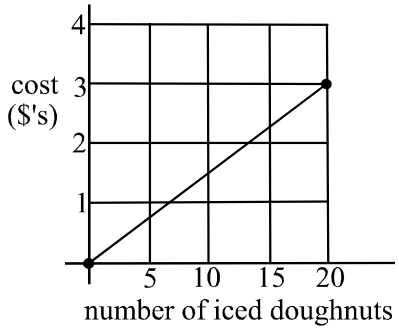
A. $x \geq 0,\ y \geq 0,\ 100x + 180y \leq 90\,000$
B. $x \geq 0,\ y \geq 200,\ 100x + 180y \leq 90\,000$
C. $x \geq 10,\ y \geq 200,\ 180x + 100y \leq 90\,000$
D. $x \geq 180,\ y \geq 0,\ 180x + 100y \geq 90\,000$
E. $x \geq 180,\ y \geq 200,\ 100x + 200y \geq 90\,000$

Question 7

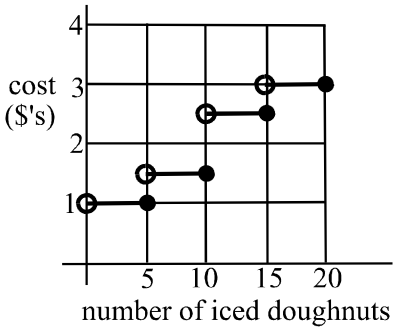
At a supermarket, iced doughnuts are sold in packets of 5 or 10. The packet of 5 costs $1.00 and the packet of 10 costs $1.50.

Which of the graphs below shows the cheapest way of buying any number of iced doughnuts up to 20?

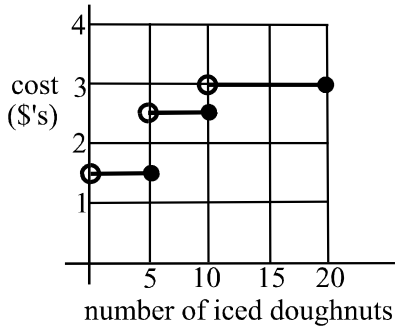
A.



B.



C.



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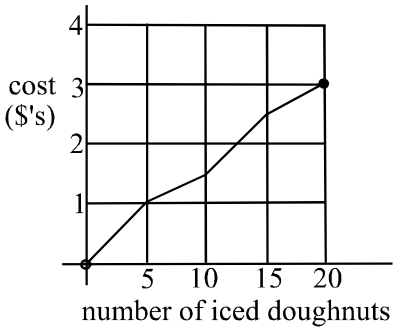

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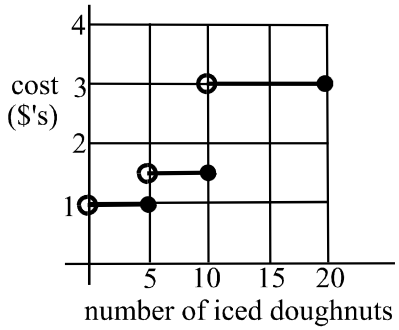
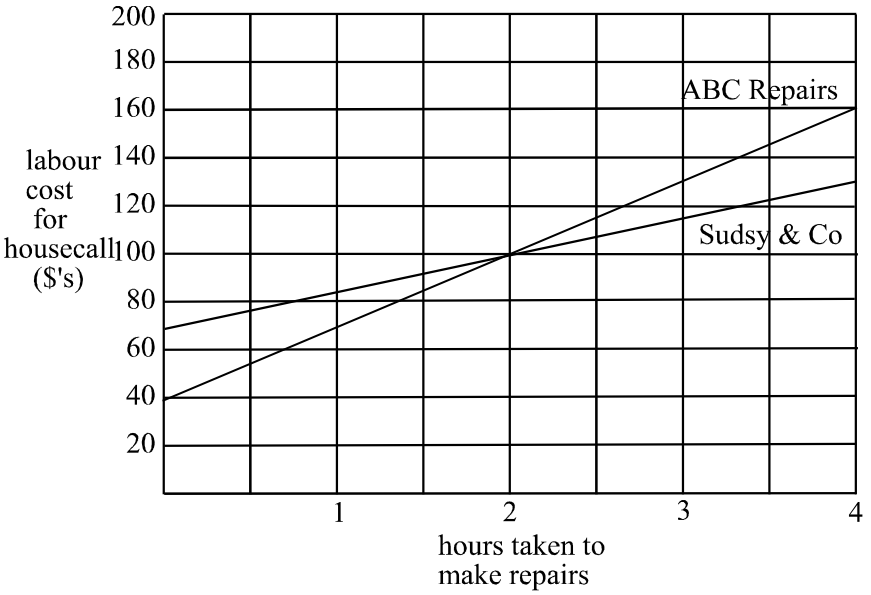


The information below relates to Questions 8 and 9.

