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FURTHER MATHEMATICS
TRIAL EXAMINATION 1
2000
SOLUTIONS

Section A Answers

Core
1. **C**
2. **E**
3. **B**
4. **A**
5. **D**
6. **D**
7. **E**
8. **A**
9. **D**
10. **B**
11. **B**
12. **C**
13. **D**

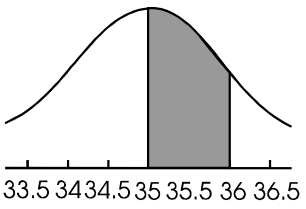
Section B : Answers

Module 1	Module 2	Module 3	Module 4	Module 5
Arithmetic and applications	**Geometry and trigonometry**	**Graphs and relations**	**Business related mathematics**	**Networks and decision mathematics**
1. **C**	1. **A**	1. **B**	1. **C**	1. **E**
2. **E**	2. **C**	2. **D**	2. **A**	2. **D**
3. **E**	3. **D**	3. **C**	3. **D**	3. **C**
4. **B**	4. **C**	4. **B**	4. **A**	4. **C**
5. **C**	5. **E**	5. **C**	5. **B**	5. **B**
6. **E**	6. **E**	6. **E**	6. **D**	6. **C**
7. **A**	7. **D**	7. **E**	7. **C**	7. **D**
8. **D**	8. **E**	8. **E**	8. **E**	8. **A**
9. **C**	9. **E**	9. **A**	9. **B**	9. **B**

Section A : Core - solutions

Question 1
On a box plot the median is shown by the vertical line drawn inside the box. So, the median weight is 95 kg. The answer is C.

Question 2
We are looking for the shaded region shown in the diagram.
We know that 95% of the data in the distribution lie within 2 standard deviations either side of the mean of 35.
So, $95\% \div 2 = 47.5\%$ of the licorice straps lie between 35 and 36 cm in length.
The answer is E.



Question 3
The category we are interested in is the non member (right hand column) who takes public transport (lighter shading at top). The lighter shading at the top of the right hand column represents 35% of non members. The answer is B.

Question 4
The direction of the association is sloping up to the right and is therefore positive. The association has linear form since the points tend to form a straight line. The strength of the association is probably best described as moderate. The answer is A.

Question 5
The variable "time" is not categorical but numerical. It is not a dependent variable since the distance covered is the dependent variable, that is, the distance is dependent on how long the tortoise decides to keep walking! The term skewed relates to describing the distribution of a set of data and the term linear relates to the form of the association between two variables. The variable "time" is independent. The answer is D.

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Question 6

The *y*-intercept of the line is 20. The gradient of the line is given by rise over run. Since the line passes through the points $(0, 20)$ and $(60, 50)$.

We have, gradient $= \dfrac{\text{rise}}{\text{run}}$

$$= \frac{50-20}{60-0}$$

$$= 0.5$$

The equation of a straight line with gradient, *m*, and *y* intercept *c*, is given by $y = mx + c$

We have $y = 0.5x + 20$

In our case the variable on the vertical axis is distance rather than *y* and the variable on the horizontal axis is time rather than *x*.

So, we have $\text{distance} = 0.5 \times \text{time} + 20$ or $\text{distance} = 20 + 0.5 \times \text{time}.$ The answer is D.

Question 7

The data on the scatterplot can be linearised by stretching out the upper end of the *y* axis or compressing the upper end of the *x* scale. This can be done by using a square transformation on the *y* axis or a log transformation on the *x* axis. Only one of these is on offer. The answer is E.

Question 8

The residual plot appears to be a random collection of data points spread evenly around zero. We can assume therefore that the association between the two variables is probably linear.
The answer is A.

