Neap

Trial Examination 2023

Suggested Solutions

QCE Specialist Mathematics Units 3&4

Paper 1 – Technology-free

SECTION 1 – MULTIPLE CHOICE QUESTIONS

	A	B	C	D
1.	○	○	○	●
2.	○	○	●	○
3.	●	○	○	○
4.	○	○	○	●
5.	○	○	●	○
6.	○	○	○	●
7.	○	●	○	○
8.	○	○	●	○
9.	○	●	○	○
10.	○	●	○	○

QUESTION 1 D

D is correct.

$f(t) = \lambda e^{-\lambda t}$

$\therefore \lambda = 2$

$$E(t) = \frac{1}{\lambda} = \frac{1}{2}$$

A is incorrect. This option may be reached by calculating $E(t)$ instead of λ and $SD(t)$ instead of $E(t)$.

B is incorrect. This option may be reached by confusing the values of λ and $E(t)$.

C is incorrect. This option may be reached by calculating $SD(t)$ instead of $E(t)$.

QUESTION 2 C

C is correct.

$$\left[\begin{array}{ccc|c} 3 & 1 & -4 & -10 \\ 1 & 1 & 0 & 10 \\ 1 & -1 & 2 & 6 \end{array}\right] \begin{array}{l} -3R_2 \\ \\ -R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 0 & -2 & -4 & -40 \\ 1 & 1 & 0 & 10 \\ 0 & -2 & 2 & -4 \end{array}\right] \begin{array}{l} \text{swap with } R_2 \\ \text{swap with } R_1 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 10 \\ 0 & -2 & -4 & -40 \\ 0 & -2 & 2 & -4 \end{array}\right]$$

A is incorrect. This option may be reached by not swapping R_1 and R_2.

B is incorrect. This option may be reached by not applying the operations to the second part of the augmented matrix.

D is incorrect. This option may be reached by subtracting R_1 from R_3, rather than subtracting R_2 from R_3.

QUESTION 3 A

A is correct.

$$\begin{aligned}(2+i)(3i-5) &= 6i - 10 + 3i^2 - 5i \\ &= i - 10 - 3 \\ &= i - 13\end{aligned}$$

B is incorrect. This option may be reached by simplifying $3i^2$ to 3.

C is incorrect. This option may be reached by ignoring the negative in the second bracket.

D is incorrect. This option may be reached by assuming that i is in the second term of the second bracket.

QUESTION 4 D

D is correct. Reducing the level of confidence will decrease the interval. Given that $\bar{x} = 14.4$, the only feasible answer is (14.0, 14.8).

A is incorrect. This option may be reached by increasing the interval.

B is incorrect. This option may be reached by decreasing the interval, but changing the value of $\bar{x}$.

C is incorrect. This option may be reached by shifting the interval up and thus changing the value of $\bar{x}$.

QUESTION 5 C

C is correct. Using the general form of a sphere, where $a = -3$, $b = 4$, $c = 0$ and $r = 3$, gives:

$$(x-a)^2 + (y-b) + (z-c)^2 = r^2$$
$$(x-(-3))^2 + (y-4)^2 + (z-0)^2 = 3^2$$
$$(x+3)^2 + (y-4)^2 + z^2 = 9$$

A is incorrect. This option may be reached by misinterpreting the placement of the centre in terms of negatives and positions.

B is incorrect. This option may be reached by misinterpreting the placement of the centre in terms of negatives and positions, and finding the cube of the radius.

D is incorrect. This option may be reached by finding the cube of the radius, rather than the square.

QUESTION 6 D

D is correct.

$\boldsymbol{n} = 2\boldsymbol{i} + 5\boldsymbol{j} - 3\boldsymbol{k}$

$\boldsymbol{a} = -\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$

Therefore:

$$\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$$
$$(x\boldsymbol{i} + y\boldsymbol{j} - z\boldsymbol{k}) \cdot (2\boldsymbol{i} + 5\boldsymbol{j} - 3\boldsymbol{k}) = (-\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}) \cdot (2\boldsymbol{i} + 5\boldsymbol{j} - 3\boldsymbol{k})$$
$$2x + 5y - 3z = 19$$

A is incorrect. This option may be reached by confusing $\boldsymbol{a}$ and $\boldsymbol{n}$.

B is incorrect. This option may be reached by incorrectly multiplying $-2 \times -3 = -6$.

