Neap

Trial Examination 2023

Suggested Solutions

QCE Mathematical Methods Units 1&2

Paper 2 – Technology-active

SECTION 1 – MULTIPLE CHOICE QUESTIONS

	A	B	C	D
1.			●	
2.				●
3.		●		
4.		●		
5.		●		
6.		●		
7.				●
8.	●			
9.			●	
10.			●	

QUESTION 1 C

C is correct. When $y = -1, \frac{2}{x-3} = 0,$ which is not possible.

A is incorrect. This option incorrectly associates the 1 with x for the asymptote.

B is incorrect. This option is the vertical asymptote.

D is incorrect. This option incorrectly associates the numerator with the asymptote.

QUESTION 2 D

D is correct.

$S_n = \frac{n}{2}\left(2t_1 + (n-1)d\right)$

This gives:

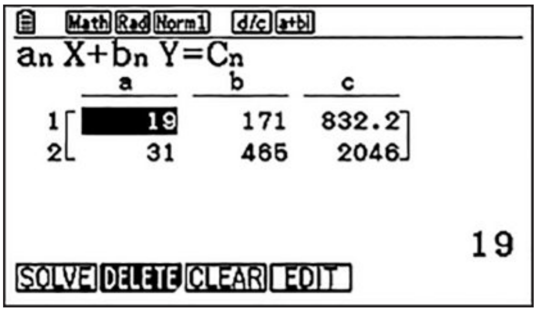
$nt_1 + \frac{n}{2}(n-1)d = S_n$

Leading to:

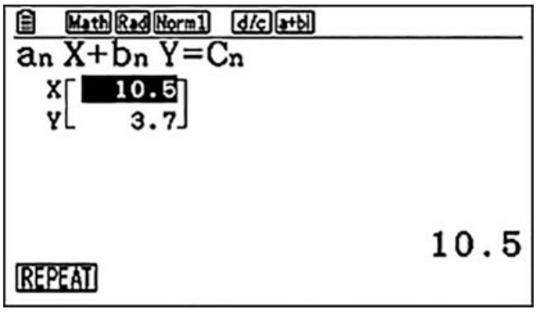
$19t_1 + 171d = 832.2$ (1)

$31t_1 + 465d = 2046$ (2)

Using a graphics calculator: Equation, Simultaneous, 2 unknowns.



Using a graphics calculator: Solve.



Therefore, t_1 = 10.50 and d = 3.70.

A is incorrect. This option swaps the values of the first term and the common difference, d.

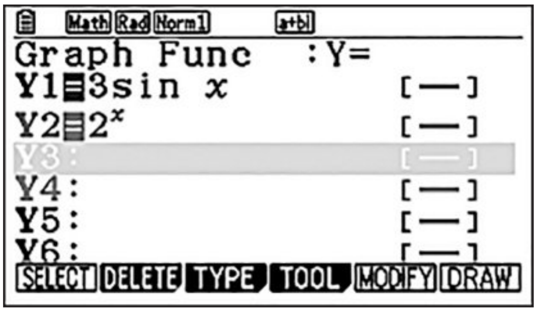
B is incorrect. This option uses n rather than $\frac{n}{2}$.

C is incorrect. This option uses n rather than $(n - 1)$.

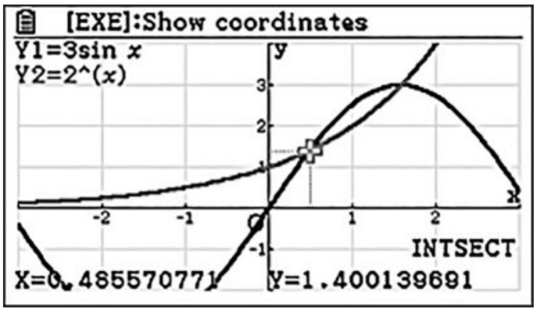
QUESTION 3 B

B is correct.

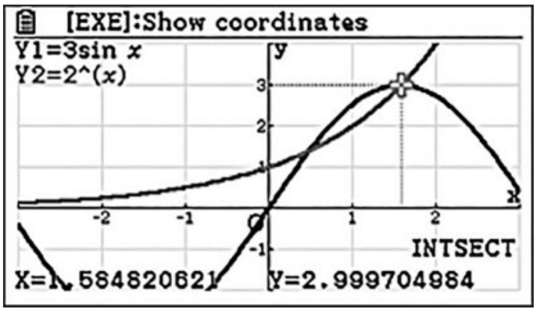
Using a graphics calculator: Graph.



