JOHN MONASH SCIENCE SCHOOL

VICTORIAN CERTIFICATE OF EDUCATION

2017

ALGORITHMICS (HESS)

Practice Exam 1

2017

SOLUTIONS

SECTION A – Multiple-choice Questions

Question	%A	%B	%C	%D	%No Answer	Comments
1	69	0	0	31	6	**Correct Answer :** A Many students failed to notice that the directedness of this graph did not lead to the creation of any cycles.
2	6	13	81	0	6	**Correct Answer :** C
3	0	0	19	81	6	**Correct Answer :** D
4	6	69	25	0	6	**Correct Answer :** B Many students were tricked by node C being connected to Node A AND Node B meaning that it still appeared immediately after B in the depth first traversal.
5	88	6	0	6	6	**Correct Answer :** A
6	0	0	6	94	6	**Correct Answer :** D
7	6	0	13	75	12	**Correct Answer :** D
8	6	6	25	63	6	**Correct Answer :** D
9	88	13	0	0	6	**Correct Answer :** A
10	19	63	0	19	6	**Correct Answer :** B
11	0	25	6	69	6	**Correct Answer :** D
12	13	44	44	0	6	**Correct Answer :** D The time complexity of Prim's algorithm was not well known by students to be $O(V^2)$.

END OF QUESTION AND ANSWER BOOK

13	81	19	0	0	6	**Correct Answer :** A
14	0	13	69	19	6	**Correct Answer :** C
15	31	19	31	19	6	**Correct Answer :** A Many students go confused by the differences between NP hard and NP complete. Answer A should have been NP hard problems are at least as hard as NP complete problems to be a true statement.
16	6	75	0	19	6	**Correct Answer :** B
17	31	63	6	0	6	**Correct Answer :** A The use of the master theorem to find the complexity of this algorithm was not well done
18	13	88	0	0	6	**Correct Answer :** B
19	13	0	88	0	6	**Correct Answer :** C
20	6	0	6	88	6	**Correct Answer :** D

SECTION B
Question 1 (3 marks)

George has started up a company producing chocolate chip cookies (mainly to profit off Curtis' recent obsession). Unfortunately, he has recently been receiving a large number of complaints that some of his chocolate chip cookies do not contain any chocolate chips.

Upon investigation of his cookie-making machine, George discovers that there is always exactly one cookie from each batch that does not contain any chocolate chips. This cookie always weighs significantly less than the cookies that do have chocolate chips.

George wishes to develop an algorithm that will check the chocolate chip cookies and identify ones that don't have chocolate chips in them. He does not want to check each chocolate chip cookie individually though.

Describe an algorithmic design pattern that George could use to successfully identify the chocolate chip cookies that do not contain any chocolate chips and explain how this algorithm will solve the problem efficiently.

Marks	**0**	**1**	**2**	**3**	**Average**
%	24	18	35	24	**1.59**

George could use a decrease and conquer approach	A1
By repeatedly dividing the batch of cookies in half and weighing them against one another the lighter half can be chosen to then be split in half again until the faulty chocolate chip cookie can be identified.	A2
This leads to a faster solution as the time complexity of this is $O(\log(n))$ rather than $O(n)$ for the worst case time complexity of the brute force approach.	A3

Question 2 (2 marks)

As a way of practising mindfulness (and avoiding eating too many of George's cookies), Angus decides to spend 5 minutes every day writing signature specifications for various ADTs. Today, he begins writing the signature specification for a graph.

```
name:        graph
import:      list, element, boolean
operations:  empty : → Graph
             addNode : graph x element → graph
             addEdge :
             adjacent : graph x element x element → Boolean
             neighbours : graph x element → list
```

Angus can't quite remember what the specification is for the addEdge operation.

In the space below, write the specification for the addEdge operation of the graph ADT.

Marks	**0**	**1**	**2**	**Average**
%	12	6	82	**1.71**

`addEdge ; graph x element x element → graph`	A1 + A2

Question 3 (17 marks)

Marcus has been following George's cookie making company with an enthusiasm that borders on being obsessive. He is intrigued by the fact that the density of the chocolate chip cookies seems to be abnormally large. Through his hacking of George's email, he learns that the chocolate chips come from a remote island in Greece. Marcus decides to investigate.

