Neap

Trial Examination 2021

Question and response booklet

QCE Specialist Mathematics Units 3&4

Paper 1 – Technology-free

Student's Name: ____________________

Teacher's Name: ____________________

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- Calculators are **not** permitted.
- Formula booklet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (55 marks)

- 8 short response questions

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

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SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has eight questions and is worth 55 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (8 marks)

Consider that ω is a non-real cube root of 1.

a) Show that $\omega^4 = \omega$ and $1 + \omega + \omega^2 = 0$. *[2 marks]*

b) Show that $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^5) = 9$. *[3 marks]*

c) Show that $(1 + \omega)(1 + 2\omega)(1 + 3\omega)(1 + 5\omega) = 21$. *[3 marks]*

QUESTION 12 (8 marks)

Consider that $U_n = \int_0^1 x^n \sqrt{1-x}\, dx$.

a) Show that $U_n = \frac{2n}{2n+3} U_{n-1}$. *[3 marks]*

b) Use the result from 12a) to prove that $U_n = \frac{n!(n+1)!}{(2n+3)!} 4^{n+1}$. *[5 marks]*

QUESTION 13 (6 marks)

A particle moves in a straight line. Its acceleration is given by $x - 4$ m s^{-2}. Initially, $v = \sqrt{5}$ m s^{-1} at $x = 7$ m.

a) Determine v in terms of x. *[4 marks]*

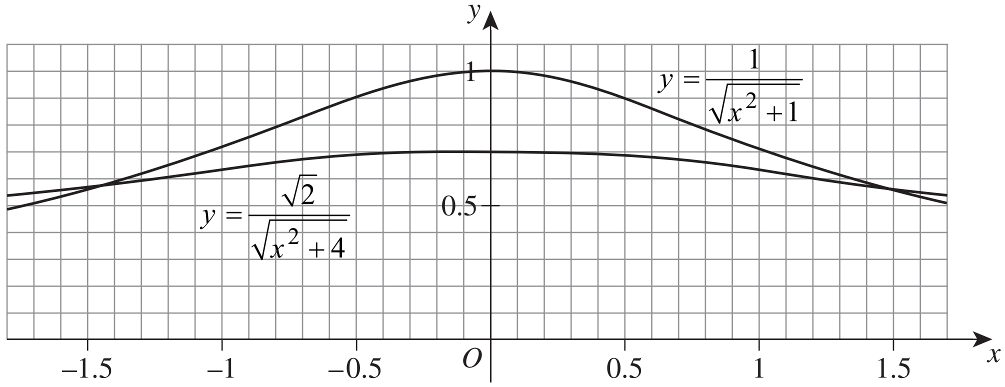
b) Describe the resultant motion, stating any restrictions on x. *[2 marks]*

QUESTION 14 (5 marks)

Use mathematical induction to prove that for all odd $n \geq 1$, $3^n + 7^n$ is divisible by 10.

QUESTION 15 (7 marks)

The curves $y = \dfrac{1}{\sqrt{x^2+1}}$ and $y = \dfrac{\sqrt{2}}{\sqrt{x^2+4}}$ are shown in the diagram below.



a) Show that the curves intersect at $x = \pm\sqrt{2}$. *[2 marks]*

b) The region bounded by the two curves is rotated about the x-axis.

By first considering the volume of one slice, show that the volume of the solid that is generated is $2\pi \arctan\left(\dfrac{1}{2\sqrt{2}}\right)$ using $\arctan(a) - \arctan(b) = \arctan\left(\dfrac{(a-b)}{(1+ab)}\right)$. *[5 marks]*

QUESTION 16 (7 marks)

a) Determine $\int \frac{1}{\sqrt{4x+3}}\,dx$. *[2 marks]*

b) Use the result from 16a) to solve the differential equation $\frac{dy}{dx} = \frac{\sqrt{4y+3}}{x^2}$ when $x = -2$ and $y = 1.5$. Express your answer in the form $y = f(x)$. *[5 marks]*

QUESTION 17 (7 marks)

In a chemical equation, a subscript indicates how many of each atom are present in the molecule.

For example, one molecule of H_3PO_4 has three atoms of H, one atom of P, and four atoms of O.

Consider the following chemical equation.

$$P_2I_4 + nP_4 + pH_2O \rightarrow 4PH_4I + qH_3PO_4$$

The coefficients for P_4, H_2O and H_3PO_4 are denoted by n, p, and q respectively.

a) Derive the equations necessary to solve for n, p, and q by equating the atoms of P, H, and O on both sides of the equation. *[2 marks]*

b) Using the result from 17a), use Gaussian elimination to solve for the values of n, p, and q. *[4 marks]*

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c) Verify that the values found in 17b) result in a balanced chemical equation. *[1 mark]*

QUESTION 18 (7 marks)

Two planes are defined by the equations $x + 2y - z = -3$ and $2x - y + z = 10$. A third plane is perpendicular to these planes and passes through the point (2, 1, 0).

a) Determine the Cartesian equation of the third plane. *[3 marks]*

b) Determine the point of intersection of all three planes. *[4 marks]*

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Formula sheet

QCE Specialist Mathematics Units 3&4

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Calculus	
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}\sin(x) = \cos(x)$	$\int \sin(x)dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$	$\int \cos(x)dx = \sin(x) + c$
$\frac{d}{dx}\tan(x) = \sec^2(x)$	$\int \sec^2(x)dx = \tan(x) + c$
$\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right) = \frac{-1}{\sqrt{a^2 - x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\tan^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2 + x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