C is incorrect. This option may be reached by adding the inside terms of 3 and 5, rather than multiplying.

QUESTION 7 B

B is correct. Completing the induction step gives:

$$\begin{aligned}2^{2(k+1)-1} + 3^{2(k+1)-1} &= 2^{2k+2-1} + 3^{2k+2-1} \\ &= 2^{2k+1} + 3^{2k+1} \\ &= 2^2 \times 2^{2k-1} + 3^2 \times 3^{2k-1} \\ &= 4 \times 2^{2k-1} + 9 \times 3^{2k-1} \\ &= 4 \times \text{ assumption step } + 5 \times 3^{2k-1}\end{aligned}$$

A is incorrect. This option could be used as a next step, but it does not lead towards substituting the assumption step.

C is incorrect. This option could be used as a next step as it attempts to get factors of 5, but it does not bring the problem closer to a solution.

D is incorrect. This option may be reached by incorrectly using indices.

QUESTION 8 C

C is correct. The shaded area is inside the complex region with modulus 3, but outside the complex region with modulus 2.

Therefore, the intersection is $|z-1| \leq 3 \cap |z-i| \geq 2$.

A is incorrect. This option may be reached by assuming the shaded area is inside both complex regions.

B is incorrect. This option may be reached by assuming the shaded area is inside the complex region with modulus 2 and outside the complex region with modulus 3.

D is incorrect. This option may be reached by assuming the shaded area is outside both complex regions.

QUESTION 9 B

B is correct. The argument of a complex number is the measurement of the angle of its direction from the positive x-axis.

$$\begin{aligned}\text{Arg}(z_1) &= \tan^{-1}\left(\frac{2}{3}\right)\\ &= 0.588\end{aligned}$$

$$\begin{aligned}\text{Arg}(z_2) &= \tan^{-1}\left(\frac{1}{-2}\right)\\ &= -0.4636 \text{ (fourth quadrant)}\\ &= \pi - 0.4636\\ &= 2.6779 \text{ (second quadrant)}\end{aligned}$$

Therefore, $\text{Arg}(z_1) > \text{Arg}(z_2)$.

A is incorrect. $|z_1| = \sqrt{5}$ and $|z_2| = \sqrt{13}$; therefore, $|z_1| < |z_2|$.

C is incorrect. $\text{Im}(z_1) = 1$ and $\text{Im}(z_2) = 2$; therefore, $\text{Im}(z_1) < \text{Im}(z_2)$.

D is incorrect. $\text{Re}(z_1) = -2$ and $\text{Re}(z_2) = 3$; therefore, $\text{Re}(z_1) < \text{Re}(z_2)$.

QUESTION 10 B

B is correct. The sample mean is $\bar{x} = 30$.

Finding the sample standard deviation gives:

$$\begin{aligned}s &= \frac{\sigma}{\sqrt{n}}\\ &= \frac{\sigma}{\sqrt{100}}\\ &= \frac{\sigma}{10}\end{aligned}$$

As $n = 100$, normality is assumed.

Note: 99.7% of data is expected to lie within 3 × s.

A is incorrect. This graph may be reached by not dividing the standard deviation by $\sqrt{n}$.

C is incorrect. This graph may be reached by not considering the expected normality of the sample.

D is incorrect. This graph may be reached by dividing the sample mean by $\sqrt{n}$.

SECTION 2

QUESTION 11 (6 marks)

a) $l_1 = (4+3\lambda)\boldsymbol{i} + 5\boldsymbol{j} + (-10-\lambda)\boldsymbol{k}$

$l_2 = (-3-\mu)\boldsymbol{i} + (1+2\mu)\boldsymbol{j} + (-1-3\mu)\boldsymbol{k}$

Therefore:

$5 = 1 + 2\mu$

$\mu = 2$

Substituting into l_2 gives:

$l_2 = (-3-2)\boldsymbol{i} + (1+2\times 2)\boldsymbol{j} + (-1-3\times 2)\boldsymbol{k}$

$= -5\boldsymbol{i} + 5\boldsymbol{j} - 7\boldsymbol{k}$

Therefore:

$4 + 3\lambda = -5$

$\lambda = -3$

$l_1 = (4+3(-3))\boldsymbol{i} + 5\boldsymbol{j} + (-10-(-3))\boldsymbol{k}$

$= -5\boldsymbol{i} + 5\boldsymbol{j} - 7\boldsymbol{k}$

Therefore, lines l_1 and l_2 intersect at the point $-5\boldsymbol{i} + 5\boldsymbol{j} - 7\boldsymbol{k}$.