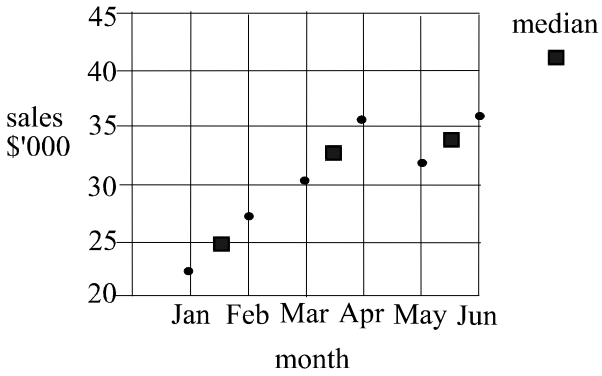
Question 9

The proportion of the variation in growth that can be explained by the variation in the amount of sunlight is given by the coefficient of determination which is equal to r^2, that is, $0.7^2 = 0.49$
So, 49% is the required proportion. The answer is D.

Question 10

The 3 medians required are indicated in the diagram.
Place a ruler to join the two outside medians and whilst maintaining the same slant on the ruler move it one third of the way towards the middle median. This gives the 3 median fit trend line.

The answer is B.



Question 11

The five term moving average for April is given by $(17 + 12 + 21 + 17 + 12) \div 5 = 15.8$
This is closest to 16. The answer is B.

Question 12

The pattern is not cyclic, that is, there are no repeating patterns. There is no seasonal behaviour. The pattern is not random nor trending downwards. The pattern is best described as trending upwards. The answer is C.

Question 13

We know that the 4 seasonal indices should add up to give 4. So, $0.85 + 1.23 + x + y = 4$
So, $x + y = 1.92$ The answer is D.

Section B : Modules - solutions

Module 1 - Arithmetic and applications

Question 1

We have an arithmetic sequence so, $S_n = \frac{n}{2}[2a + (n-1)d]$

Now, $n = 28$, $a = 11$ and $d = 4$, therefore $S_{28} = \frac{28}{2}[2 \times 11 + (28-1) \times 4]$

$= 1820$

The answer is C.

Question 2

The ratio of the drug, in mg, compared to the stock solution, in ml, is 0.25 : 1
Expressing this in whole numbers, we have, 1 : 4
Since the patient requires 5 mg of the drug, and maintaining the same ratio, we have 5 : 20
So the amount of stock solution required to be injected, in order to give the patient 5 mg of the drug is 20 ml. The answer is E.

Question 3

The ratio 2 : 3 : 3 : 4 : 6 has a total of 18 parts. Kindergarten C receives 3 of those parts and that amount is \$10 500. The ratio of parts received to money received is 3 : 10 500
So, maintaining that ratio, we have 1 : 3500
So 1 part is worth \$3500 and therefore 18 parts are worth $18 \times \$3500 = \$63\,000$
The sum of money allocated by council to the five kindergartens is \$63 000. The answer is E.

Question 4

The series is geometric with $a = 4$ and $r = \frac{-2}{4} = \frac{-1}{2}$

Now $S_\infty = \frac{a}{1-r}$

$= \frac{4}{1 - \frac{-1}{2}}$

$= 4 \div (1 + \frac{1}{2})$

$= 4 \times \frac{2}{3}$

$= \frac{8}{3}$ The answer is B.

Question 5

Since the production of wine increases by 2.5% each year, the multiplication factor is 1.025
So, the difference equation which represents the production of wine at this winery is

$P_{n+1} = 1.025P_n$ where $P_1 = 40000$ The answer is C.

Question 6

The sequence generated by the difference equation $f_{n+1} = 3f_n - 1$ where $f_1 = 2$ is $2,\ 5,\ 14,\ 41,\ 122,\ldots$

The sequence is increasing which eliminates option A. It is neither arithmetic (since $5-2 \neq 14-5$ etc) nor geometric (since $\frac{5}{2} \neq \frac{14}{5}$ etc) which eliminates options B and D. The difference equation is first order and all first order difference equations can be solved (remember that solving a difference equation means that it is possible to write down an expression for the *n*th term of the sequence in terms of *n*) and this eliminates option C. The answer is E.

