Student Name...

FURTHER MATHEMATICS

TRIAL EXAMINATION 2

(ANALYSIS TASK)

2000

Reading Time: 15 minutes
Writing Time: 90 minutes

Instructions to Students

This exam consists of Section A and Section B.
Section A contains a set of extended answer questions from the core, "Data Analysis".
Section A is compulsory and is worth 15 marks.
Section B consists of 5 modules. You should choose 3 of these modules and answer every question in each of your chosen modules. Each of the modules is worth 15 marks.
There is a total of **60** marks available for this exam.
Students may bring up to two A4 pages of pre-written notes into the exam.

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Section A

Answer every question in Section A.

Question 1

A company opens a website on the Internet where customers can place orders for the company's product.
The data in Table 1 below shows the number of orders the company receives per day for the first 15 days.

38	65	67	52	61	54	69	49	53	62
		59	60	65	43	57			

Table 1

a. Circle **two** of the following terms which correctly describe the type of data shown in Table 1 above.

numerical   continuous

discrete   categorical

2 marks

b. Complete the stemplot below which displays the data given in Table 1 above.

stem	leaf
	8
	3 9
	2 3 4 7 9
	0 1 2 5 5 7 9

2 marks

c. Use the shape of the stemplot to describe the distribution of the data.

_____

_____

1 mark

d.　　　For the data given in Table 1, write down

　　　i. the mean (to 2 decimal places)

_____

_____

　　　ii. the standard deviation (to 2 decimal places)

_____

_____

　　　iii. the median

_____

_____

　　　**iv.** the interquartile range

_____

_____

4 marks

Question 2

The company which runs the website is interested to know whether the orders they receive over the Internet will affect the number of orders they will receive through the fax.
Table 2 below shows the number of orders the company receives over the net in a day together with the number of orders they receive through the fax for the first 15 days after the Internet site is established.

number of orders over internet	number of orders by fax
38	130
65	93
67	98
52	106
61	90
54	108
69	87
49	115
53	105
62	95
59	101
60	93
65	88
43	121
57	106

Table 2

A scatterplot of this data is shown below.
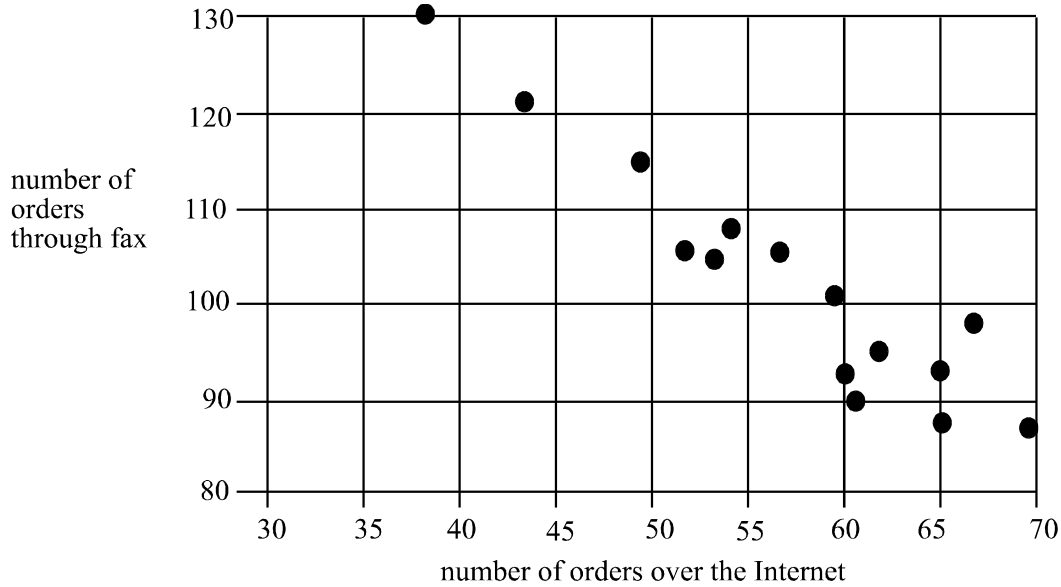
a. i. Use the scatterplot to explain why it is appropriate to use Pearson's product moment correlation coefficient when investigating the relationship between the number of orders received over the Internet and the number of orders received by fax.

