



The Mathematical Association of Victoria

2001

FURTHER MATHEMATICS

Trial Examination 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Directions to students

This examination consists of **Core** material and five **Modules**.

Answer **all** of the **Core** questions.

Select three modules and answer **all** questions in each of your selected modules.

All working and answers should be written in the spaces provided.

The marks allotted to each part of each question appear at the end of each part.

There are **60 marks** available for this task.

A formula sheet is attached.

These questions have been produced to assist students in their preparation for the 2001 Further Mathematics Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority (VCAA) Assessing Panels. The Association gratefully acknowledges the permission of the VCAA to reproduce the formula sheet.

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Further Mathematics Formulas

Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\frac{2n}{n+1} \times \text{flat rate}$

annuities: $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$, where $R = 1 + \frac{r}{100}$

Geometry and trigonometry

area of a triangle: $\frac{1}{2}bh$

area of a triangle: $\frac{1}{2}bc \sin A$

area of circle: r^2

volume of sphere: $\frac{4}{3}r^3$

volume of cone: $\frac{1}{3}r^2h$

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Graphs and relations

Straight line graphs

gradient: $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y - y_1 = m(x - x_1)$ gradient-point form

$y = mx + c$ gradient-intercept form

$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ two-point form

Number patterns and applications

arithmetic series: $a + (a + d) + \dots + (a + (n - 1)d) = \frac{n}{2} [2a + (n - 1)d] = \frac{n}{2} (a + l)$

geometric series: $a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \dots = \frac{a}{1 - r}, |r| < 1$

linear difference equations: $t_n = at_{n-1} + b = a^{n-1} t_1 + b \frac{(a^{n-1} - 1)}{a - 1}, a \neq 1$
 $= a^n t_0 + b \frac{(a^n - 1)}{a - 1}$

Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Statistics

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

END OF FORMULA SHEET

Specific instructions

This paper consists of a core and five modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

Core : Data analysis

Question 1

A group of smokers consisting of 15 males and 15 females were asked to record how many cigarettes they had smoked on a particular day.

The data (in raw form) is as follows

Males	9	18	29	46	8	22	3	8	27	5	10	3	12	14	6
Females	8	12	27	33	23	13	7	9	16	1	19	18	19	20	21

In an attempt to present the data in a more useful form for analysis, a back to back stemplot was produced.

- a. Complete the following sentence:

This form of displaying data has been chosen as number of cigarettes smoked is a numerical variable and gender (male/female) is a variable.

1 mark

- b. The data for males was entered by Joe into the stemplot below. **In order**, enter the data for the females.

	Male		Female
	9 8 8 6 5 3 3	0	
	8 4 2 0	1	
	9 7 2	2	
		3	
	6	4	

1 mark

- c. Write a brief sentence comparing the **shapes** of the two distributions.

1 mark

- d. Calculate the median for the group of females.

1 mark

It can be seen that the highest observation of 46 in the male group is considerably higher than the others. Joe decided to perform a test to decide if this was an outlier. He calculated that the lower quartile was 6 and the upper quartile was 22.

- e. Determine the interquartile range for the male group.

1 mark

- f. Considering the result from e. , determine whether 46 is an outlier.

2 marks

Question 2

A second group of 90 people were surveyed as to whether they believed smoking should be allowed in casinos. The results are displayed in the following two way frequency table to determine if gender differences occur.

Attitude to smoking	Gender	
	Female	Male
For smoking	32	
Against smoking	17	19

- a. Which of the 2 variables (attitude to smoking or gender) is the independent variable ? Explain your answer briefly.

1 mark

- b. Fill in the missing value in the previous table.

1 mark

In order to decide whether there is a relationship between attitude to smoking and gender, column percentages are to be calculated.

- c. Calculate the required percentages, correct to 1 decimal place, and enter in the table below.

Attitude to smoking	Gender	
	Female	Male
For smoking		
Against smoking		
	100%	100%

1 mark

- d. Is there a relationship between attitude to smoking and gender. Explain your answer with reference to the table in part c.

2 marks

Question 3

A group of casino patrons was studied in an attempt to determine the average amount of money lost over a period of time. An equation using their data was found to be

$$M = 5.02 \times H - 2.65$$

where M represents money lost in dollars
H represents hours gambled

- a. Using the equation, predict to the nearest dollar the loss incurred by a patron gambling for 2 hours.