Question 7

The graph of an increasing arithmetic sequence should go up in equal "steps", that is, the interval between each of the terms is the same. The answer is A.

Question 8

The sequence generated by this difference equation is $7,21,63,189,...$

We notice that the difference equation represents a geometric sequence for which $MW = 93 - 45 = 48\text{m}$ So, the geometric sequence can be given by $t_n = ar^{n-1}$

In our case, we have $(BD)^2 = 4^2 + 4^2$ The answer is D.

Question 9

The common ratio, *r*, of this geometric sequence is given by $\frac{t_5}{t_4} = \frac{54}{162} = \frac{1}{3}$

So, 

Now, for $r < 1,\ S_n = \frac{a(1-r^n)}{1-r}$

So, $WZ = 49$ The answer is C.

Module 2 : Geometry and trigonometry

Question 1

Since $MN = NP = 2$ cm, then $\angle MPN = \angle NMP = 60°$. So, ΔMNP is an equilateral triangle and therefore $MP = 2$ cm. The answer is A.

Question 2

Area of triangle $= \frac{1}{2}bc\sin A$

We have, area $= \frac{1}{2} \times 6 \times 9 \times \sin 80°$

$= 26.6$ (to one decimal place) The answer is C.

Question 3

Triangle ABC is similar to triangle ADE and the ratio of sidelengths is 1 : 2 since $AB = BD$.
So, the area of $ADE = 2^2 \times$ area of ΔABC

$= 4 \times 4 = 16$ The answer is D.

Question 4

Using the cos rule, we have $10^2 = 5^2 + 11^2 - 2 \times 5 \times 11 \times \cos\theta,$ where $\theta = \angle XZY$

So, $\cos\theta = \frac{100 - 25 - 121}{-110}$

$\theta = 65°17'$ The answer is C.

Question 5
The scale is 1 : 2000
So, 10 mm on the map will be $10 \text{ mm} \times 2000$

$= 20\,000$ mm

$= 20$ m of the actual building

And, 15 mm on the map will be $15 \text{ mm} \times 2000$

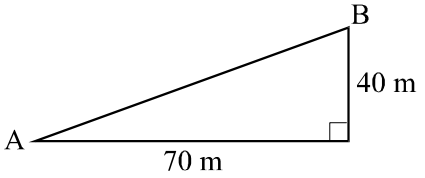
$= 30\,000$ mm

$= 30$ m of the actual building

The area of the actual building is $20 \text{ m} \times 30 \text{ m} = 600 \text{ m}^2$ The answer is E.

Question 6

Using the contour lines we know that point B is 40 m higher than point A.
We have a right angled triangle as indicated in the diagram.
The distance flown by the bird is AB.



Using Pythagoras, $(AB)^2 = 70^2 + 40^2$

$= 4900 + 1600$

So, $AB = 81$ to the nearest metre.

The answer is E.

Question 7

Construct a diagram. Let M be vertically below point W lying on the horizontal line from Z to AB.

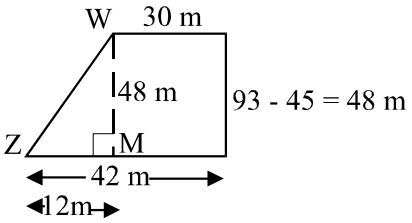
The side $MZ = 42 - 30 = 12$ m

The side $MW = 93 - 45 = 48$ m

Using Pythagoras $(WZ)^2 = 48^2 + 12^2$

So $WZ = 49$ to the nearest metre

The answer is D.



Question 8

Let M be the midpoint of BC and O be the centre of the square ABCD.

Find the height of the pyramid.