[2 marks]
1 mark for determining μ OR λ.
1 mark for providing the correct solution.

b) $\boldsymbol{d}_1 \cdot \boldsymbol{d}_3 = (3\boldsymbol{i} - \boldsymbol{k}) \cdot (-\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k})$

$= 3\times(-1) + (-1)\times(-3)$

$= 0$

Therefore, l_1 and l_2 are perpendicular.

[1 mark]
1 mark for showing that the dot product is equal to 0 and therefore the lines are perpendicular.

c)
$$d_1 \times d_3 = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 0 & -1 \\ -1 & 2 & -3 \end{vmatrix}$$

$= (0\times(-3) - (-1)\times 2)\boldsymbol{i} - \left(3\times(-3) - (-1)\times(-1)\right)\boldsymbol{j} + \left(3\times 2 - 0\times(-1)\right)\boldsymbol{k}$

$= 2\boldsymbol{i} + 10\boldsymbol{j} + 6\boldsymbol{k}$

$r \cdot n = a \cdot n$

$(x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}) \cdot (2\boldsymbol{i} + 10\boldsymbol{j} + 6\boldsymbol{k}) = (-5\boldsymbol{i} + 5\boldsymbol{j} - 7\boldsymbol{k}) \cdot (2\boldsymbol{i} - 10\boldsymbol{j} + 6\boldsymbol{k})$

$2x + 10y + 6z = -10 + 50 - 42$

$2x + 10y + 6z = -2$

$x + 5y + 3z = -1$

[3 marks]
1 mark for using an appropriate plane formula and substituting the correct values.
1 mark for using an appropriate technique using d_1 and d_2 to find the perpendicular vector.
1 mark for providing the equation of the plane.
*Note: Consequential on answer to **Question 11b)**.*

QUESTION 12 (4 marks)

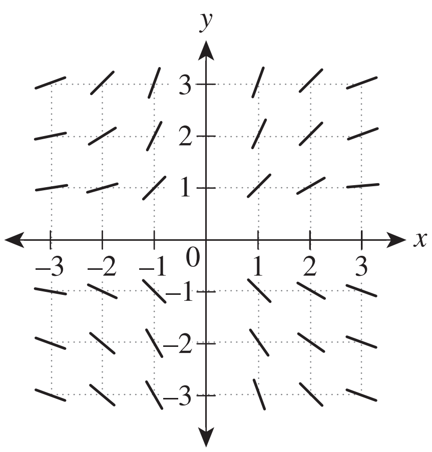
a) Slope field A represents $\frac{dy}{dx} = x\cos(y)$.

Slope field B represents $\frac{dy}{dx} = x + y^2$.

In slope field A, each coordinate on the *y*-axis has a gradient of 0, which matches the differential equation $\frac{dy}{dx} = x\cos(y)$ because each point on the *y*-axis has an *x*-value of 0.

[2 marks]
1 mark for linking each differential equation with its slope field.
1 mark for explaining reasoning.
Note: Accept any suitable considerations of key differences, such as the negative versus positive gradient values in quadrant 1 and 4; a series of individual points (at least two); observation of non-zero values on the y-axis of slope field B; or gradients of zero at points (–1, 1) and (–1, –1) in slope field B.

b)



[2 marks]
1 mark for providing gradient marks for all values in each quadrant.
1 mark for sketching all gradient marks suitably to show approximate patterns.
Note: Be considerate of the slope of each mark and accept some error; consider where the slopes should be increasing or decreasing, as shown in the diagram.

QUESTION 13 (5 marks)

a) If $p(2) = 0$, then $(z - 2)$ is a factor.