1 mark

- b. Complete the following sentence.

As the number of hours gambled increases by 1, money lost increases by

1 mark

- c. Explain the significance (if any) of -2.65 in the equation $M = 5.02 \times H - 2.65$

1 mark

Total: 15 marks

Module 1 : Number patterns and applications

Regular maintenance of an underground road tunnel has revealed a crack that is leaking water.

Question 1

Dave, the maintenance person, has monitored the leak for three hours and has collected the following amounts, in litres, for each of the first three hours :

1 st hour	2 nd hour	3 rd hour
1.8	2.4	3

Assume that the amount leaking from the crack continues to increase in this manner.

- a. How much water will leak out in the fifth hour?

1 mark

- b. Write a rule for L_n , the amount leaking from the crack in the n^{th} hour after monitoring began, in terms of n .

2 marks

- c. The tunnel will have to be closed if the amount leaking is 1 litre per minute (60 litres per hour) or more. In which hour, after Dave started monitoring, would the amount leaking from the crack first exceed 1 litre per minute?

2 marks

- d. How much water, in total, will have leaked from the crack in the 24 hours after monitoring began?

2 marks

Question 2

Dave tells Tony, the foreman, about the leak and Tony decides to do his own monitoring. Over the next three hours Tony collects the following amounts, in litres.

1 st hour	2 nd hour	3 rd hour
5	7	9.8

Use Tony's values and assume that the pattern continues.

- a. Calculate the percentage increase in the amount leaking from the 1st to the 2nd hour.

1 mark

- b. Find the value of a if the amount leaking, T_n , in the n^{th} hour after Tony started monitoring, is represented by the difference equation $T_n = aT_{n-1}$; $T_1 = 5$

1 mark

- c. Find the hour, after Tony started monitoring, in which the amount leaking will first exceed 1 litre per minute.

2 marks

- d. Find the total amount that will leak from the crack in the 10 hours after Tony started monitoring. Give your answer to the nearest litre.

2 marks

Question 3

Tony decides to consult the engineer, Deidre.

Deidre also does her own monitoring of the amount leaking from the crack and collects the following amounts, in litres :

1 st hour	2 nd hour	3 rd hour
20	17	14.45

Deidre concludes that the amount leaking is decreasing and that no action needs to be taken to stop the leak.

If Deidre’s pattern continues, find the total volume of water that will leak from the crack from the time Deidre starts monitoring. Give your answer to the nearest litre.

2 marks

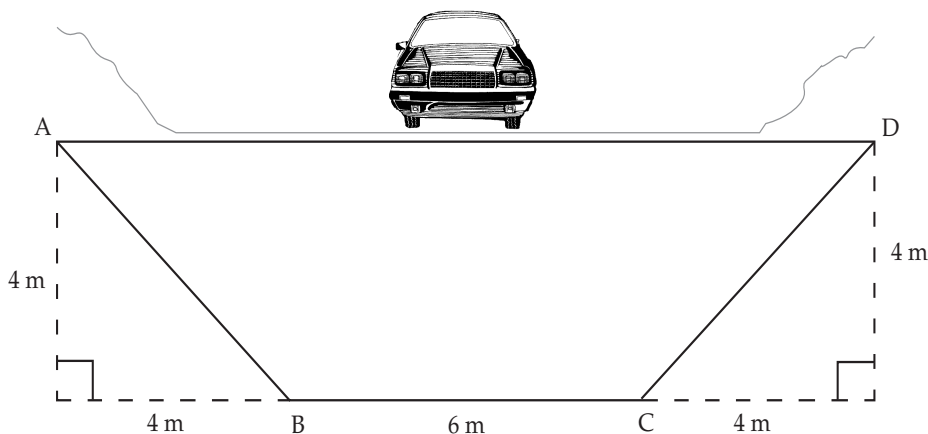
Total 15: marks

Module 2 : Geometry and trigonometry

The 'Trygo' Golf Course is being redeveloped requiring a number of surveying tasks.

Question 1

A pedestrian tunnel for the golfers is planned to go under a roadway to connect two of the holes on the course. The tunnel is serviced by two ramps, one at each end. A simplified diagram of the proposed tunnel is shown below.



- a. What is the angle of depression for the ramp?