Using a graphics calculator: Draw, V-Window (set x from –3 to 3 and y from –2 to 4).



Using a graphics calculator: G-Solv, INTSECT.



A is incorrect. This option is a domain where $y_2 \geq y_1$.

C is incorrect. This option is a domain that uses one x boundary and one y boundary.

D is incorrect. This option gives the y boundaries of the same section as option **B**.

QUESTION 4 B

B is correct.

Method 1:

Social media visits	Frequency	Relative frequency
0	16	$\frac{16}{29}$
1	8	$\frac{8}{29}$
2	3	$\frac{3}{29}$
4	1	$\frac{1}{29}$
6	1	$\frac{1}{29}$

$$E\left(X^2\right)=\left(0^2\times\frac{16}{29}+1^2\times\frac{8}{29}+2^2\times\frac{3}{29}+4^2\times\frac{1}{29}+6^2\times\frac{1}{29}\right)$$
$$=\frac{72}{29}$$

$$E(X)=\left(0\times\frac{16}{29}+1\times\frac{8}{29}+2\times\frac{3}{29}+4\times\frac{1}{29}+6\times\frac{1}{29}\right)$$
$$=\frac{24}{29}$$

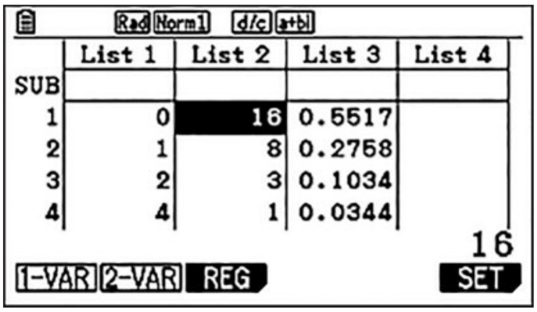
$$Var(X)=E\left(X^2\right)-\left(E(X)\right)^2$$
$$=\frac{72}{29}-\left(\frac{24}{29}\right)^2$$
$$=1.7979$$

$$\sigma(X)=\sqrt{Var(X)}$$
$$=1.3408$$

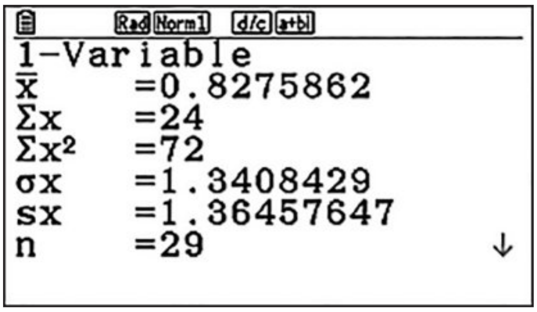
Method 2:

Alternatively, this is a one-variable statistics calculation.

Using a graphics calculator: Statistics, enter data.



Using a graphics calculator: Calc, SET (1Var XList: List 1, 1Var Freq: List 2), Exit, 1-VAR.



A is incorrect. This is the standard deviation of the probability values.

C is incorrect. This is the variance of the distribution.

D is incorrect. This is the standard deviation of the two-variable list.

QUESTION 5 B

B is correct.

$t_n = t_1 r^{(n-1)}$ and $A = P(1+i)^n$

$$\begin{aligned} r &= 1 + i \\ &= 1 + \frac{6.2}{12 \times 100} \\ &= 1.0052 \end{aligned}$$

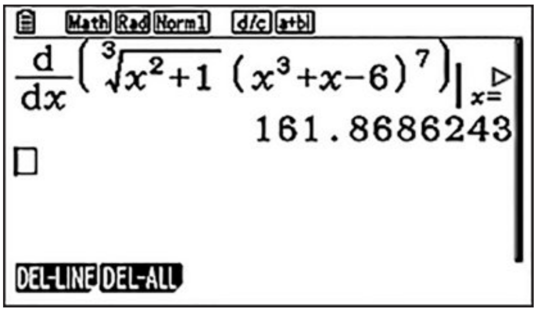
A is incorrect. This option calculates the rate using fortnightly compounds.