Marcus begins to pack a bag for his trip. He has a wide variety of items that each have a weight and value associated with them.

Marcus wants to find out what the best combination of items he should put in his bag to maximise the total value of his bag, whilst keeping it under a certain weight limit.

At first Marcus uses a greedy approach towards selecting items to put in the bag.

a. Describe what a greedy approach towards solving this problem might look like and explain why it won't necessarily lead to an optimal solution. 3 marks

Marks	0	1	2	3	Average
%	12	0	18	71	**2.47**

A Greedy approach would follow some rule whereby Marcus might always take the most valuable object	A1
available given the remaining weight restriction on the bag until the bag is full or no other objects can fit in the bag.	A2
It doesn't consider the possibility of two lower value items adding together to have a larger value than another large item.	A3

b. How might a dynamic programming approach be used to find a solution to Marcus' bag packing problem in reasonable time? 3 marks

Marks	0	1	2	3	Average
%	30	24	24	24	**1.29**

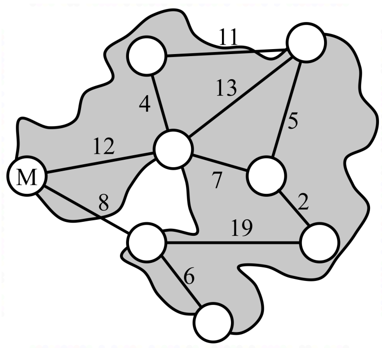
Dynamic Programming involves storing the outcomes of calculations that can then be recalled rather than recalculated over and over again.	A1
In this situation, dynamic programming could be used to construct a table of values that determine what the optimum selection will be for a 1kg weight limit, a 2kg weight limit, and so on.	A2
By starting at the lower end and working upwards, it becomes possible to use previous calculations to determine what the best combination will be.	A3

c. Explain why Marcus' bag packing problem is a NP-hard problem and not an Non-deterministic Polynomial-time (NP) problem. 3 marks

Marks	0	1	2	3	Average
%	35	18	29	18	**1.29**

An NP problem is one for which it is not possible to get a solution to in polynomial time but that it is possible to verify the solution of in polynomial time.	A1
In this case, because Marcus wants to find the optimum solution, the only way of determining if it is optimum is to compare the given solution with every other available solution to see if it is better than them all.	A2
As a result, it is not verifiable in polynomial time and hence is NP-hard.	A3

Marcus uses a travel algorithm created by Alex to determine the most effective way of getting to the remote island where the chocolate chips originate. When he arrives at the island he finds the following map of the island.



Each vertex on this map represents a different community living on the island. Marcus is initially located at the vertex labelled M. The weightings on each edge connecting the vertices represent the time, in hours, it will take Marcus to walk from one community to another.

At this moment, Marcus doesn't know the location of the community where the chocolate chips are made. He remembers back to an Algorithmics class where he learnt about Prim's Algorithm, Dijkstra's Algorithm, the Bellman-Ford Algorithm and the Floyd-Warshall Algorithm.

d. **i.** Which of these four algorithms would be most suitable for Marcus to use to find the shortest path from node M to all other nodes in the map? Give reasons for your answer. 3 marks

Marks	0	1	2	3	Average
%	24	29	35	12	**1.35**

The Bellman-Ford Algorithm is the most appropriate algorithm to use for this problem.	A1
As Marcus does not know which community to travel to but does know his starting position, he is looking for the shortest path from a node to all other nodes which is the problem solved by the Bellman-Ford Algorithm.	A2
He could use Dijkstra's but would be required to do this repeatedly to get the shortest path to all other nodes and he could use Floyd-Warshall but this would give the shortest path for all pairs of nodes which is more information than is required.	A3

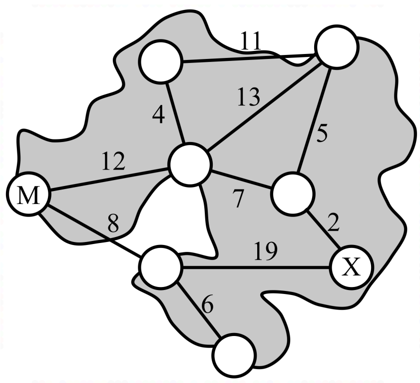
Marcus hears a church bell ringing in the distance and so decides to use the Bellman-Ford algorithm.