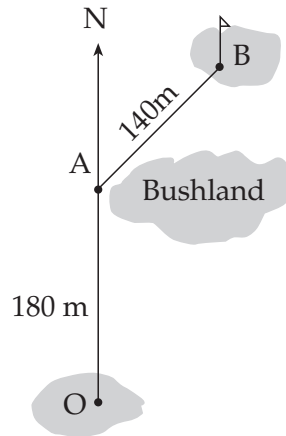
1 mark

- b. What is the total length of the tunnel as given by lengths \overline{AB} , \overline{BC} and \overline{CD} (to 1 decimal place)?

2 marks

Question 2

The layout of the 2nd hole at 'Trygo' Golf Course is shown below.



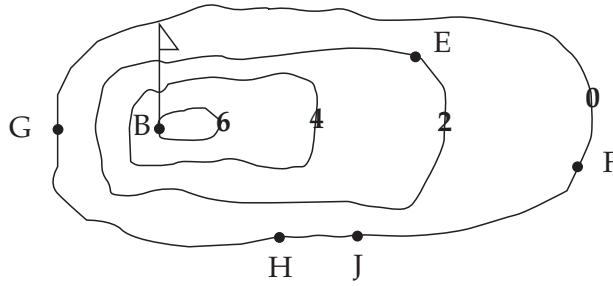
- a. If the second shot from A to B is on a bearing of $N 27^\circ E$, give the value of the obtuse angle at A.

1 mark

- b. Trigger Woods wishes to play a single shot from tee (O) to green (B). From the diagram, calculate the length of the shot (to the nearest metre) and its bearing from O (to the nearest degree).

4 marks

A contour map of the above hole is shown below.



- c. From which position (E, F, G, H or J) is the steepest slope to the top of the green? Explain why.

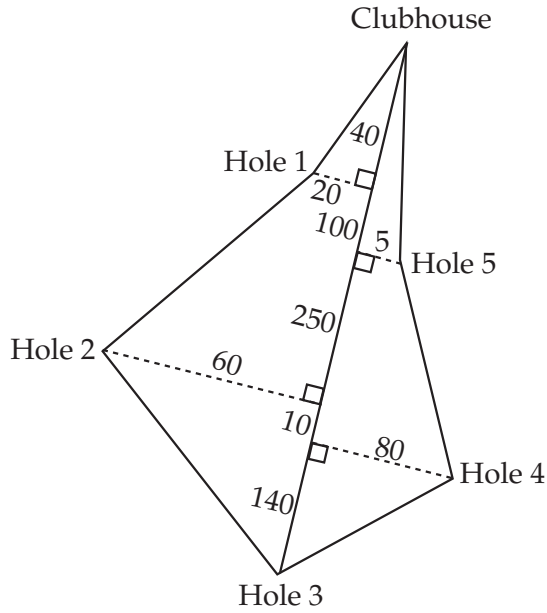
1 mark

- d. If the horizontal distance from E to B is 15 metres, state the gradient from E to B.

1 mark

Question 3

The region bounded by the Clubhouse and the first five holes is shown below.



- a. Complete the surveyor’s notes below using traverse surveying.

Clubhouse
0
540
Hole 3

2 marks

- b. If Hole 3 to Clubhouse is on N 10° E traverse line, calculate the bearing from Hole 3 to Hole 2 (to the nearest degree).

3 marks

Total: 15 marks

Module 3 : Graphs and relations

Question 1

Moca's Enterprises is planning to bring a new graphing calculator on to the market. The cost of production in any week consists of a fixed cost of \$10 000 plus a cost of \$40 per calculator. The owner of Moca's Enterprises constructs a rule which represents the cost per week (C) in terms of the number produced (n).

He writes the rule in the form $C = an + b$

- a. Write down the values of a and b .