C is incorrect. This option calculates the rate based on 10 months per year.

D is incorrect. This option does not take the compounding period into account.

QUESTION 6 B

B is correct. Using a graphics calculator: Run-Matrix, Option, Calc, d/dx.



A is incorrect. This option uses $x^2 - 1$ instead of $x^2 + 1$.

C is incorrect. This option uses the square root rather than the cube root.

D is incorrect. This option uses x^2 rather than x^3.

QUESTION 7 D

D is correct.

$$2.4 \times \frac{180}{\pi} = 137.51$$

A is incorrect. This is the supplementary solution to the correct answer (180 – 137.51 = 42.49°).

B is incorrect. This option halves the correct answer.

C is incorrect. This is the rounded value obtained by using the approximate conversion of 57° per radian (2.4 × 57 = 136.80°).

QUESTION 8 A

A is correct.

$$\begin{aligned} P(\bar{N} \cap \bar{C}) &= P(\bar{N}) \times P(\bar{C}) \\ &= \left(1 - \frac{4}{7}\right) \times \left(1 - \frac{7}{9}\right) \\ &= \frac{2}{21} \end{aligned}$$

B is incorrect. This is the probability of both teams winning.

C is incorrect. This option adds $P(\bar{N})$ and $P(\bar{C})$ rather than multiplying.

D is incorrect. This is the probability of at least one team winning.

QUESTION 9 C

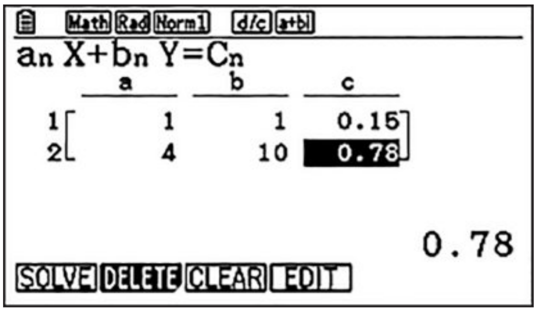
C is correct. The sum of $P(X = x)$ is 1.

$$0.5+0.2+0.15+a+b=1$$
$$a+b=0.15$$

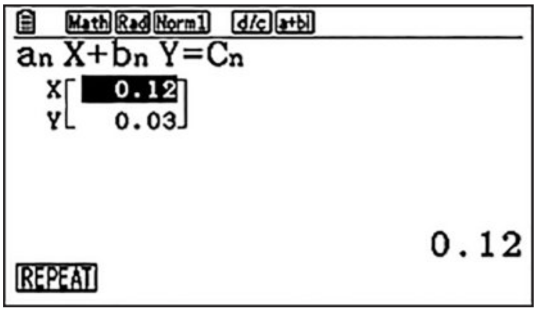
The expected value, $E(X)$, is 1.28.

$$0.5\times0+0.2\times1+0.15\times2+a\times4+b\times10=1.28$$
$$4a+10b=0.78$$

Using a graphics calculator: Equation, Simultaneous, 2 unknowns.



Using a graphics calculator: Solve.



Therefore, $a = 0.12$ and $b = 0.03$.

A is incorrect. This option has an error in the calculation for $E(X)$.

B is incorrect. This option is an expected distribution based on patterns associated with the table.

D is incorrect. This option incorrectly calculates $0.5 \times 0 = 0.5$ in the calculation for $E(X)$.

QUESTION 10 C

Equation R can also be written as $y = 2x^2 + 9$ and is a quadratic function.

Equation S can also be written as $y = \dfrac{11}{3x}$ and is a hyperbolic function.

Equation T is already written in the form of a circle and so it is not a function.

SECTION 2

QUESTION 11 (4 marks)

a) $\frac{4^2 \times 8}{2^5} = \frac{(2^2)^2 \times 2^3}{2^5}$

$= \frac{2^4 \times 2^3}{2^5}$

$= \frac{2^7}{2^5}$

$= 2^2$

$= 4$

[2 marks]
1 mark for equating 4 and 8 into numbers with base 2.
1 mark for simplifying to 4 or equivalent.

b) $\log_3(2x - 1) = 1.7$

$3^{1.7} = 2x - 1$

$x = \frac{3^{1.7} + 1}{2}$

$= 3.7$

[2 marks]
1 mark for rearranging into exponential form.
1 mark for providing the solution correct to one decimal place.