ii. How many iterations of the Bellman-Ford Algorithm will this graph require to find the shortest distance from M to all other nodes? 1 mark

Marks	**0**	**1**	**Average**
%	84	18	**0.18**

3 after 3 iterations the distances assigned to each node will remain unchanged.	A1

Marcus talks to some locals and learns that the community he is looking for is located at Node X in the diagram below.



iii. After two iterations of the Bellman-Ford Algorithm what distance, from M, will the algorithm have stored for node X? 1 mark

Marks	**0**	**1**	**Average**
%	35	65	**0.65**

27	A1

iv. Which algorithm would be most suitable for Marcus to use to find the shortest path from node M to node X? Give reasons justifying your choice. 3 marks

Marks	**0**	**1**	**2**	**3**	**Average**
%	12	12	41	35	**2.00**

Dijkstra’s would be the most suitable algorithm for this problem	A1
Dijkstra’s is used to find the shortest distance between two nodes	A2
As the graph does not contain any negative edges, Dijkstra’s will be the most effective algorithm to use.	A3

Upon arriving at the community, Marcus discovers Tom leading the chocolate chip making process. Tom explains that the reason the chocolate chips are so dense is because they have been made out of a combination of chocolate and dark matter to make a ‘dark chocolate matter chip’.

How Tom and George came by the dark matter was not made clear but Marcus knew better than to ask too many questions.

Question 4 (9 marks)

Kyle was caught on facebook by Mr Chisholm during Chemistry the other day and, as punishment, was forced to do some Geography work. Mr Chisholm got the coloured pencils and asked Kyle to colour in a map of the distribution of different types of tourism and tourist destinations in Eastern Europe. Mr Chisholm insisted that Kyle colour it in so that no two adjacent regions were the same colour and using as few colours as possible (remember Mr Chisholm is colour blind).

a. Describe the decision version of the graph colouring problem and explain why it is a Non-deterministic Polynomial-Time (NP) problem. 3 marks

Marks	**0**	**1**	**2**	**3**	**Average**
%	35	12	24	29	**1.47**

The decision version of this problem would be, "Is it possible to colour the map in *n* colours or less?"	A1
This is NP as while finding the solution cannot be done in polynomial time	A2
It can be verified in polynomial time as once you have a solution it is easy to determine if it has coloured the map in *n* colours or less.	A3

b. How could Kyle make use of Randomised Heuristics to aid him in finding a solution to Mr Chisholm's problem. 3 marks

Marks	**0**	**1**	**2**	**3**	**Average**
%	35	24	24	18	**1.24**

He could randomly colour the map in with *x* number of colours	A1
He could then look for parts of the map that do not meet the conditions of the task and start changing them in an attempt to find a valid solution	A2
Making all required changes until the map meets the specified conditions.	A3

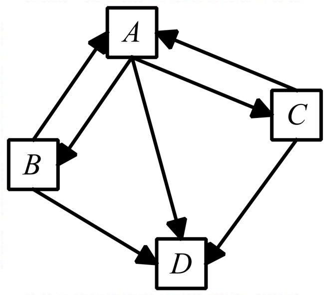
c. Would the use of Randomised Heuristics lead to Kyle finding the optimal solution to the problem? 3 marks

Marks	**0**	**1**	**2**	**3**	**Average**
%	6	18	24	53	**2.24**

It might	A1
But this is unlikely	A2
As it might find a 'good' solution but the chance of it being the optimal solution is very small.	A3

Question 5 (6 marks)

The following graph represents links between a series of web pages.