Joyce rings two businesses about having her washing machine repaired.
For labour only, the businesses charge a call-out fee plus a fee per hour for the time it takes to make the repair.

The graph below shows the labour cost of each of the companies, ABC repairs and Sudsy and Co., for a house call.



Question 8

ABC Repairs charges a call-out fee of $40 plus $30 per hour.
Sudsy and Co charges a call-out fee of

A. $15 plus $70 per hour
B. $40 plus $15 per hour
C. $40 plus $30 per hour
D. $70 plus $15 per hour
E. $70 plus $30 per hour

Question 9

If the repair takes between two to three hours, and assuming Joyce would choose the cheaper option, then the least amount that she would pay for labour charges is between

A. $40 and $100
B. $70 and $100
C. $100 and $115
D. $100 and $130
E. $100 and $160

Module 4: Business-related mathematics

If you choose this module all questions must be answered.

Question 1

Robyn invested $2 500 in an account that earned simple interest of 4% per annum. After 4 years, Robyn's investment was worth

A. $100
B. $400
C. $2500
D. $2750
E. $2900

Question 2

Bob bought some machinery valued at $1800 for his business. Bob intends replacing the machinery when it has a depreciated to a value of $150.
If the machinery depreciates according to the straight line method of depreciation at $275 per annum, in how many years will Bob have to replace it?

A. 6
B. 6.5
C. 9.5
D. 10
E. 12

The information below relates to questions 3 and 4.

James invests $9 500 in an account earning compound interest of 6% per annum compounding quarterly.

Question 3

After 2 years, James' investment is worth

A. $9933.94
B. $9787.14
C. $10543.53
D. $10701.68
E. $12563.75

Question 4

How much interest did James earn in the ninth quarter of the investment?

A. $36.50
B. $160.52
C. $446.74
D. $908.49
E. $1362.20

Question 5

Kate invests $5 000 for 3 years in an account earning compound interest of 4% per annum compounding annually.

Her brother Pete invests the same amount of money for the same time in an account earning simple interest. Given that Kate and Pete earn the same amount of interest on their investments. What annual rate of interest did Pete obtain?

A. 2.4%
B. 2.66%
C. 3.75%
D. 4.16%
E. 37.5%

Question 6

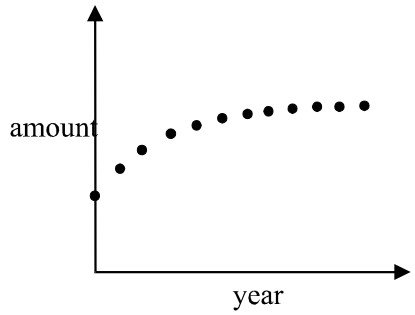
Jane has entered a loan agreement whereby she pays $2 400 every six months for 5 years. The amount Jane borrowed was $19 000. The effective rate of interest per annum that Jane is paying on this loan is closest to

A. 5.3%
B. 9.6%
C. 13.2%
D. 26%
E. 48%

Question 7

George invests a one off amount of $15 000 in an account that earns interest which compounds each year.
The amount of George's investment after a few years is best shown by

A.



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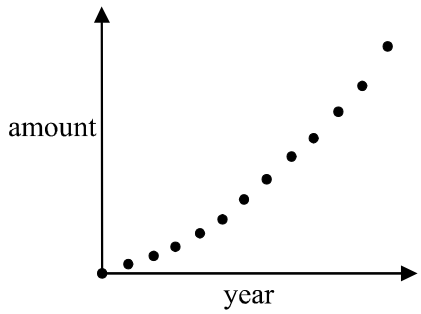
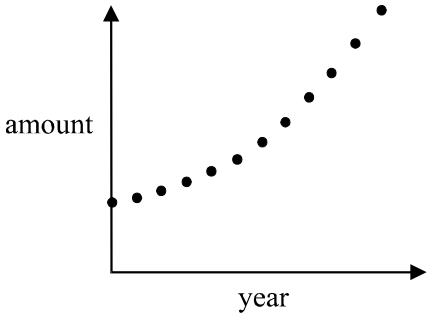
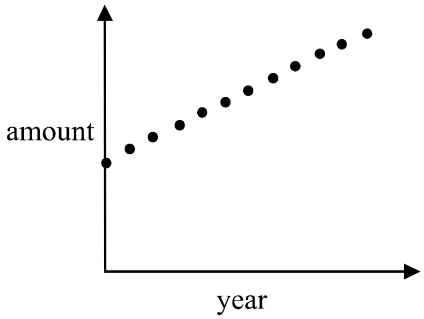


C.