QUESTION 12 (4 marks)

a) $2x^2 - 8x + 7 = 0$

$a = 2, b = -8, c = 7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 2 \times 7}}{2 \times 2}$$

$$= \frac{8 \pm \sqrt{64 - 56}}{4}$$

$$= \frac{8 \pm 2\sqrt{2}}{4}$$

$$= \frac{4 \pm \sqrt{2}}{2}$$

[2 marks]

1 mark for substituting into the quadratic formula.

1 mark for providing the solution in any simplified form. Note: Acceptable forms are $2 \pm \frac{\sqrt{2}}{2}$, *2 ± 0.7071, 1.2929 or 2.7071.*

b)

$$x^2 + 8x = 5$$

$$x^2 + 8x + 16 - 16 = 5$$

$$(x + 4)^2 = 21$$

$$x + 4 = \pm\sqrt{21}$$

$$x = \pm\sqrt{21} - 4$$

[2 marks]

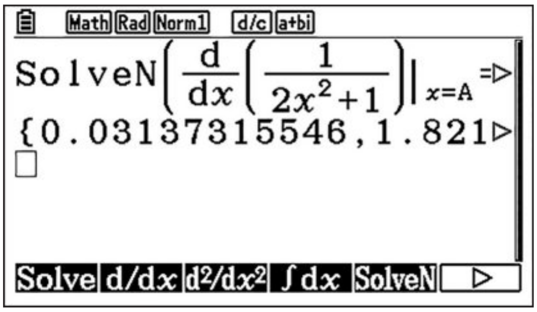
1 mark for determining $(x + 4)^2$.

1 mark for providing the solution.

QUESTION 13 (4 marks)

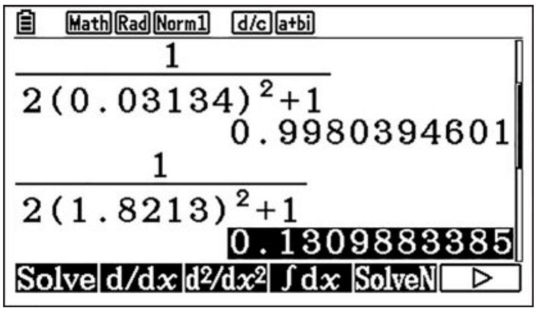
Method 1:

Using a graphics calculator: Run-Matrix, SolveN.



The x-values that give a gradient of $-\frac{1}{8}$ are $x = 0.0314$ and $x = 1.8213$ (correct to four decimal places).

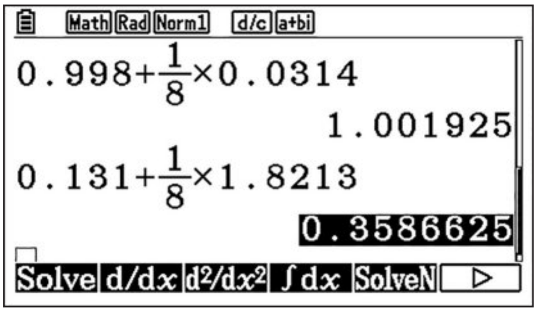
Substituting into the original function using a graphics calculator gives:



At $x = 0.0314$, $y = 0.998$.

At $x = 1.8213$, $y = 0.131$.

Substituting into the tangent function using a graphics calculator gives:



Therefore, the c-values are $c = 1.002$ and $c = 0.3587$, respectively.

[4 marks]

1 mark for showing that solutions for x must be found where $\frac{dy}{dx} = -\frac{1}{8}$.

Note: This may be implied by subsequent working.

1 mark for determining the two x-values that provide the appropriate gradient.

1 mark for determining the respective y-values that match the determined x-values.

1 mark for finding both c-values.