1 mark

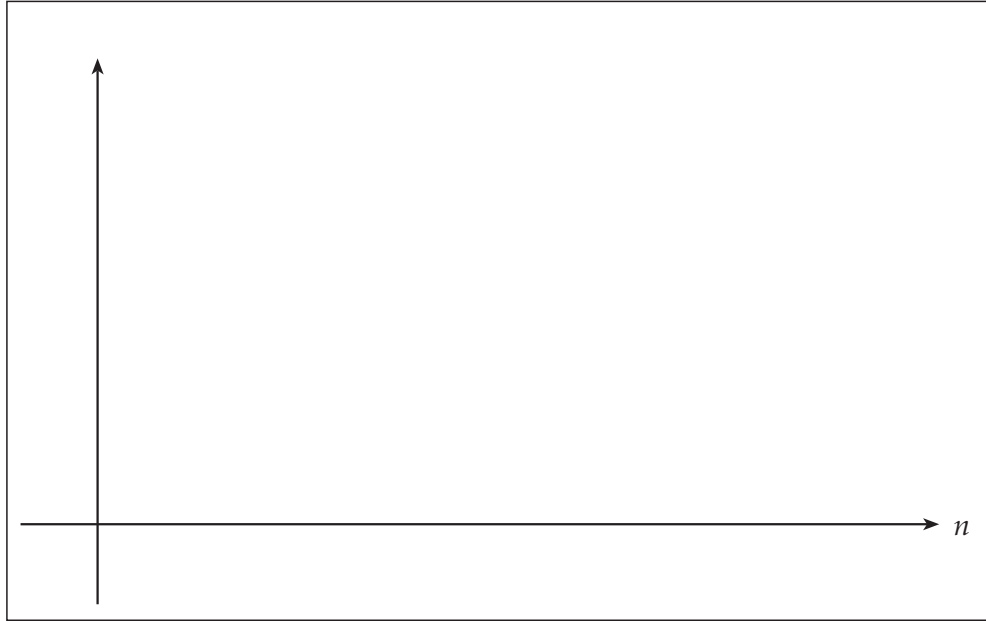
- b. Calculate the cost in a week when 140 calculators were produced.

1 mark

- c. In one particular week, the cost of production was \$22 600. How many calculators were produced?

1 mark

d. Using your previous results or otherwise, sketch a graph of C versus n



2 marks

The selling price of each calculator is \$140

e. Write an expression for the revenue received, R , from the sale of n calculators.

1 mark

f. Determine the number of calculators that must be sold in order that Moca's Enterprises breaks even.

2 marks

g. Draw and label/name the revenue function on the same set of axes as in part d. labelling the co-ordinates of the intersection point.

1 mark

Question 2

Moca's Enterprises also produces computers – both desktop and laptop. In any week the production is subject to the following constraints:

1. The total number of computers produced will not exceed 70
2. The total number of computers produced will not be less than 30
3. The number of desktop computers produced will be no more than 50
4. The number of laptop computers produced will be no more than 30

By letting x represent desktop computers and y represent laptop computers, the constraints, in addition to $x \geq 0$ and $y \geq 0$, can be listed as follows:

1.
2. $x + y \geq 30$
3. $x \leq 50$
4. $y \leq 30$

- a. Fill in the missing constraint above. 1 mark
- b. On the axes provided, draw a graph showing all the constraints and clearly indicate the feasible region.



3 marks

c. An expression for the profit on computers is $P = 120x + 80y$

Using your graph and the equation, calculate the maximum profit that can be made in any week.

2 marks

Total: 15 marks

Module 4 : Business related mathematics

WW Wide Internet Company initially offered debentures (term deposits) to the general public that would, on investment of \$10 000, return 8% per annum over 3 years.

Question 1

- a. Calculate the interest earned over 3 years.

2 marks

- b. Hence calculate the total value of the investment after 3 years.

1 mark

To encourage further investments, WW Wide Internet Company now calculate and credit the interest at the end of each year rather than at end of the third year. A table was prepared to illustrate that the investment would show a greater return.

- c. Complete the table below.

Time (years)	Balance at start of the year (\$)	Interest earned (\$)	Balance at the end of the year (\$)
1	10 000	800	
2	10 800		11 664
3	11 664		