In ΔABD,

$(BD)^2 = 4^2 + 4^2$ (Pythagoras)

So, $BD = 5.6569$ (to 4 places)

So, $BO = 2.8284$ (to 4 places)

In triangle BEO, $(EO)^2 = (EB)^2 - (BO)^2$

$(EO)^2 = 6^2 - (2.8284)^2$

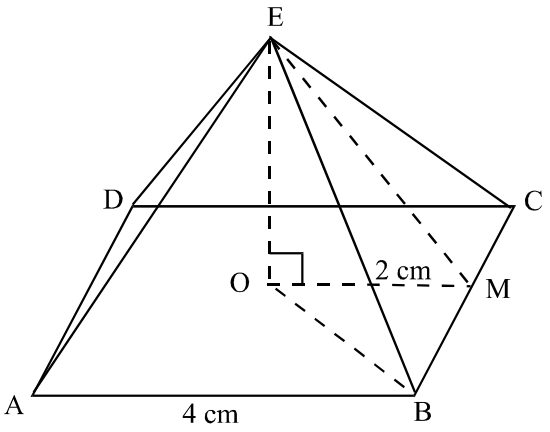
$EO = 5.2915$

In triangle EMO, $\tan(\angle EMO) = \dfrac{EO}{MO}$

$= \dfrac{5.2915}{2}$

So, $\angle EMO = 69°18'$

So BCE makes an angle of $69°18'$ (to the nearest minute) with the square base. The answer is E.



Question 9

Draw a diagram.

To find the bearing of L, the lunch spot from C, the campsite, we need to first find angle ACL.

Now, from the diagram, we see that

$\angle CAL = 90° - 40° - 10° = 40°$

Using the cos rule, we have

$(CL)^2 = 3^2 + 4^2 - 2 \times 3 \times 4 \times \cos 40°$

So, $CL = 2.5720$ (to 4 places)

Now using the sin rule, we have

$\dfrac{\sin(\angle ACL)}{4} = \dfrac{\sin 40°}{2.5720}$

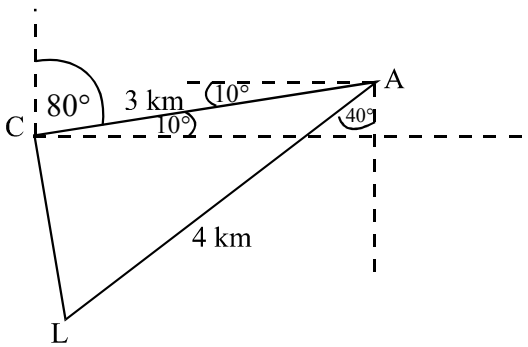
So, $\angle ACL = 88°32'$

So the bearing of L from C is

$88°32' + 80° = 168°32'$

The closest answer is $169°$.

The answer is E.



Module 3 : Graphs and relations

Question 1

The horizontal line lies below the *x* axis. The equation must be of the form *y* = *a negative number*. Only option B offers this possibility. The answer is B.

Question 2

Do a quick sketch graph on your calculator. Option A is clearly wrong. As *x* increases, *y* decreases. So option B is wrong. The *y* intercept of the graph is negative so option C is wrong. The gradient of the graph is $-\frac{1}{2}$. So option D is correct. The answer is D.

Question 3

The depth decreased most rapidly between 8 and 10 hours, that is, the gradient was negative (depth was decreasing) and steepest around this time. The answer is C.

Question 4

We want to find the *n* value of the point of intersection. Now, $R = 5n$ and $C = 3n + 200$
At the point of intersection, that is, at break even point, $R = C$ so, $5n = 3n + 200$

$$2n = 200$$
$$n = 100$$

The owner will break even after 100 lunches have been ordered in a week. The answer is B.

Question 5

For the values of *x* for which $0 \le x \le 2$ we have a straight line with a *y* intercept of 5 and a gradient of $\frac{5-1}{0-2} = \frac{4}{-2} = -2$. So the equation $y = mx + c$ becomes $y = -2x + 5$.
Only options C and E offer this.
For the values of *x* for which $2 < x \le 6$ we have a straight line with a *y* intercept (if the line were continued back to the y axis) of -1 and a gradient of $\frac{1-5}{2-6} = \frac{-4}{-4} = 1$. So the equation $y = mx + c$ becomes $y = x - 1$.
Only option C offers both parts correctly. The answer is C.