Method 2:

$$y = \frac{1}{2x^2+1}$$

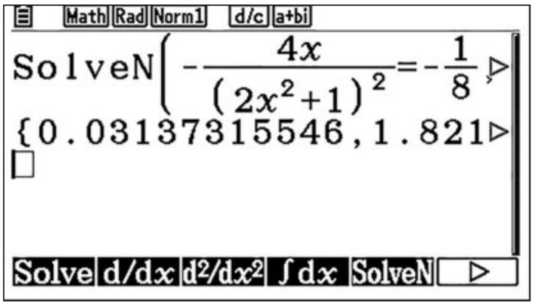
$$= \left(2x^2+1\right)^{-1}$$

$$y' = -1 \times \left(2x^2+1\right)^{-1} \times \frac{d}{dx}\left(2x^2+1\right)$$

$$= -\frac{4x}{\left(2x^2+1\right)^2}$$

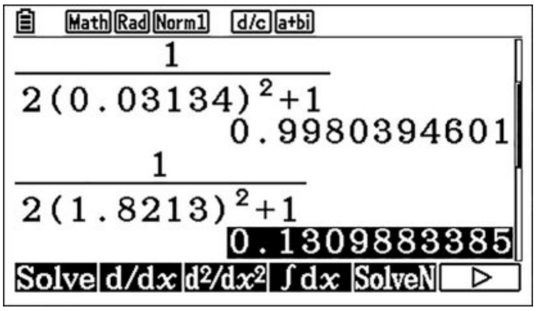
At some value of x, $y' = -\frac{1}{8}$.

Using a graphics calculator: Run-Matrix, SolveN.



x-values that give a gradient of $-\frac{1}{8}$ are $x = 0.0314$ and $x = 1.8213$ (correct to four decimal places).

Substituting into the original function using a graphics calculator gives:



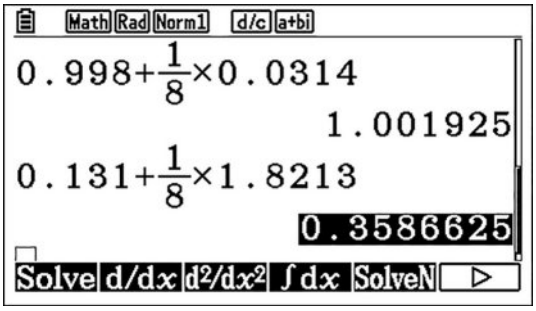
At $x = 0.0314$, $y = 0.998$.

At $x = 1.8213$, $y = 0.131$.

(continues on next page)

(continued)

Substituting into the tangent function using a graphics calculator gives:



Therefore, the c-values are $c = 1.002$ and $c = 0.3587$, respectively.

[4 marks]
1 mark for determining the gradient function.
1 mark for determining the two x-values that provide the appropriate gradient.
1 mark for determining the respective y-values that match the determined x-values.
1 mark for finding both c-values.

QUESTION 14 (4 marks)

a) $P(L \mid R) = \dfrac{P(L \cap R)}{P(R)}$

$$\begin{aligned} P(L \cap R) &= P(L \mid R) \times P(R) \\ &= 0.45 \times 0.22 \\ &= 0.099 \end{aligned}$$

[2 marks]
1 mark for rearranging the formula. Note: This may be implied by subsequent working.
1 mark for determining $P(L \cap R)$.

b) **Method 1:**

Independent events: $P(L) \times P(R) = P(L \cap R)$

In this case:

$$\begin{aligned} P(L) \times P(R) &= 0.15 \times 0.22 \\ &= 0.033 \\ &\neq 0.099\ (P(L \cap R)) \end{aligned}$$

Therefore, events L and R are not independent, as the rule for independence produces an inconsistency.

[2 marks]
1 mark for using either the independence law or $P(L \mid R)$.
1 mark for showing a calculation leading to inconsistency.

Method 2:

For the events to be independent, $P(L)$ must equal $P(L \mid R)$.

In this case, the information provided states that $P(L) = 0.15$ and $P(L \mid R) = 0.45$.

Therefore, $P(L) \neq P(L \mid R)$ and the events are not independent.

[2 marks]
1 mark for identifying the relationship between P(L) and P(L | R). Note: This may be implied by subsequent working.
1 mark for demonstrating that the values of P(L) and P(L | R) are not equal and stating that the events are not independent.