_____

1 mark

ii. Calculate Pearson's correlation coefficient correct to 2 decimal places.

_____
1 mark

iii. Does Pearson's correlation coefficient suggest to us that the introduction of orders over the Internet has caused a drop in the number of orders coming in over the fax?

_____

_____

_____

1 mark

b. i. Write down the equation of the least squares regression line by filling in the boxes below. Express the values correct to 2 decimal places.

orders by fax = ☐ - ☐ × orders over Internet

1 mark

ii. Draw this regression line on the scatterpot.

1 mark

iii. Draw a box around the data point on the scatterplot with the minimum residual.

1 mark

Total 15 marks

Section B

Section B consists of 5 modules. Choose 3 of those modules and answer every question in each of those modules.

Module 1 - Number patterns and applications

If you choose this module **all** questions are to be answered.

Question 1

As you leave the town of Burkeville on the highway going south, the first speed limit sign on the left side of the road that you come across is 1 km from the Burkeville post office. The second speed limit sign is 6 km and the third is 11 km from the Burkeville post office.
The distance from the Burkeville post office of each successive speed limit sign on the left side of the road forms an increasing arithmetic sequence.

a. **i.** How far from the Burkeville post office would the next speed limit sign on the left side of the road be situated?

_____
1 mark

   ii. Write down the common difference of the sequence formed.

_____
1 mark

   iii. If the distance of the nth speed limit sign to the Burkeville post office is t_n kilometres, write down a formula for t_n in terms of n.

_____

_____

_____

_____

2 marks

Question 2

As you leave Burkeville on this same highway going south, the first emergency phone on the left hand side of the road is situated 10 km from the Burkeville post office. The next is situated 15 km from the post office.
The distances of successive emergency phones from the Burkeville post office form a geometric sequence.

a. i. Write down the common ratio of this geometric sequence.

_____

1 mark

ii. Write down the distance, to the nearest kilometre, from the Burkeville post office of the seventh emergency phone.

_____

_____

1 mark

b. When these emergency phones had to be upgraded, a technician travelled from Burkeville to each of the emergency phones, in order, beginning with the closest emergency phone. After upgrading each phone, the technician would return directly to Burkeville for more parts before going on to the next emergency phone.

What was the total distance, to the nearest kilometre, the technician had travelled to and from the phones, after having returned to Burkeville after having upgraded the 11th emergency phone?

_____

_____

_____

_____

_____

_____

2 marks

Question 3

Write down the ratio, expressed in simplest form, of the number of emergency phones compared to the number of speed limit signs you would pass on the left hand side of the road, on the first 1000 km of this highway as you travelled south from Burkeville.

_____

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4 marks

Question 4

Billboards about fire restrictions are erected on this stretch of highway heading south of Burkeville. The distance of each of these billboards from the Burkeville post office forms a sequence described by the difference equation

$$t_{n+1} = 2t_n + 1 \qquad n = 1, 2, 3 \ldots\ldots$$

Each of these signs cost the council $455. Given that the second of these billboards is situated 11 km from the Burkeville post office, what is the cost to the council for signs along the first 500 km of this stretch of highway?

_____

_____

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_____

3 marks

Total 15 marks

Module 2 : Geometry and trigonometry

If you choose this module **all** questions are to be answered.