Question 6

Corner points of the feasible region are (0, 0), (0, 30), (20, 40), (60, 30), (80, 30) and (80, 0).
So, for (0, 0), $2x + 3y = 0$
for (0, 30), $2x + 3y = 90$
for (20, 40), $2x + 3y = 160$
for (60, 30), $2x + 3y = 210$
for (80, 30), $2x + 3y = 250$
for (80, 0), $2x + 3y = 160$
The maximum value is 250. The answer is E.

Question 7

If 3 trips are made in a week, the cost per trip is \$130 per trip. So in one week the cost is $3 \times \$130 = \$390$
If 5 trips are made in a week, the cost per trip is \$110 per trip. So in the next week the cost is $5 \times \$110 = \$550$ Over the 2 weeks, the total cost is \$940. The answer is E.

Question 8

The graphs for options A and B would have 2 separate branches. For option A, they would be in the first and second quadrants. For option B they would be in the first and third quadrants. Option C gives a straight line. Option D gives a parabola. To double check , substitute the given points into the equation. The answer is E.

Question 9

The vertical borders on the left and right can be described by
$x \geq 0$ and $x \leq 6$, that is, $0 \leq x \leq 6$. Only options A and B offer this.
The horizontal border at the bottom of the shaded area is described by $y \geq 0$. Only options A, B and D offer this.
The diagonal straight line border nearest the origin passes through (0, 2) and (2, 0). It has a y intercept of 2 and a gradient of $\frac{2-0}{0-2} = -1$
The equation of the line $y = mx + c$ becomes $y = -x + 2$ or $x + y = 2$
The area above this line is given by the inequation $x + y \geq 2$
Only options A and C offer this. Only option A can be the correct answer now. To confirm the other boundary, the diagonal line passes through (0, 5) and (6, 2). Gradient $= \frac{5-2}{0-6} = \frac{3}{-6} = -\frac{1}{2}$
The y intercept is 5. The equation is $y = -\frac{1}{2}x + 5$
Which when rearranged becomes $2y = -x + 10$
and so $x + 2y = 10$
The area below this is described by the inequation $x + 2y \leq 10$
The answer is A.

Module 4 :Business related mathematics

Question 1

The value of the phone system after 6 years $= 25\ 000(1-\frac{13}{100})^6$

$= 10\ 840.66$

The closest answer is \$10 840. The answer is C.

Question 2

The minimum balance in May is \$790.53 (that is, this is the balance up until 7th May)

So, the interest is $\$790.53 \times \frac{0.3}{100}$

$= \$790.53 \times 0.003$ The answer is A.

Question 3

The effective rate of interest $= \frac{2n}{n+1} \times$ flat rate

$= \frac{2 \times 52}{53} \times 15\%$

$= 29.4\%$ (to one decimal place) The answer is D.

Question 4

$A = PR^n$

So, $A = 20\ 000(1.005)^9$

$= 20\ 918.21$

$R = 1 + \frac{r}{100}$

$= 1 + \frac{6/12}{100}$

$= 1.005$

The answer is A.

Question 5

For Sam, after 3 years his interest was worth $5000(1.04)^3$

Sam's interest is therefore $5000(1.04)^3 - 5000$

Jack's interest is given by $\frac{5000 \times r \times 3}{100}$

Since the interest after 3 years was the same, we have

$$\frac{5000 \times r \times 3}{100} = 5000(1.04)^3 - 5000$$

$$r = 4.16 \text{ (to 2 places)}$$

The answer is B.

Question 6

From the graph, after 1 year, the amount in the account was \$1050

So, $1050 = 1000(1 + \frac{r}{100})^1$

$1050 = 1000(1 + \frac{r}{100})$

$1050 = 1000 + 10r$

$r = 5$

The annual interest rate is 5%. The answer is D.