The PageRank of Page A is given by

$$\mathrm{PR}(A) = \frac{(1-d)}{N} + d\left(\frac{\mathrm{PR}(B)}{\mathrm{L}(B)} + \frac{\mathrm{PR}(C)}{\mathrm{L}(C)} + \frac{\mathrm{PR}(D)}{\mathrm{L}(D)}\right)$$

where $\mathrm{PR}(x)$ is the PageRank of Page x, N is the number of pages in this network and $\mathrm{L}(x)$ is the number of outgoing links from Page x.

a. Explain the purpose of d in the PageRank. 2 marks

Marks	**0**	**1**	**2**	**Average**
%	35	47	18	**0.82**

d is the damping factor	A1
It represents the probability of a surfer following the links on the webpage rather than just visiting a page randomly.	A2

b. What does $\frac{(1-d)}{N}$ represent in the PageRank? 1 mark

Marks	**0**	**1**	**Average**
%	94	6	**0.06**

This represents the probability of a surfer randomly clicking on one of the pages rather than following a link to the page.	A1

c. What does $d\left(\frac{\mathrm{PR}(B)}{\mathrm{L}(B)} + \frac{\mathrm{PR}(C)}{\mathrm{L}(C)} + \frac{\mathrm{PR}(D)}{\mathrm{L}(D)}\right)$ represent in the PageRank? 1 mark

Marks	**0**	**1**	**Average**
%	88	12	**0.12**

The probability of a surfer following a link on existing pages to arrive at the page	A1

d. Explain how PageRank would handle the values attributed to node D given that there are no outbound links from Page D? 2 marks

Marks	**0**	**1**	**2**	**Average**
%	94	0	6	**0.12**

It would evenly distribute the weighting of Node D between all other nodes	A1 + A2

Question 6 (9 marks)

Marc and Faris are having an argument about Neural Networks and how they can be used to overcome soft limits of computability.

Marc states

"Neural Networks emulate the way in which information is processed in the human brain. As such, they do not contain algorithms that can be written down in the traditional sense and are therefore no longer Turing Complete."

Faris retorts with

"You're wrong Marc. Neural Networks, while able to handle a variety of problem types that traditional algorithms would find challenging, are no threat to the Church-Turing Thesis."

a. What is the Church-Turing Thesis and what does it mean to say that something is Turing Complete? 3 marks

Marks	**0**	**1**	**2**	**3**	**Average**
%	12	29	6	53	**2.00**

The CT thesis states that a problem is algorithmically computable if (and only if) it is computable by a Turing Machine	A1
A real world computer can be said to be Turing complete if it is possible to simulate a Turing Machine on it and hence a Turing Machine can simulate the real world computer.	A2
This implies that all forms of computation can be reduced to the basic operations of a Turing Machine.	A3

b. Explain what Marc means when he states that Neural Networks "do not contain algorithms that can be written down in the traditional sense". 2 marks

Marks	**0**	**1**	**2**	**Average**
%	18	47	35	**1.18**

Neural networks have an initial connection of nodes to one another that allow for the transmission of data from one node to another.	A1
The weightings on these nodes do not follow the traditional algorithm set up with for, if and while loops.	A2

c. Give an example of a problem type that Neural Networks are better suited at solving than traditional algorithms? 1 mark

Marks	**0**	**1**	**Average**
%	18	82	**0.82**

Facial recognition	A1

d. Explain how Neural Networks operate and how they relate to the concept of machine learning. 3 marks

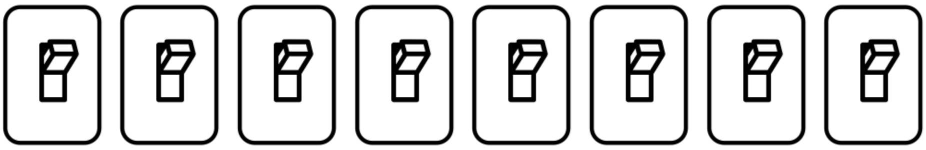
Marks	0	1	2	3	Average
%	12	24	35	29	**1.82**

Neural networks involve a network of nodes that communicate information to one another via weighted links	A1
Initially these weights might not lead to the neural network outputting the correct response but during training, the weights of edges are modified to lead to improved answers	A2
Through training, the machines can thus 'learn' to output the correct result.	A3

Question 7 (3 marks)

Tim was on his way home one night when he spotted a new escape room. Being an escape room enthusiast, he quickly handed over some money to get locked in the room.

Once in the room, he discovered that he could only escape by turning off a series of *n* switches. The switches are all arranged in a line as shown below.



After some experimentation he learns that the switches follow the following set of rules:

1. The rightmost switch may be turned on and off at will
2. Any other switch can only be turned on or off if the switch to its immediate right is on **and** all other switches to its right are turned off.
3. Only one switch may be toggled at a time.