Question 1

Shoppers at a suburban shopping centre can access the shops from the carpark using a ramp or a lift. The ramp has four straight sections indicated in Figure 1 below as AB, BC, CD and DE.
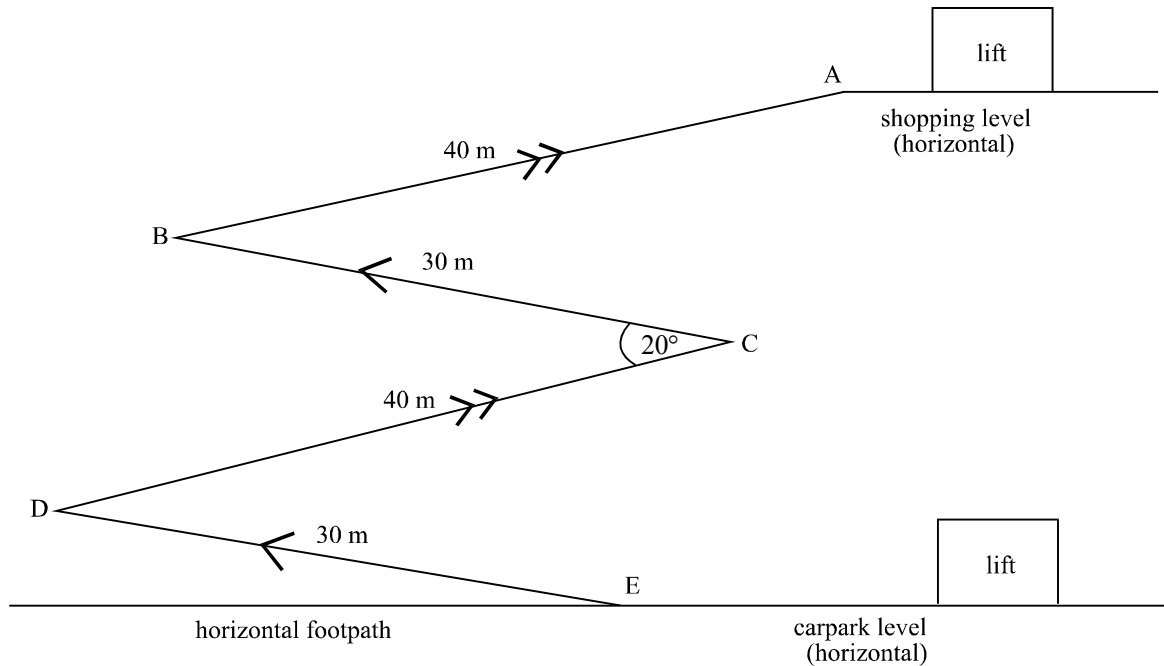

Figure 1

Sections AB and CD run parallel and are 40 metres in length. Sections BC and DE run parallel and are 30 metres in length. Angle BCD $= 20°$.

a. i. A boy standing at point D on the ramp drops his toy 5.724 metres vertically below. How far does he have to walk along the footpath from point E to reach his toy? (Express your answer correct to 3 decimal places.)

_____

_____

_____

1 mark

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ii. What angle does ramp DE make with the horizontal? (Express your answer to the nearest minute.)

_____

_____

_____

<div align="right">1 mark</div>

iii. What angle does ramp CD make with the horizontal? (Express your answer to the nearest minute.)

_____

_____

_____

<div align="right">1 mark</div>

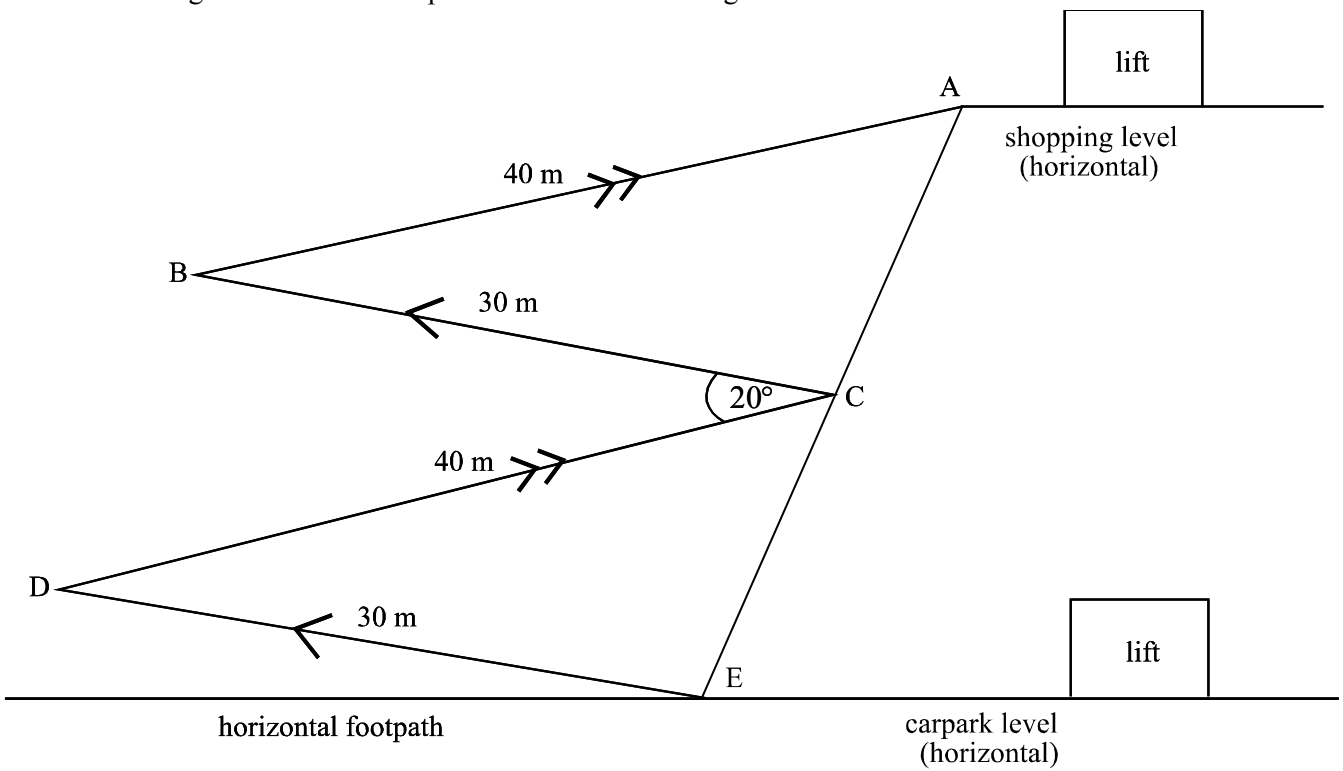The straight line AE touches point C as indicated in Figure 2 below.