QUESTION 15 (3 marks)

Line 1 passes through (0, 5) and (5, 15.5).

$$m = \frac{15.5 - 5}{5 - 0}$$
$$= 2.1$$

$c = 5$

$C = 2.1x + 5$

Line 2 passes through (5, 15.5) and (10, 20).

$$m = \frac{20 - 15.5}{10 - 5}$$
$$= 0.9$$

$$c = 20 - 10 \times 0.9$$
$$= 11$$

$C = 0.9x + 11$

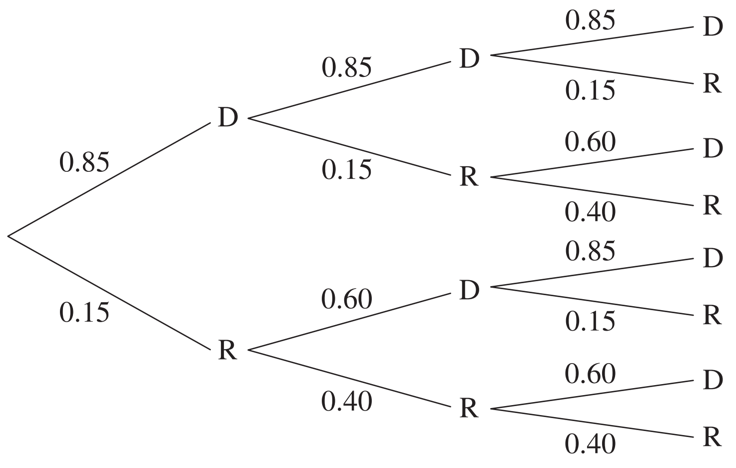
Therefore:

$$C = \begin{cases} 2.1x + 5 & 0 \le x \le 5 \\ 0.9x + 11 & 5 < x \end{cases}$$

[3 marks]
1 mark for determining the first linear equation.
1 mark for determining the second linear equation.
1 mark for writing the piecewise function with appropriate domains and notation.
Note: Do not accept a response that includes 5 in both domains. The second equation does not require an upper bound of 14, but do not deduct marks for responses that include this.

QUESTION 16 (5 marks)

a)



[2 marks]
1 mark for sketching a probability tree with three days and eight outcomes.
1 mark for showing the correct probability on each branch.

b) i) $P(D=3)=P(D)\times P(D)\times P(D)$

$=0.85^3$

$=0.61$

[1 mark]
1 mark for determining the probability.

ii) $P(R\geq 1)=1-P(D=3)$

$=0.39$

[1 mark]
1 mark for determining the probability.
Note: Consequential on answer to ***Question 16bi)****.*

iii) $P(R \text{ on day } 3)=P(DDR)+P(DRR)+P(RDR)+P(RRR)$

$=0.85\times 0.85\times 0.15+0.85\times 0.15\times 0.40+0.15\times 0.60\times 0.15+0.15\times 0.40\times 0.40$

$=0.20$

[1 mark]
1 mark for determining the probability.

QUESTION 17 (5 marks)

a) $V=2x\times x\times h=670 \text{ cm}^3$

$h=\dfrac{670}{2x^2}$

$=\dfrac{335}{x^2}$

$S=2\times 2x\times x+2\times 2x\times h+2\times x\times h$

$=4x^2+6xh$

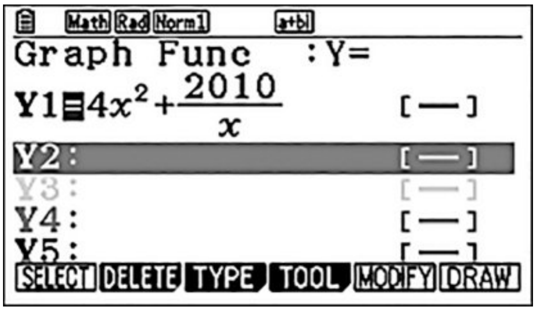
$=4x^2+6x\times\dfrac{335}{x^2}$

$=4x^2+\dfrac{2010}{x}$

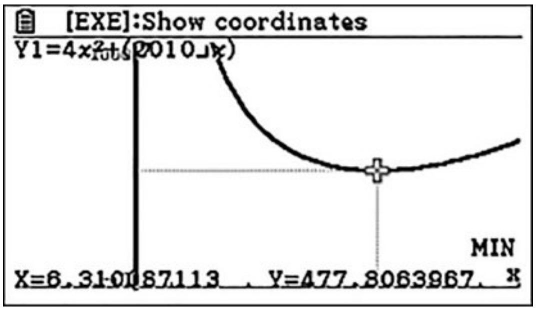
[2 marks]
1 mark for determining h in terms of x.
1 mark for substituting into the surface area formula and simplifying to show the given equation.

b) **Method 1:**

Using a graphics calculator: Graph.