D.



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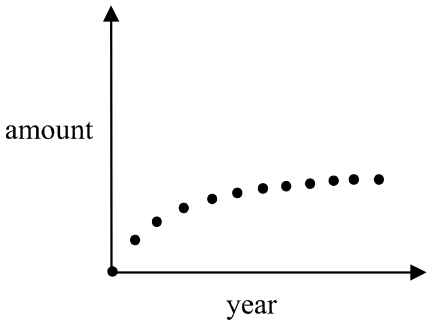


Question 8

Tonia takes out a reducing balance loan of $12 000 with interest compounding quarterly. She makes repayments of $500 each quarter. Tonia's first statement for this account is shown below.

Date	Amount Repaid	Interest Charged	Amount Owing
01 Oct	0	0	12 000
24 Dec	500	0	11 500
31 Dec	0	230	11 730
23 Mar	500	0	11 230
31 Mar	0	224.60	11005.40

The annual interest rate charged for this loan is closest to

A. 1.96%
B. 2%
C. 4.6%
D. 7.8%
E. 8%

Question 9

The following equation gives the annuities formula for a loan of $40 000 which has compounding interest of 9% per annum. The amount of the periodic repayment Q, can be calculated for this loan, using the following formula

$$0 = 40\,000(1 \cdot 0225)^{24} - \frac{Q(1 \cdot 0225^{24} - 1)}{(1 \cdot 0225 - 1)}$$

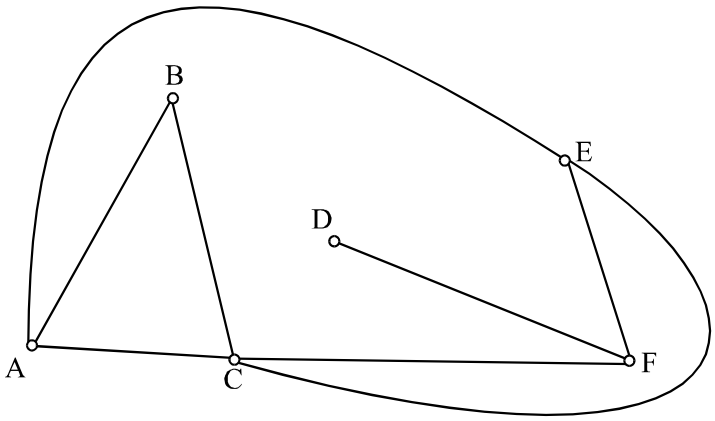
This calculation assumes that the loan will be paid off after

A. 2.25 years
B. 4 years
C. 6 years
D. 12 years
E. 24 years

Module 5: Networks and decision mathematics

If you choose this module all questions must be answered.

Question 1

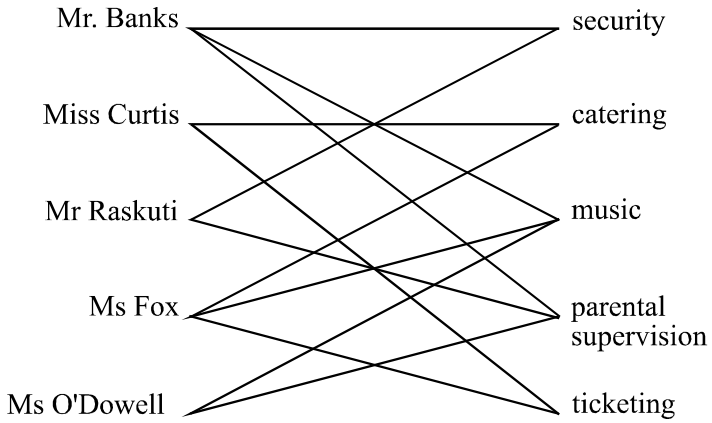


Which one of the following shows a Hamiltonian path for the graph above?

A. ABCFDFEA
B. ABCFDFE
C. ABCFECFD
D. ABCEFD
E. AECFD

Question 2

Five teachers are on a committee to organise a school disco. The graph below shows the teachers and the areas they are prepared to coordinate.



If each teacher coordinates one area, which one of the following assignments will **not** be acceptable?

A.

Mr Banks	Security
Miss Curtis	Catering
Mr Raskuti	Parental supervision
Ms Fox	Ticketing
Ms O'Dowell	Music

B.