Figure 2

b. i. Write down $\angle CDE$.

_____

<div align="right">1 mark</div>

ii. Find CE correct to 3 decimal places.

_____

_____

_____

_____

2 marks

iii. Hence write down the length of AE correct to 3 decimal places.

_____

_____

1mark

iv. Use the sin rule to find $\angle CED$ to the nearest minute.

_____

_____

_____

_____

2 marks

v. Using your answers to parts **b.**, **iii.** and **iv.**, and part **a. ii.**, calculate the vertical distance that the lift moves to get from the carpark level to the shop level. Express your answer correct to two decimal places.)

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3 marks

Question 2

The lift is cubic with internal side lengths of 2.5 metres. A man places in the lift a straight piece of timber he has purchased of length 2.7 metres. One end of the timber rests in the corner of the lift at point A and the other end rests against the back wall of the lift at point I. The distance from point I vertically to the floor and horizontally to HD is equal.
Find that distance. (Express your answer correct to 2 decimal places.)

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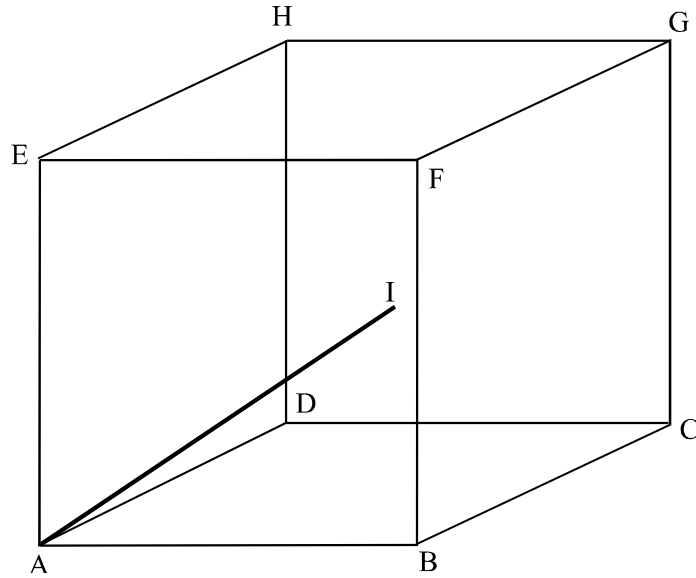
3 marks
Total 15 marks

Module 3 : Graphs and relations

If you choose this module **all** questions are to be answered.

Question 1

A bowling club can have a maximum of 200 members because of the number of bowling greens it has available.

In order to survive, the club must raise a minimum of $15 300 in fees per year and a minimum of $20 400 in fundraising per year. It is club policy that female members pay $90 each per year in fees and male members pay $102 each per year in fees.

The club has found on average that female members contribute $150 each per year to fundraising and male members contribute $102 each per year to fundraising.

Let x be the number of female members at the club in a year.
Let y be the number of male members at the club in a year.