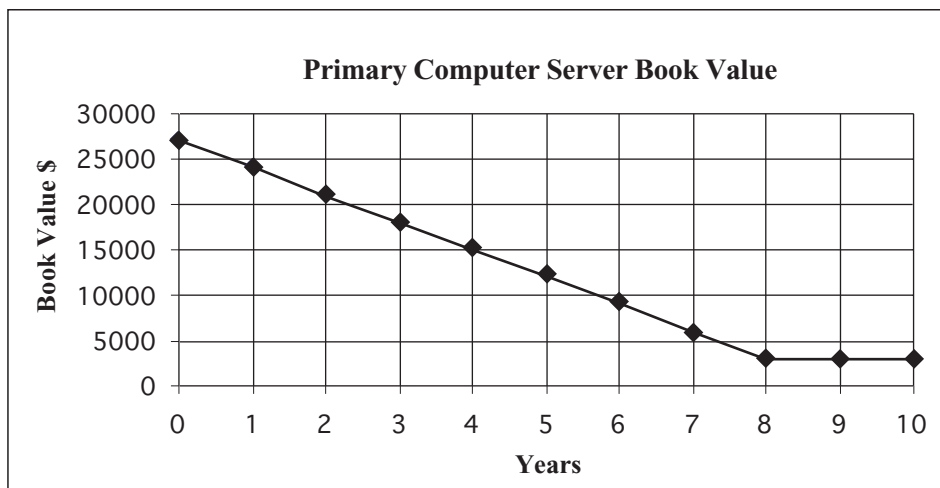
2 marks

- d. The type of investment shown by the table in part c can be described as

1 mark

Question 2

The primary computer server for the WW Wide Internet Company was originally purchased for \$27 000. After 8 years a new computer server is purchased. The computer is depreciated over the first 8 years and its book value can be shown graphically as below.



- a. After 8 years the graph indicates the book value remains a constant \$3 000. What does this represent?

1 mark

- b. Calculate the rate of depreciation of the computer server and express it as a percentage of the purchase price (to 1 decimal place).

2 marks

- c. On the above graph, approximately sketch the book value if a Reducing Balance Method was used to depreciate the computer server rather than the straight-line method shown.

1 mark

Question 3

The replacement server is valued at \$35 000 with the old computer server used as a trade-in. The owner wishes to pay off the unit in 5 years with monthly repayments.

Option A is to finance it using hire purchase with interest at 5% per annum.

- a. Calculate the monthly repayments (to the nearest cent).

2 marks

- b. Calculate the effective interest rate (to one decimal place).

1 mark

Option B is a reducing balance loan from a bank. The interest rate is 8.7% per annum, compounded monthly.

- c. Calculate the monthly repayment to the bank (to the nearest cent).

2 marks

Total: 15 marks

Module 5 : Networks and decision mathematics

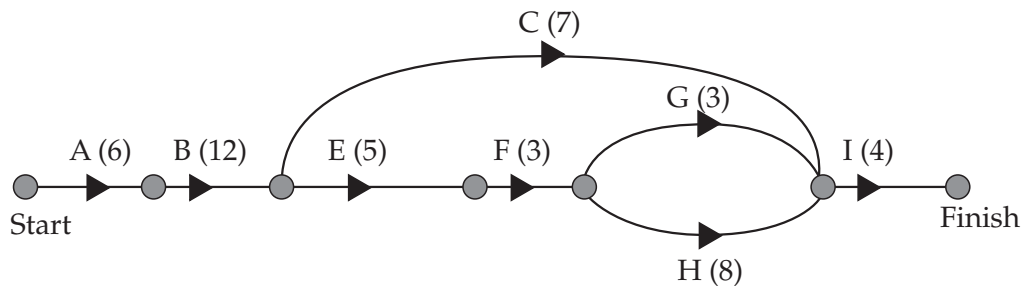
A garden designer has been contracted to design and construct a large garden complete with plants, paving and watering system.

The designer has identified 9 tasks involved in the project. The tasks, and the time it takes to complete these tasks, are given in the table below.

Task	Description	Predecessor	Duration (days)
A	Consult client	–	6
B	Draw-up plans	A	12
C	Order trees and shrubs	B	7
D	Order watering system	B	4
E	Install drainage system	B	5
F	Construct garden beds	E	3
G	Install watering system	D, F	3
H	Construct paths	D, F	8
I	Plant trees and shrubs	C, G, H	4

Question 1

- a. Use the table to complete the project network shown below with the tasks listed on the arcs.



1 mark

- b. Complete the following table of earliest start times (EST) and latest start times (LST) in the grey spaces below.

Task	EST	LST
A	0	0
B	6	6
C	18	
D	18	22
E	18	18
F		23
G	26	31
H	26	26
I	34	34

2 marks

- c. Determine the float (slack time) for task G.

1 mark

- d. Write down the critical path for this project network.

1 mark

- e. Determine the time of completion of the garden.