Mr Banks	Security
Miss Curtis	Catering
Mr Raskuti	Ticketing
Ms Fox	Music
Ms O'Dowell	Parental Supervision

C.

Mr Banks	Music
Miss Curtis	Ticketing
Mr Raskuti	Security
Ms Fox	Catering
Ms O'Dowell	Parental Supervision

D.

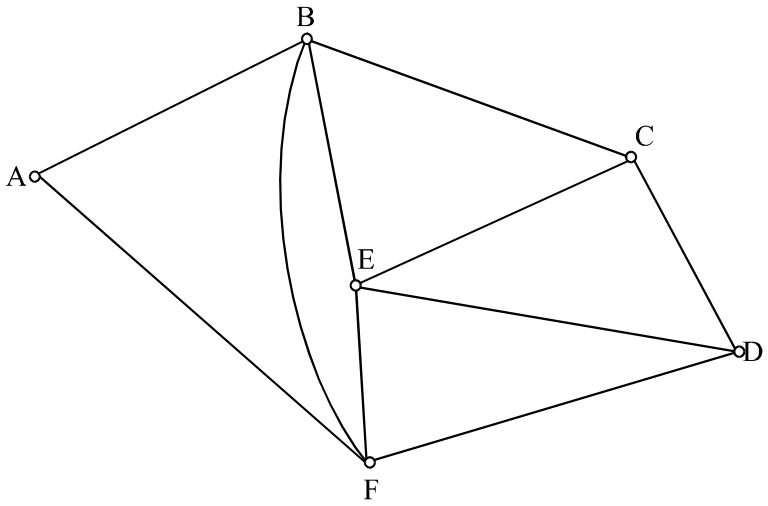
Mr Banks	Music
Miss Curtis	Catering
Mr Raskuti	Security
Ms Fox	Ticketing
Ms O'Dowell	Parental supervision

E.

Mr Banks	Parental Supervision
Miss Curtis	Ticketing
Mr Raskuti	Security
Ms Fox	Catering
Ms O'Dowell	Music

Question 3

To create an Euler circuit on the graph below an arc needs to be added.



That arc is

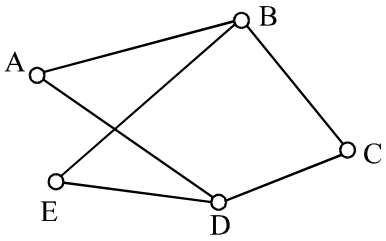
A. A C
B. A F
C. B E
D. B D
E. C D

Question 4

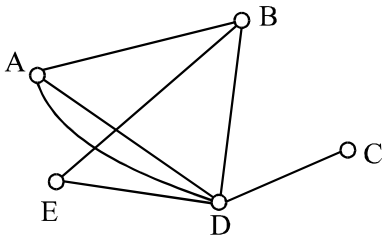
$$\begin{bmatrix} 0 & 1 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Which one of the following graphs could have the adjacency matrix above?

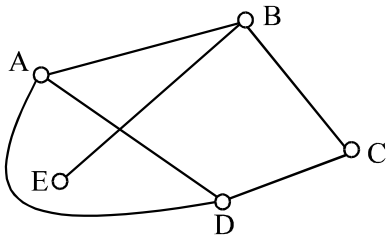
A.



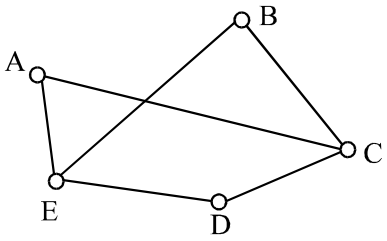
B.



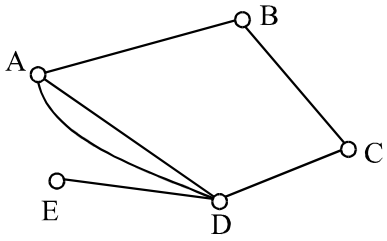
C.



D.



E.