To calculate the minimum number of club members required to keep the club going, the committee works out the constraints imposed by these factors.

The constraint imposed by the fees is given by $90x + 102y \geq 15\ 300$

a.   Write down, in terms of x and y, the constraint imposed by
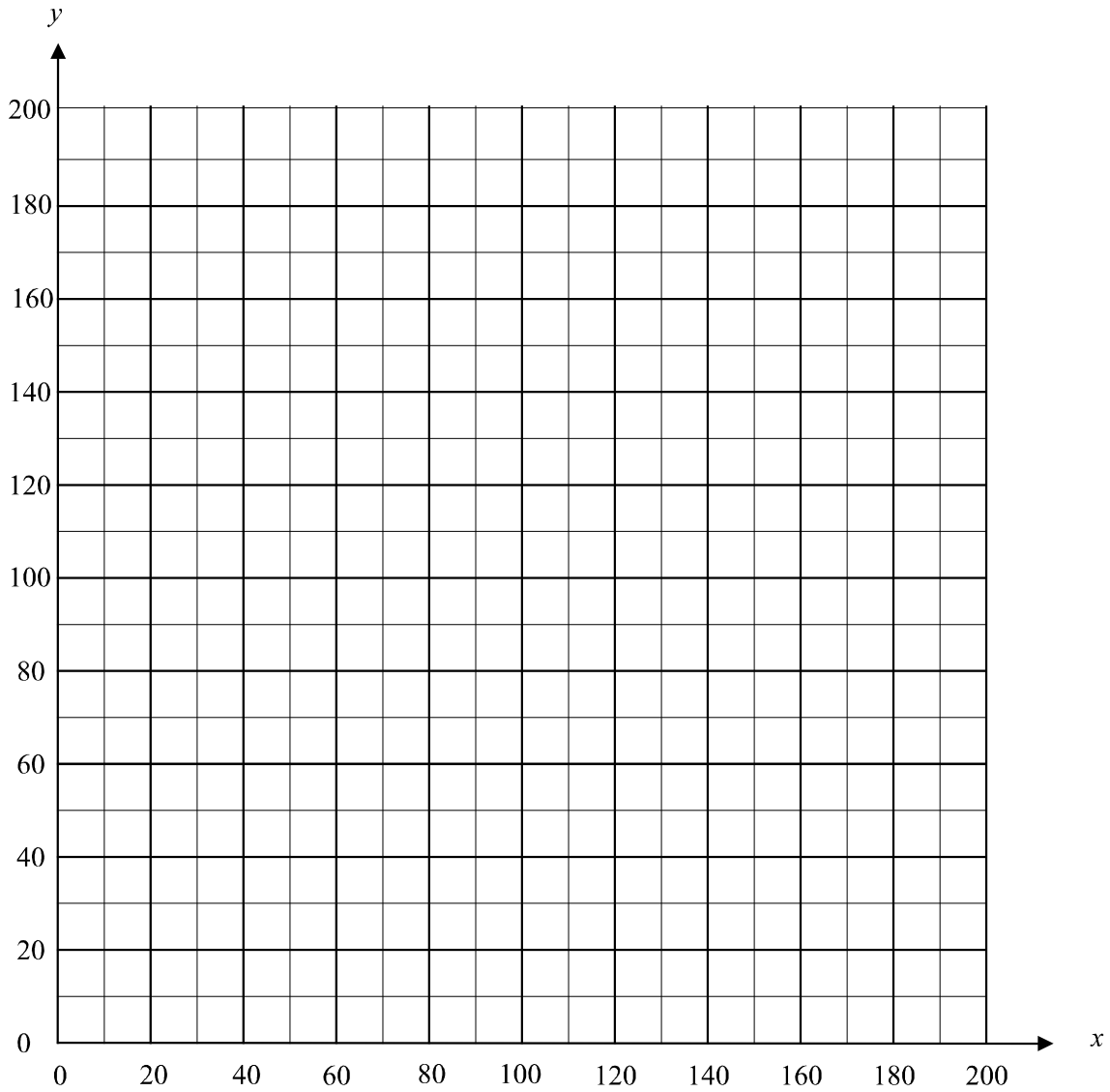
i. the number of bowling greens the club has available.

_____
<div align="right">1 mark</div>

ii. the fundraising amount which has to be raised by the club.