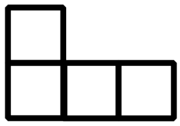
Write the pseudocode for an algorithm that would turn off all of the switches for Tim whilst obeying the set of rules outlined above.

Marks	0	1	2	3	Average
%	47	29	6	18	**0.94**

Algorithm finds a partial solution to the problem	A1
Algorithm uses an approach that will result in all of the switches being turned off	A2
Algorithm returns the sequence of light switches that Tim must turn on and off in order to result in all switches being turned off	A3

Question 8 (4 marks)

Dylan, after many competitions and trips overseas, has decided to settle down and become a tradie. In particular, after seeing some of the amazing mosaics in Spain, he is looking at becoming a tiler. As part of his apprenticeship with George (the Scottish maintenance worker at JMSS) Dylan is asked to tile a 16x16 sized grid with the unique L-shaped tile made up of four 1x1 squares as shown below.



Describe two different approaches that Dylan could use to find a possible arrangement of the L-shaped tiles on the 16x16 sized grid.

Marks	0	1	2	3	4	Average
%	12	6	41	12	29	2.41

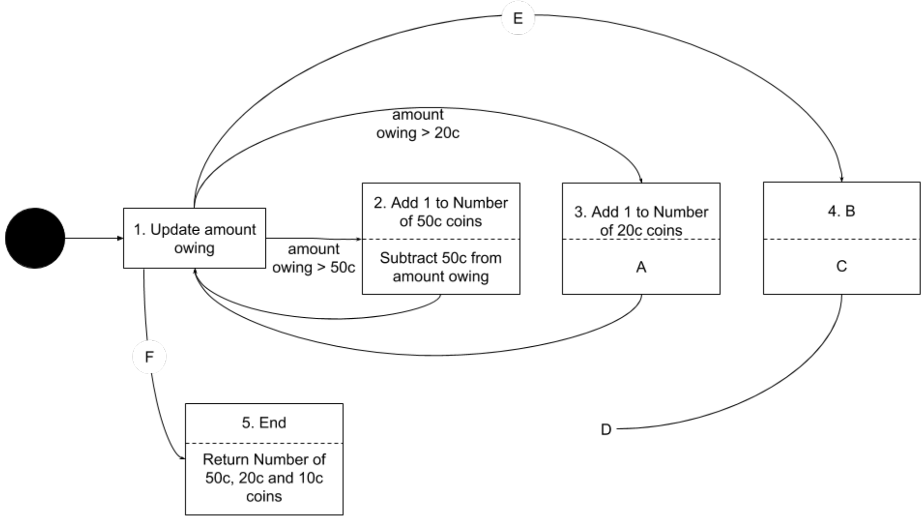
Decrease and Conquer approach	A1
Dylan could reduce the size of the grid to a 4x4 grid and find a solution to tiling this with the L-shaped tile and then use this solution to tile the entire 16x16 sized grid.	A2
Brute force approach	A3
Dylan could try every single combination of tiles until he has successfully tiled the entire grid. ****Accept any other valid approach that could have been used that includes an appropriate justification.**	A4

Question 9 (6 marks)

Noelani has been practising designing algorithms in the lead up to the exams. She has created an algorithm that will determine the amount of change a vending machine should give a customer using only 50c, 20c, and 10c coins.

In order to understand her algorithm better she draws a state diagram to represent the process her algorithm.

The diagram that Noelani has drawn so far is shown below. The upper case letters A through to F represent components that Noelani has not completed yet.



a. If state 3 is reached, Noelani expects an action to take place. What action should take place at A? 1 mark

Marks	0	1	Average
%	6	94	**0.94**

Subtract 20c from amount owing	A1

b. **i.** Noelani has identified a 4th state at B. What should the label of B be?. 1 mark

Marks	0	1	Average
%	6	94	**0.94**

Add 1 to Number of 10c coins	A1

ii. If state 4 is reached, Noelani expects an action to take place. What action should take place at C? 1 mark

Marks	0	1	Average
%	12	88	**0.88**

Subtract 10c from amount owing	A1

c. Noelani has started another transition labelled D. To which state should this transition go? 1 mark