[1 mark]
1 mark for stating that $(z - 2)$ is a factor.
Note: Accept $(z + 3 - 2i)$ or $(z + 3 + 2i)$ if they are provided.

b)
$$\begin{aligned} p(z) &= z^3 + 4z^2 + z - 26 \\ &= (z-2)\left(Az^2 + Bz + C\right) \\ &= Az^3 + (B - 2A)z^2 + (C - 2B)z - 2C \end{aligned}$$

Equating cubic terms gives:

$A = 1$

Equating the constant terms gives:

$-26 = -2C$

$C = 13$

Equating the quadratic terms gives:

$B - 2A = 4$

$B = 6$

Therefore:

$$\begin{aligned} p(z) &= (z-2)\left(z^2 + 6z + 13\right) \\ &= (z-2)\left(z^2 + 6z + 9 + 4\right) \\ &= (z-2)\left((z+3)^2 + 4\right) \\ &= (z-2)\left((z+3)^2 - (2i)^2\right) \\ &= (z-2)(z+3-2i)(z+3+2i) \end{aligned}$$

[4 marks]
1 mark for showing $(z - 2)$ as a factor with a quadratic multiple.
1 mark for determining the quadratic factor in the expression for $p(z)$.
1 mark for using a suitable technique to factorise the quadratic.
1 mark for providing the complete factorisation.
*Note: Consequential on answer to **Question 13a)**.*

QUESTION 14 (5 marks)

a) $\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 2 & -3 & 3 & -5 \\ 0 & 2 & -1 & 8 \end{array}\right]$

[1 mark]
1 mark for providing the correct solution.

b) $\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 2 & -3 & 3 & -5 \\ 0 & 2 & -1 & 8 \end{array}\right] \begin{array}{c} \\ -2R_1 \\ \\ \end{array}$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & -1 & 8 \end{array}\right] \begin{array}{c} \\ \\ -2R_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -3 & 6 \end{array}\right] \begin{array}{c} \\ \\ \div -3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$

Therefore:

$z = -2$

$y + z = 1$

$y = 3$

$x - 2y + z = -3$

$x = 5$

[4 marks]
1 mark for subtracting $2R_1$ to simplify row 2.
1 mark for using row-echelon form.
1 mark for finding the value of one variable.
1 mark for finding the values of the two remaining variables.
Note: Allow any suitable Gaussian technique.

QUESTION 15 (7 marks)

a) $\int \tan^3(x)\sec^2(x)\,dx$

Letting $u = \tan(x)$ gives:

$$\frac{du}{dx} = \sec^2(x)$$

$$dx = \frac{du}{\sec^2(x)}$$

$$\begin{aligned}\int \tan^3(x)\sec^2(x)\,dx &= u^3 \cancel{\sec^2(x)}\frac{du}{\cancel{\sec^2(x)}} \\ &= \int u^3\,du \\ &= \frac{u^4}{4} + c \\ &= \frac{1}{4}\tan^4(x) + c\end{aligned}$$

[3 marks]
1 mark for choosing the correct substitution.
1 mark for using $\frac{du}{dx}$ to eliminate $\sec^2(x)$.
1 mark for providing the correct final integral.

b) $\int_2^4 \frac{\ln x}{x^2}\,dx$

If $u = \ln x,\ u' = \frac{1}{x}$.

If $v' = x^{-2}$:

$$\begin{aligned} v &= -x^{-1} \\ &= -\frac{1}{x} \end{aligned}$$

$$\begin{aligned} \int_2^4 \frac{\ln x}{x^2}\,dx &= \left[-\frac{\ln x}{x}\right]_2^4 - \int_2^4 \frac{1}{x} \times -\frac{1}{x}\,dx \\ &= \left[-\frac{\ln x}{x}\right]_2^4 - \int_2^4 -x^{-2}\,dx \\ &= \left[-\frac{\ln x}{x}\right]_2^4 + \left[-\frac{1}{x}\right]_2^4 \\ &= \left(-\frac{\ln 4}{4} - \left(-\frac{\ln 2}{2}\right)\right) - \left(\frac{1}{4} - \frac{1}{2}\right) \\ &= -\frac{\ln 2}{2} + \frac{\ln 2}{2} + \frac{1}{4} \\ &= \frac{1}{4} \text{ units}^2 \end{aligned}$$

[4 marks]

1 mark for identifying integration by parts and finding the appropriate u and v′ values.

1 mark for substituting into the integration by parts rule.

1 mark for completing the integration up to the substitution of boundaries.

1 mark for providing the correct solution.