_____
<div align="right">1 mark</div>

_____

b. Graph these 3 constraints on the set of axes below showing clearly the feasible region defined by these constraints.

c. **i.** What is the minimum number of members required in a year for the club to survive?

_____

_____

_____

_____

_____

3 marks

ii. How many of these are women?

_____

1 mark

Question 2

The greenkeeper at the bowling club sews two different types of seed. On one of his bowling greens he sews seed A which contains the usual fertilisers and on the second green he sews seed B which contains a slow release fertilizer.
The height, in cm, of the grass grown from seed A is measured at weekly intervals after the seed has been sewn. A graph is plotted using this data and is shown below.
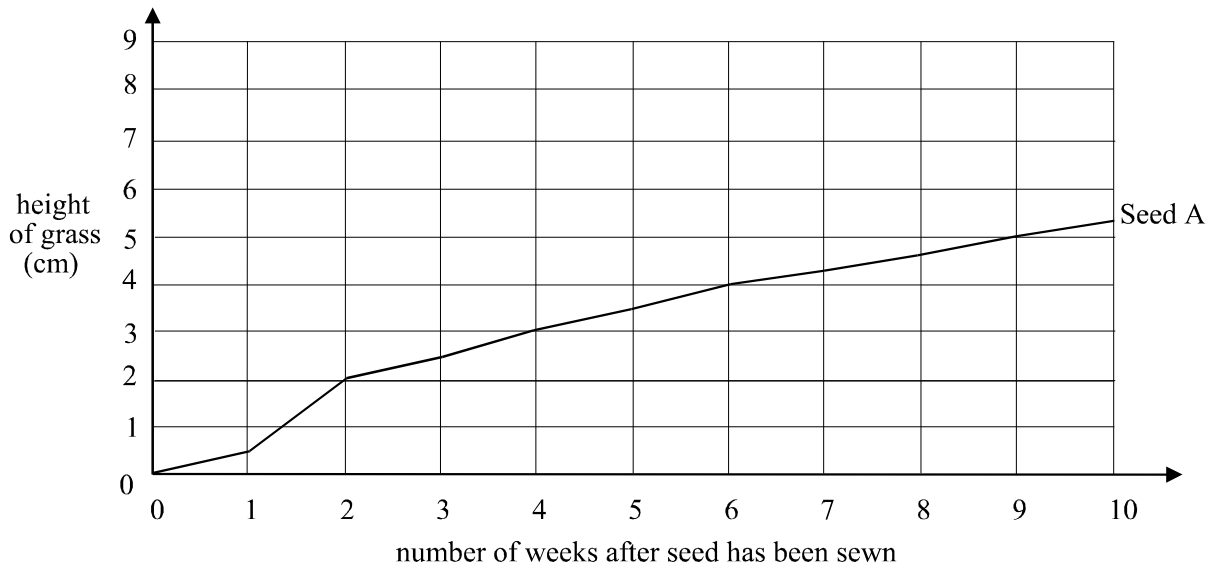
The height of the grass sewn using seed B is also measured at weekly intervals after sewing and is recorded in Table 1 below.

Number of weeks after sewing	0	1	2	3	4	5	6	7	8	9	10
Height (cm)	0	0.25	1	2.5	5	6.5	7	7.5	8	8.25	8.5

Table 1

a.　Using the data from Table 1, plot the graph of height of grass grown from seed B against the number of weeks after being sewn on the set of axes above.

1 mark

b.   From the graphs answer the following questions.

i. How many weeks after planting is the height of the grass on each of the bowling greens the same?

_____

<div align="right">1 mark</div>

ii. During which week did the height of the grass sewn using seed B first exceed the height of the grass sewn using seed A?

_____

<div align="right">1 mark</div>

iii. Estimate the height of the grass sewn using seed A, 12 weeks after sewing.

_____

<div align="right">1mark</div>

iv. During which week did the grass sewn using seed B grow the most?

_____

<div align="right">1 mark</div>

<div align="right">Total 15 marks</div>

Module 4 : Business related mathematics

If you choose this module **all** questions are to be answered.

Question 1

Andrew starts his own business and allocates $1500 in his business plan to purchasing office furniture. The furniture that he wishes to buy, however, costs $1590.

a.   **i.** How long would Andrew need to leave his $1500 in an account earning simple interest of 3% per annum for him to have $1590?