Question 7

The interest would be calculated on the amount owing at that time, that is, \$21 250. Interest is charged 6 monthly, so the annual interest rate of 10% becomes 5% for 6 months. So, interest charged is 5% of \$21 250 = \$1062.50
The answer is C.

Question 8

The annuities formula is $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$ where $R = 1 + \frac{r}{100}$

We have $P = 30\ 000, \quad R = 1 + \frac{5}{100}, \quad n = 6, \quad Q = 5000$

So, $A = 30\ 000 \times (1.05)^6 - \frac{5000(1.05^6 - 1)}{0.05}$

The answer is E.

Question 9

On 6 monthly repayments over 5 years, Joy pays $5 \times 2 = 10$ repayments of \$1295.05 She repays \$12 950.50
So, she pays \$2950.50 in interest.
To make yearly repayments and still have the loan repaid in 5 years, she would have to make yearly repayments of Q, where, using the annuities formula

$$0 = 10\ 000 \times (1.1)^5 - \frac{Q(1.1^5 - 1)}{0.1}$$

So, $\frac{Q(1.1^5 - 1)}{0.1} = 10\ 000 \times (1.1)^5$

$Q = 2637.97$

Five of these yearly repayments amount to \$13 189.87 She would pay \$3189.87 in interest. The difference in interest is \$239.37
The answer is B.

Module 5 : Networks and decision mathematics

Question 1

The sum of the degrees of the vertices, beginning at the bottom left vertex and working clockwise, is 3 + 2 + 3 + 2 + 2 =12
The answer is E.

Question 2
Option A is not correct since the chocolate drive was as unpopular as the bulb drive. Option B is incorrect since Neil and Lee both chose the raffle. Option C is incorrect since Glenys chose the bulb drive. Option D is correct since three people chose the cake stall. Only one or two people chose any of the other activities. The answer is D.

Question 3

Using Euler's formula we have $v - e + f = 2$
Now, $v = 10$, $f = 5$ and so $10 - e + 5 = 2$
$e = 13$
There are 13 edges. The answer is C.

Question 4

An Euler circuit is a path that starts and finishes at the same vertex and passes over every edge just once. Since a connected graph where all the vertices have an even degree will have an Euler circuit, we look for a graph without this. Graph C is such a graph. The answer is C.

Question 5

A Hamiltonian circuit is a path that starts and finishes at the same vertex and passes through each vertex exactly once.
Option A repeats vertex E.
Option C repeats vertices E and G.
Option D repeats vertex G.
Option E leaves out vertex B.
Option B is correct. The answer is B.

Question 6

The capacity of the cut is 6 + 4 + 9 + 2 = 21
The flow on arc CD flows in the opposite direction across the cut of the other flows and is therefore not counted. The answer is C.

Question 7

Option C is incorrect since task A and B have no immediate predecessors. Option A is incorrect since task D does not have task B as its immediate predecessor. Option B is incorrect since task E has task B as its immediate predecessor. Option E is incorrect since task I does not have task C as its immediate predecessor. The correct option is D. The answer is D.
Question 8

There is a directed arc from vertices 1 to 2, 1 to 3, 1 to 4, 2 to 3, 2 to 4 and 4 to 2. Only matrix A reflects this. The answer is A.

Question 9

Row 1 has 1 zero. Box this and cross out the other zeros in the same column.

	W	X	Y	Z
A	0	5	10	8
B	0	0	11	6
C	9	0	4	0
D	0	0	0	7

Row 2 now has 1 zero. Box this and cross the other zeros in the same column.

	W	X	Y	Z
A	[0]	5	10	8
B	~~0~~	0	11	6
C	9	0	4	0
D	~~0~~	0	0	7

The other two rows now have 1 zero each.

	W	X	Y	Z
A	[0]	5	10	8
B	~~0~~	[0]	11	6
C	9	~~0~~	4	0
D	~~0~~	~~0~~	0	7

The allocation is : courier A to destination W.
courier B to destination X.
courier C to destination Z.
courier D to destination Y.

The answer is B.