Marks	0	1	Average
%	18	82	**0.82**

1	A1

d. For the transition from state 1 to 4, what should Noelani write at E? 1 mark

Marks	0	1	Average
%	18	82	**0.82**

Amount owing > 10c	A1

e. For the transition from state 1 to 5, what should Noelani write at F? 1 mark

Marks	0	1	Average
%	41	59	**0.59**

Amount owing <10c	A1

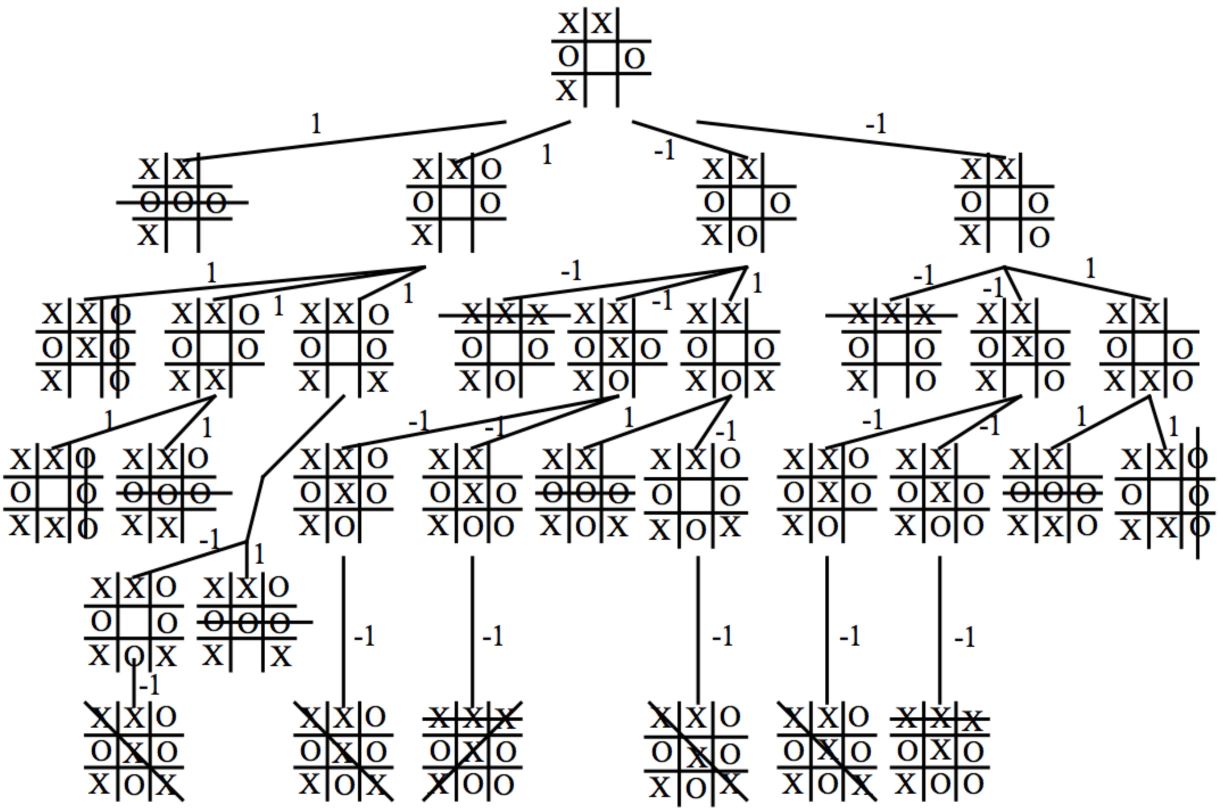
Question 10 (5 marks)

Percival and Riley are procrastinating and decide to play a game of tic-tac-toe. Because they have been studying tic-tac-toe in algorithmics and both know how to reach a draw with ease, they decide to play with different rules. The game is still played by Player 1 writing a cross in an unoccupied square of their choosing followed by Player 2 writing a naught in an unoccupied square of their choosing. This process is then repeated until one of the players either gets three naughts or three crosses in a row. In the new version of the game, however, the player who has three naughts or crosses in a row, loses.

After playing a few turns, Percy is struggling to figure out what to do next. He decides to distract Riley with a laser pointer whilst he uses the minimax algorithm to determine what move he should make next.