_____

_____

_____

2 marks

ii. What would Andrew's $1500 amount to after 2 years if it were invested in an account earning compound interest of 3% per annum compounding annually?

_____

_____

_____

1 mark

b.   Andrew enters an agreement to purchase the office furniture which cost $1590. He pays a deposit of $590 and pays $100 a month for 1 year.

i. How much interest would Andrew pay under this agreement on the amount he owed?

_____

_____

_____

1 mark

ii. What flat rate of interest does this represent?

_____

_____

_____

1 mark

c. Andrew wonders whether he would have been better off investing his $1500 in an account which attracts compound interest of 4% compounding quarterly. How long would it have taken for Andrew's money to amount to $1590 if he had taken this course? (Express your answer to the nearest quarter.)

_____

_____

_____

_____

_____

_____

_____

_____

3 marks

Question 2

The car that Andrew purchases for his business depreciates at the rate of 30 cents per kilometer travelled. Given that the car cost $32 000, after how many kilometers would the book value of the car be zero?

_____

_____

_____

_____

_____

2 marks

Question 3

To set his business up, Andrew takes out a reducing balance loan of $80 000 which he plans to pay off completely over 5 years. The lending institution he is borrowing from charges annual interest of 12% on the reducing monthly balance.
The annuities formula

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$$

is used to work out the monthly repayments.

   i. For Andrew's loan,

$$R =$$

$$n =$$

2 marks

   ii.   What will Andrew's monthly repayment be on this loan?

_____

_____

_____

_____

_____

2 marks

   iii.   Use your result in part **ii** to find how much interest Andrew is paying on this loan?

_____

_____

_____

1 mark

Total 15 marks

Module 5 : Networks and decision mathematics

If you choose this module **all** questions are to be answered.

Question 1
The graph below shows a school campsite made up of 4 student bunkrooms, indicated as W, X, Y and Z, 1 meals area, indicated as M, a bathroom block indicated as B, a teachers block indicated as T and a games room indicated as G.
Each of these are connected by concrete paths to avoid mud being carried inside the buildings.
The distances, in metres, between the various buildings are indicated in Figure 1 below.
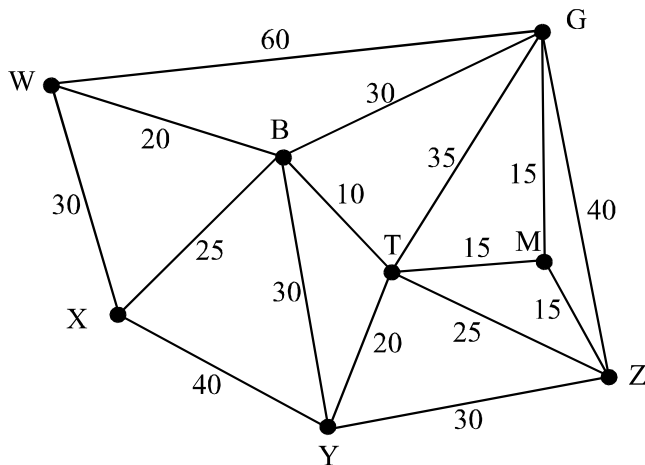
Figure 1

a.   What is the shortest distance from bunkroom Y to the games room?

_____

_____
1 mark

The concrete paths were laid in stages. At the end of the first stage a minimum spanning tree had been formed.
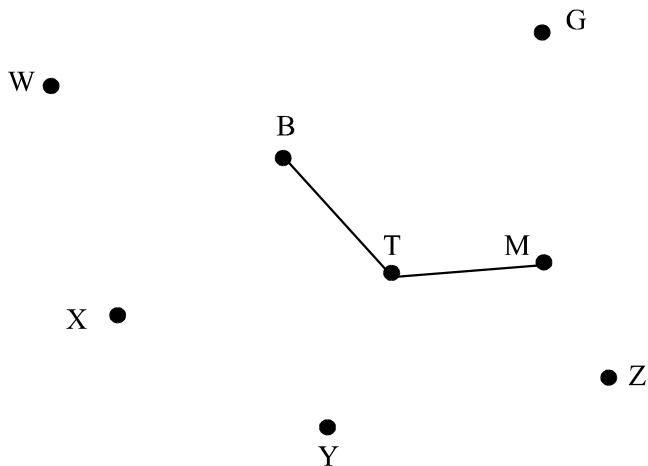Part of that minimum spanning tree is shown in Figure 2 below.