Question 5

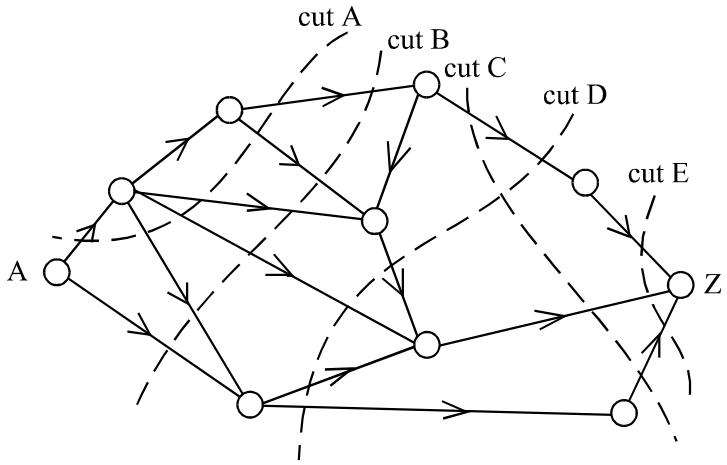
The formula

$$v + f = e + 2$$

is Euler's formula and describes the relationship between the number of edges, vertices and regions for some graphs. For Euler's formula to apply, a graph must be connected and

A. undirected
B. be a circuit
C. be a spanning tree
D. planar
E. complete.

Question 6

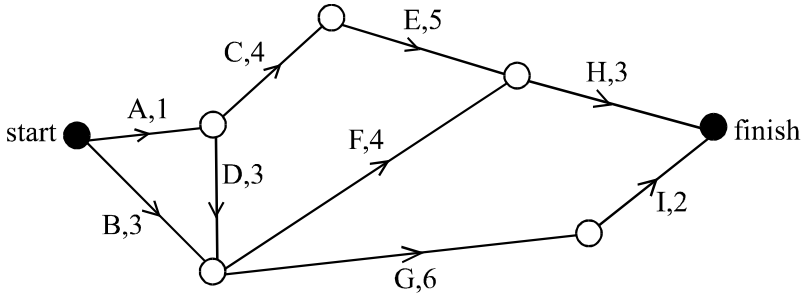


For the network above, which of the cuts shown does not enable us to measure a flow from A to Z

A. cut A
B. cut B
C. cut C
D. cut D
E. cut E

The information below relates to questions 7, 8 and 9.

A project involves nine tasks, A – I being completed.
The network below shows these tasks together with the time it will take in days to complete each task.



Question 7
Which of the tables below correctly shows the correct predecessor(s) for each of the tasks?

A.

Task	Immediate Predecessor(s)
A	-
B	-
C	A, B
D	A, B
E	C
F	A, B, D
G	B, D
H	E,F
I	B, G

B.

Task	Immediate Predecessor(s)
A	-
B	-
C	A
D	A
E	C
F	B, D
G	B, D
H	E, F
I	G

C.

Task	Immediate Predecessor(s)
A	-
B	-
C	A
D	A
E	A, C
F	A, B, D
G	B, D
H	E,F
I	G

D.

Task	Immediate Predecessor(s)
A	B
B	A
C	A
D	A
E	A, C
F	B, D
G	B, D, F
H	E,F
I	G

E.

Task	Immediate Predecessor(s)
A	-
B	-
C	A
D	A
E	C
F	B, D
G	B, D, F
H	E,F
I	G

Question 8

The minimum number of days in which this project can be completed is

A. 9
B. 10
C. 11
D. 12
E. 13

Question 9

If the time it takes to complete a task could be reduced by 1 day, which one of the following tasks would you reduce in order to complete the project one day earlier?

A. task B
B. task D
C. task E
D. task F
E. task G

Further Mathematics Formulas

Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

annuities: $A = PR^n - \dfrac{Q(R^n - 1)}{R - 1}$, where $R = 1 + \dfrac{r}{100}$

Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bh$

area of a triangle: $\dfrac{1}{2}bc\sin A$

area of circle: πr^2

volume of sphere: $\dfrac{4}{3}\pi r^3$

volume of cone: $\dfrac{1}{3}\pi r^2 h$

Pythagoras' theorem $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Graphs and relations

Straight line graphs

gradient: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y - y_1 = m(x - x_1)$ gradient-point form

$y = mx + c$ gradient-intercept form

$\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$ two-point form

Number patterns and applications

arithmetic series: $a + (a+d) + \ldots + (a+(n-1)d) = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}, \quad r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r}, \quad |r| < 1$

linear difference equations: $t_n = at_{n-1} + b = a^{n-1}t_1 + b\frac{(a^{n-1}-1)}{a-1}, \quad a \neq 1$

$$= a^n t_0 + b\frac{(a^n - 1)}{a-1}$$

Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Statistics

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

FURTHER MATHEMATICS

TRIAL EXAMINATION 1

MULTIPLE- CHOICE ANSWER SHEET

STUDENT NAME:..

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: A ■ C D E

The answer selected is B. Only one answer should be selected.

Section A - Core

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E

Section B - Modules

Module Number ___

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E

Module Number ___

1. A B C D E
2. A B C D E
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4. A B C D E
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9. A B C D E