The current state of the board is shown below and it is Percy's turn to write a naught somewhere on the grid.

Using the minimax algorithm, complete the game tree until you can determine what move Percy should make next.



Marks	0	1	2	3	4	5	Average
%	18	41	29	12	0	0	1.35

Identification of left most path as optimal choice	A1
Identification of the second from the left path as another possible choice	A2
Partial documentation of the game tree shown	A3
Full documentation of the game tree shown	A4
Correctly labelled paths according to mini-max algorithm (1's and 0's)	A5

Question 11 (7 marks)

Jess has been growing rather concerned watching Curtis eating the cookies he has been purchasing from George every day. She is worried that Curtis will overdose on cookies if he is not careful. She comes up with a cunning plan to reduce the number of cookies that Curtis eats.

Every time Curtis takes a cookie from his box of cookies he becomes so distracted by the enjoyment of eating the cookies that Jess is able to steal the jar of cookies and remove some of the cookies before putting it back without him realising. She doesn't want to make the removal of the cookies obvious though so every time Curtis eats a cookie, she removes one third of the remaining cookies from the cookie jar and returns the rest.

a. Write the pseudocode for an algorithm, `operationCookie(jar)`, that describes Jess' recursive theft of cookies from Curtis' cookie jar. 3 marks

Marks	0	1	2	3	Average
%	18	41	18	24	**1.47**

Algorithm finds a partial solution to the problem or one that is not recursive	A1
Algorithm finds a solution to the problem that is recursive	A2
Base conditions for recursive algorithm have been included	A3

An example of a possible algorithm

```
operationCookie(jar)
    if jar has less than two cookies
        Halt
    else
        remove 1/3 of the cookies from the jar
        jar = ⌊2/3 * jar⌋
        place jar back in the original position
        if Curtis takes another cookie
            operationCookie(Jar)
```

b. Write a recurrence relation for the time complexity of `operationCookie(jar)` and use the Master Theorem to solve the time complexity. 4 marks

Marks	0	1	2	3	4	Average
%	41	18	35	6	0	**1.06**

Correct identification of base operation	A1
Correct identification of relation of one step of the recurrence relation	A2
Correct identification of general recurrence relation	A3
Accurate use of the master theorem to identify the time complexity of the algorithm from part a.	A4

Question 12 (9 marks)

William has been pondering the meaning of intelligence. Unfortunately, he is finding it difficult to ponder this during class when people keep talking about Chinese speakers. He finds himself a quiet room with a large book in it where he can ponder intelligence in peace. Just as he has made himself comfortable, people start sliding notes written in chinese under the door. Frustrated, he puts his headphones in and listens to some ‘Rage Against the Machine’.

If William had been paying more attention, he might have realised that the class was actually discussing John Searle’s argument regarding artificial intelligence.

a. In the space below, describe John Searle’s Chinese Room Argument and John Searle’s position regarding the possibility of artificial intelligence. 3 marks

Marks	0	1	2	3	Average
%	24	24	24	29	**1.59**

The Chinese Room argument places a non-Chinese speaking person inside a room that contains a book of instructions on what to do when presented with certain Chinese characters.	A1
Chinese speakers can send messages to the person in the room who then uses the instruction book to respond. To the Chinese speakers outside the room, the person inside appears to understand Chinese but the person inside the room actually does not.	A2
Searle uses this argument to suggest that computers, like the person in the room, are not capable of understanding what they are doing and hence are not capable of ever achieving strong artificial intelligence and the appearance of intelligence that they might exhibit are simply examples of ‘weak artificial intelligence’, that is, simulated intelligence.	A3

b. Discuss two possible counter arguments to John Searle’s Chinese Room Argument. 6 marks

Marks	0	1	2	3	4	5	6	Average
%	35	12	6	6	24	12	6	**2.29**

The system’s reply While the individual in the room has no understanding of what they are doing,	A1
the entire system of the book, person and room can be considered to understand what it is doing.	A2
It is akin to an individual neuron in a brain not understanding English but the whole collection of neurons leading to a brain that can understand English.	A3
The Robot reply If the room were to possess additional sensory inputs	A4
Then the person in the room might be able to learn the meaning of what they are doing through observing the outcome of each response	A5
This is akin to how babies learn language	A6