Figure 2

b.   Complete the minimum spanning tree shown in Figure 2 above.

2 marks

_____

c.   What length of concrete paths was laid in stage 1?

_____

_____

<div align="right">1 mark</div>

Figure 3 below shows the system of concrete paths after the final stage of laying concrete paths is completed.
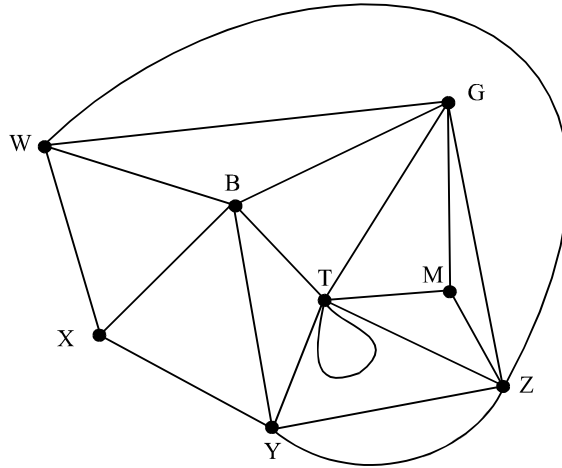

Figure 3

d.   Complete the 8 missing entries for the matrix below which represents the graph shown in Figure 3.

	B	G	M	T	W	X	Y	Z
B	0	1	0	1	1	1	1	–
G	1	0	1	1	1	0	0	1
M	0	1	0	1	0	0	0	1
T	1	1	1	-	0	0	1	1
W	1	1	0	0	0	1	0	–
X	1	0	0	0	1	–	1	0
Y	1	0	0	1	0	1	0	–
Z	0	1	1	1	–	0	–	–

<div align="right">2 marks</div>

Question 2

The Year 7 students who use this camp go on a day hike. Various groups of students are assigned to activities involved in preparing for such a hike. Those activities are shown in Table 1 below together with the order in which the activities can be undertaken (except for activity G) and the time they take to complete.

Activity	Description	Immediate predecessor	Duration (mins)
A	prepare breakfast	–	14
B	obtain and check equipment from games room	–	23
C	supply and check sunscreen has been applied	–	5
D	clean up breakfast dishes	A	10
E	fill water bottles	B	7
F	make lunches	D	8
G	pack packs		12
H	clean bathroom block	A	13
I	hang out shower curtains to dry	H	6
J	report to teacher at front gate	G, I	4

Table 1

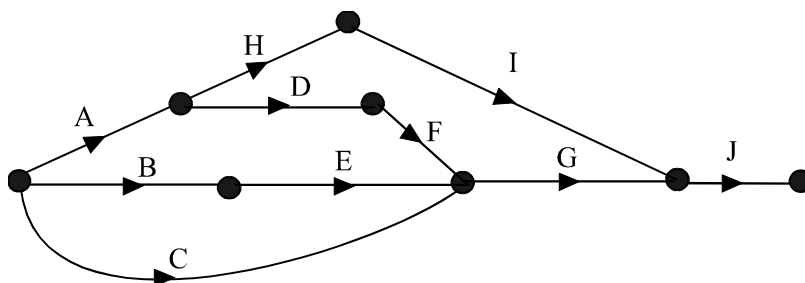A directed graph showing the activities involved in preparing for the day hike is shown in Figure 4 Below.

Figure 4

a.   Write down the activity or activities which immediately preceed activity G.

_____

_____
<div align="right">1 mark</div>

b. Use the information in Table 1 to complete Table 2 below.

Task	Earliest start time	Latest start time
A	0	0
B	0	2
C	0	27
D	14	
E	23	25
F	24	24
G		32
H	14	
I	27	38
J	44	44

Table 2

3 marks

c. Write down the critical path of the network.

_____

_____

2 marks

d. What is the earliest completion time for the Year 7 students to prepare for the day hike?

_____

1 mark

e. If the teachers wanted to shorten the length of time it takes for the Year 7 students to prepare for their day hike, which activities should they try to shorten the duration of?

_____

_____

1 mark

f. Write down the slack (or float time) for the activity which involves supplying and checking that sunscreen has been applied.

_____

_____

1 mark

Total 15 marks