1 mark

Question 2

The client would like the garden completed as quickly as possible so the designer has constructed a table showing the additional costs (hiring extra workers, equipment, etc.) that would shorten the completion time.

Task	Time (days) that can be shortened	Additional cost per day (\$)
A	0	–
B	5	100
C	2	30
D	1	30
E	1	200
F	0	–
G	1	200
H	4	150
I	2	150

- a. Which tasks should have their completion times changed so that the project is completed as quickly as possible?

1 mark

- b. Determine the new completion time of the garden.

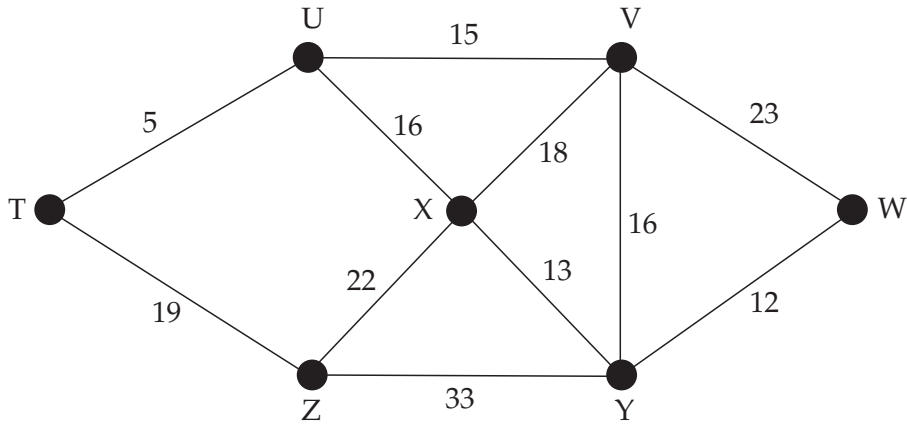
1 mark

- c. What is the additional cost caused by these changes?

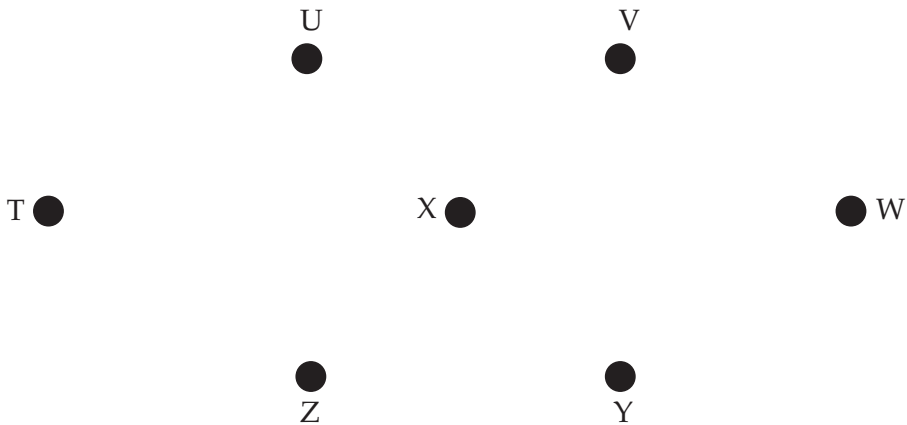
1 mark

Question 3

The diagram below shows the relative positions of the outlets for the watering system for the garden.
 The watering system is to be controlled from the point T and needs to be connected to the other points by pipe.
 Distances, in metres, of possible links between the other points, U, V, W, X, Y and Z, are shown in the diagram below.



- a. Draw the minimal-length spanning-tree for the watering system pipe in the space provided below.



2 marks

- b. What is the minimum length of pipe required for the watering system?

1 mark

Question 4

Andrew, Tom, Carlo and Eric are four of the designer’s regular contractors. The designer has split the tasks into four jobs, P, Q, R and S. Each of the contractors is asked to quote a price for completing each of the four jobs.

The following table sets out their quotes, in dollars, for each of the jobs.

Job	Andrew	Tom	Carlo	Eric
P	2200	2000	1800	2300
Q	660	750	500	900
R	2600	3000	1500	2500
S	800	1000	1200	900

Using the Hungarian algorithm or otherwise, assign the jobs to the contractors so as to minimise the cost to the designer.

3 marks

Total: 15 marks