Note: Consequential on answer to ***Question 15a)****.*

QUESTION 16 (5 marks)

Initial statement:

$$\begin{aligned} f(1) &= 5^{1+1} - 4\times 1 + 11 \\ &= 25 - 4 + 11 \\ &= 32 \\ &= 16\times 2 \end{aligned}$$

Assumption step:

$f(k) = 5^{k+1} - 4k + 11 = 16A, A \in Z$

Inductive step:

Required to prove that $f(k+1) = 5^{(k+1)+1} - 4(k+1) + 11 - 16B, B \in Z$

$$\begin{aligned} \text{LHS} &= 5^{(k+1)+1} - 4(k+1) + 11 \\ &= 5^{5+2} - 4k - 4 + 11 \\ &= 5\times 5^{k+1} - 4k + 7 \\ &= 5\times 5^{k+1} - 20k + 55 + 16k - 48 \\ &= 5(5^{k+1} - 4k + 11) + 16(k-3) \\ &= 5\times 16A + 16(k-3) \\ &= 16(5A + k - 3) \\ &= 16B \end{aligned}$$

$\therefore B = 5A + k - 3,\ B \in Z$

Conclusion:

It has been shown that if the rule works for $n = k$, then it must also work for $n = k + 1$. Thus, since step 1 proved that it was true for $n = 1$, it must also be true for $n = 2$. Additionally, if it is true for $n = 2$, it must be true for $n = 3$ and so on…

[5 marks]

1 mark for proving the initial statement.

1 mark for stating the assumption step and proof requirements for the inductive step.

1 mark for using the assumption step as part of the proof step.

1 mark for providing the evidence and reasoning used to identify the result as a multiple of 16.

1 mark for communicating the key steps of completing the proof.

QUESTION 17 (6 marks)

a) Writing $4-4\sqrt{3}i$ in polar form:

$$R=\sqrt{(4)^2+\left(4\sqrt{3}\right)^2}$$
$$=\sqrt{16+48}$$
$$=8$$

$$\theta=\tan^{-1}\left(\frac{-4\sqrt{3}}{4}\right)$$
$$=-\frac{\pi}{3}+2\pi k,\ k\in Z$$

Therefore:

$$z^3=8\text{ cis}\left(-\frac{\pi}{3}+2\pi k\right)$$

$$z=\left(8\text{ cis}\left(-\frac{\pi}{3}+2\pi k\right)\right)^{\frac{1}{3}}$$
$$=2\text{ cis}\left(-\frac{\pi}{9}+\frac{2\pi k}{3}\right)$$

For $k=0,\ z=2\text{ cis}\left(-\frac{\pi}{9}\right)$

For $k=1,\ z=2\text{ cis}\left(\frac{7\pi}{9}\right)$

For $k=-1,\ z=2\text{ cis}\left(\frac{13\pi}{9}\right)$

[3 marks]

1 mark for correctly writing z^3 in polar form.

1 mark for correctly applying De Moivre's theorem to determine a general solution for z.

1 mark for determining three unique solutions.

b) $$\left(\cos\left(\frac{5\pi}{8}\right)+i\sin\left(\frac{5\pi}{8}\right)\right)^a=i$$

$$\left(\text{cis}\left(\frac{5\pi}{8}\right)\right)^a=i$$

$$\text{cis}\left(\frac{5\pi a}{8}\right)=i$$

$$\text{cis}\left(\frac{5\pi a}{8}\right)=\text{cis}\left(\frac{\pi}{2}+2\pi k\right)$$

$$\frac{5\pi a}{8}=\frac{\pi}{2}+2\pi k$$

$$a=\frac{4}{5}+\frac{16}{5}k$$

As the first positive integer value of a occurs at $k=1$, $a=4$.

[3 marks]

1 mark for using De Moivre's theorem to bring a into the argument.

1 mark for applying any suitable rule to find a relevant value for a, in terms of k or otherwise.

1 mark for identifying a = 4 as the first positive integer.

QUESTION 18 (5 marks)

If $|z - w| = |z + w|$:

$$\left|(z_1 + z_2 i) - (w_1 + w_2 i)\right| = \left|(z_1 + z_2 i) + (w_1 + w_2 i)\right|$$

$$\left|(z_1 - w_1) + (z_2 - w_2)i\right| = \left|(z_1 + w_1) + (z_2 + w_2)i\right|$$

$$\sqrt{(z_1 - w_1)^2 + (z_2 - w_2)} = \sqrt{(z_1 + w_1)^2 + (z_2 + w_2)}$$

$$z_1^2 - 2z_1 w_1 + w_1^2 + z_2^2 - 2z_2 w_2 + w_2^2 = z_1^2 + 2z_1 w_1 + w_1^2 + z_2^2 + 2z_2 w_2 + w_2^2$$