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Calculus		
chain rule	If $h(x)=f\left(g(x)\right)$ then $h'(x)=f'\left(g(x)\right)g'(x)$	If $y=f(u)$ and $u=g(x)$ then $\dfrac{dy}{dx}=\dfrac{dy}{du}\times\dfrac{du}{dx}$
product rule	If $h(x)=f(x)g(x)$ then $h'(x)=f(x)g'(x)+f'(x)g(x)$	$\dfrac{dy}{dx}(uv)=u\dfrac{dv}{dx}+v\dfrac{du}{dx}$
quotient rule	If $h(x)=\dfrac{f(x)}{g(x)}$ then $h'(x)=\dfrac{f'(x)g(x)-f(x)g'(x)}{\left(g(x)\right)^2}$	$\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$
integration by parts	$\int f(x)g'(x)dx=f(x)g(x)-\int f'(x)g(x)dx$	$\int u\dfrac{dv}{dx}dx=uv-\int v\dfrac{du}{dx}dx$
volume of a solid of revolution	about the x-axis	$V=\pi\int_a^b\left[f(x)\right]^2dx$
	about the y-axis	$V=\pi\int_a^b\left[f(y)\right]^2dy$
Simpson's rule	$\int_a^b f(x)dx\approx\dfrac{w}{3}\left[f\left(x_0\right)+4\left[f\left(x_1\right)+f\left(x_3\right)+\ldots\right]+2\left[f\left(x_2\right)+f\left(x_4\right)+\ldots\right]+f\left(x_n\right)\right]$	
simple harmonic motion	If $\dfrac{d^2x}{dt^2}=-\omega^2x$ then $x=A\sin\left(\omega t+\alpha\right)$ or $x=A\cos\left(\omega t+\beta\right)$	
	$v^2=\omega^2\left(A^2-x^2\right)$	$T=\dfrac{2\pi}{\omega}$; $f=\dfrac{1}{T}$
acceleration	$a=\dfrac{dv}{dt}=\dfrac{d^2x}{dt^2}=v\dfrac{dv}{dx}=\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$	

Real and complex numbers	
complex number forms	$z=x+yi=r\left(\cos(\theta)+i\sin(\theta)\right)=r\,\text{cis}(\theta)$
modulus	$\lvert z\rvert=r=\sqrt{x^2+y^2}$
argument	$\arg(z)=\theta,\ \tan(\theta)=\dfrac{y}{x},-\pi<\theta\le\pi$
product	$z_1z_2=r_1r_2\,\text{cis}\left(\theta_1+\theta_2\right)$
quotient	$\dfrac{z_1}{z_2}=\dfrac{r_1}{r_2}\text{cis}\left(\theta_1-\theta_2\right)$
De Moivre's theorem	$z^n=r^n\text{cis}\left(n\theta\right)$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
permutation	${}^nP_r = \frac{n!}{(n-r)!} = n\times(n-1)\times(n-2)\times\ldots\times(n-r+1)$	
combination	${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$	
sample means	mean	μ
	standard deviation	$\frac{\sigma}{\sqrt{n}}$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$	

Trigonometry	
Pythagorean identities	$\sin^2(A) + \cos^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \text{cosec}^2(A)$
angle sum and difference identities	$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
double-angle identities	$\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $= 1 - 2\sin^2(A)$ $= 2\cos^2(A) - 1$
product identities	$\sin(A)\sin(B) = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$ $\cos(A)\cos(B) = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$ $\sin(A)\cos(B) = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$ $\cos(A)\sin(B) = \frac{1}{2}\left(\sin(A+B) - \sin(A+B)\right)$

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Vectors and matrices		
magnitude	$\lvert \boldsymbol{a} \rvert = \left\lvert \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \right\rvert = \sqrt{a_1^2 + a_2^2 + a_3^2}$	
scalar (dot) product	$\boldsymbol{a} \,.\, \boldsymbol{b} = \lvert \boldsymbol{a} \rvert \lvert \boldsymbol{b} \rvert \cos(\theta)$	
	$\boldsymbol{a} \,.\, \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3$	
vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + k\boldsymbol{d}$	
Cartesian equation of a line	$\frac{x - a_1}{d_1} = \frac{y - a_2}{d_2} = \frac{z - a_3}{d_3}$	
vector (cross) product	$\boldsymbol{a} \times \boldsymbol{b} = \lvert \boldsymbol{a} \rvert \lvert \boldsymbol{b} \rvert \sin(\theta)\hat{\boldsymbol{n}}$	
	$\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$	
vector projection	$\boldsymbol{a}$ on $\boldsymbol{b} = \lvert \boldsymbol{a} \rvert \cos(\theta)\hat{\boldsymbol{b}} = \left(\boldsymbol{a} \,.\, \hat{\boldsymbol{b}}\right)\hat{\boldsymbol{b}}$	
vector equation of a plane	$\boldsymbol{r} \,.\, \boldsymbol{n} = \boldsymbol{a} \,.\, \boldsymbol{n}$	
Cartesian equation of a plane	$ax + by + cz + d = 0$	
determinant	If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(\mathbf{A}) = ad - bc$	
multiplicative inverse matrix	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(\mathbf{A})}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, $\det(\mathbf{A}) \neq 0$	
linear transformations	dilation	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
	rotation	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
	reflection (in the line $y = x\tan(\theta)$)	$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$

Physical constant	
magnitude of mean acceleration due to gravity on Earth	$g = 9.8 \text{ m s}^{-2}$