$$-2z_1 w_1 - 2z_2 w_2 = -2z_1 w_1 + 2z_2 w_2$$

$$-4z_1 w_1 = 4z_2 w_2$$

$$z_1 w_1 = -z_2 w_2$$

$$\frac{w}{z} = \frac{w_1 + w_2 i}{z_1 + z_2 i}$$

$$= \frac{w_1 + w_2 i}{z_1 + z_2 i} \times \frac{z_1 - z_2 i}{z_1 - z_2 i}$$

$$= \frac{w_1 z_1 + z_1 w_2 i - z_2 w_1 i - z_2 w_2 i^2}{z_1^2 - z_2^2 i^2}$$

$$= \frac{w_1 z_1 + z_2 w_2 + (z_1 w_2 - z_2 w_1)i}{z_1^2 + z_2^2}$$

Using $z_1 w_1 = -z_2 w_2$ gives:

$$= \frac{-w_2 z_2 + z_2 w_2 + (z_1 w_2 - z_2 w_1)i}{z_1^2 + z_2^2}$$

$$= \frac{(z_1 w_2 - z_2 w_1)i}{z_1^2 + z_2^2}$$

$$= \left(\frac{z_1 w_2 - z_2 w_1}{z_1^2 + z_2^2}\right)i$$

Therefore, $\frac{w}{z}$ is purely imaginary, as required.

[5 marks]

1 mark for converting $|z - w| = |z + w|$ into a statement involving square roots.

1 mark for finding a relevant relationship between z_1, z_2, w_1 and w_2 from the statement involving square roots.

1 mark for using the complex conjugate to simplify the denominator of $\frac{w}{z}$ into a real number.

1 mark for using $z_1 w_1 = -z_2 w_2$ to simplify $\frac{w}{z}$.

1 mark for providing the final imaginary representation and concluding statement.

QUESTION 19 (7 marks)

$$\frac{dC}{dt} = r - kC$$

$$\left(\frac{1}{r-kC}\right)dC = dt$$

$$\int\left(\frac{1}{r-kC}\right)dC = \int dt$$

$$\frac{\ln|r-kC|}{-k} = t + c$$

At $t = 0$, $C = C_0$.

$$\frac{\ln|r-kC_0|}{-k} = 0 + c$$

$$c = -\frac{1}{k}\ln|r-kC_0|$$

Therefore:

$$-\frac{\ln|r-kC|}{k} = t - \frac{1}{k}\ln|r-kC_0|$$

$$\ln|r-kC| - \ln|r-kC_0| = -kt$$

$$\ln\left|\frac{r-kC}{r-kC_0}\right| = -kt$$

$$\frac{r-kC}{r-kC_0} = e^{-kt}$$

$$r-kC = (r-kC_0)e^{-kt}$$

$$C = \frac{1}{k}\left(r-(r-kC_0)e^{-kt}\right)$$

$$= \frac{r}{k} - \frac{(r-kC_0)}{k}e^{-kt}$$

To find $\lim_{t\to\infty}(C)$, the following components of the function must be considered.

- Both r and k are positive.
- $r - kC > 0$ due to the logarithm; therefore, both C and $C_0 < \frac{r}{k}$.

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(continued)

$$\lim_{t\to\infty}\left(e^{-kt}\right)=0$$

$$\begin{aligned}\lim_{t\to\infty}(C) &= \lim_{t\to\infty}\left(\frac{r}{k}-\frac{\left(r-kC_0\right)}{k}e^{-kt}\right)\\ &= \lim_{t\to\infty}\left(\frac{r}{k}\right)-\lim_{t\to\infty}\left(\frac{\left(r-kC_0\right)}{k}e^{-kt}\right)\\ &= \frac{r}{k}-\frac{r-kC_0}{k}\times\lim_{t\to\infty}\left(e^{-kt}\right)\\ &= \frac{r}{k}-\frac{\left(r-kC_0\right)}{k}\times 0\\ &= \frac{r}{k}\end{aligned}$$

[7 marks]

1 mark for rearranging the differential equation into an integral.
1 mark for solving the integral.
1 mark for determining a suitable value for the constant of integration, c.
1 mark for rearranging the equation to reach C = f(t).
1 mark for applying a limit argument to any suitable function.
1 mark for determining the correct limit for C.
1 mark for communicating key steps and using logical working.