Using a graphics calculator: Draw, G-Solv, MIN.



Therefore, the x-value is a minimum.

$$h = \frac{335}{x^2}$$
$$= \frac{335}{6.31^2}$$
$$= 8.41 \text{ cm}$$

[3 marks]
1 mark for calculating the x-value at the stationary point.
1 mark for determining that the stationary point is a minimum.
1 mark for determining the height.
Note: Consequential on working for ***Question 17a)****.*

Method 2:

$\frac{dS}{dx} = 8x - \frac{2010}{x^2}$

Let $\frac{dS}{dx} = 0$.

$8x - \frac{2010}{x^2} = 0$

$8x^3 = 2010$

$x^3 = 251.25$

$x = \sqrt[3]{251.25}$

$= 6.3101 \text{ cm}$

$h = \frac{335}{x^2}$

$= \frac{335}{6.3101^2}$

$= 8.41 \text{ cm}$

Therefore, the height of a brick with a minimum surface area is 8.41 cm.

[3 marks]
1 mark for setting the derivative to equal 0.
1 mark for calculating the x-value at the stationary point.
1 mark for determining the height.
*Note: Consequential on working for **Question 17a)**.*

QUESTION 18 (5 marks)

a) $t_8 = 3 \times t_3$

$t_1 r^{8-1} = 3 \times t_1 r^{3-1}$

$r^7 = 3 \times r^2$

$r^5 = 3$

$r = \sqrt[5]{3}$

$= 1.2457$

[2 marks]
1 mark for using the t_n formula to determine a relationship with r only.
1 mark for determining the value of r in any form.

b) $r = 1.2457$

$$t_1 r^{8-1} = 3 + t_1 r^{3-1}$$

$$t_1 r^7 - t_1 r^2 = 3$$

$$t_1 = \frac{3}{r^7 - r^2}$$

$$= 0.9666$$

$$S_{10} = t_1 \frac{r^n - 1}{r - 1}$$

$$= 0.9666 \times \frac{\left(\sqrt[5]{3}\right)^{10} - 1}{\sqrt[5]{3} - 1}$$

$$= 31.4683$$

[3 marks]

1 mark for using the t_n formula to determine a relationship with t_1 and r.

1 mark for determining t_1.

1 mark for determining S_{10}.

*Note: Consequential on answer to **Question 18a)**.*

QUESTION 19 (5 marks)

$$(3+2x)^6 = \binom{6}{0} \cdot 3^6 + \binom{6}{0} \cdot 3^5 \cdot (2x)^1 + \binom{6}{2} \cdot 3^4 \cdot (2x)^2 + \binom{6}{3} \cdot 3^3 \cdot (2x)^3 + \ldots$$

$$= 729 + 2916x + 4860x^2 + 4320x^3 + \ldots$$

$$(1+ax)+(3+2x)^6 = (1+ax)\left(729 + 2916x + 4860x^2 + 4320x^3 + \ldots\right)$$

$$= 729 + 2916x + 729ax + 4860x^2 + 2916ax^2 + 4320x^3 \ldots$$

$$= 729 + (2916 + 729a)x + (4860 + 2916a)x^2 + (4320)\ldots$$

Therefore, equating x^2 terms gives:

$$4860 + 2916a = -243$$

$$a = -1.75$$

$$= -\frac{7}{4}$$

[5 marks]

1 mark for expanding $(3 + 2x)^6$ to include at least the linear and quadratic terms.

1 mark for expanding $(1 + ax)(3 + 2x)^6$ to include at least the quadratic terms.

1 mark for collecting like terms to identify the coefficients of x^2.

1 mark for equating the coefficients of x^2 to determine an equation with a.

1 mark for determining the value of a.

QUESTION 20 (6 marks)

For example:

Step 1: The original model

Let P be the population of trout and t be the time in years from 2010.

$P = A \times b^t$

At $t = 0$, $P = 3000$. Therefore, $A = 3000$.

At $t = 13$, $P = 1000$.

$$1000 = 3000 \times b^{13}$$
$$b^{13} = \frac{1}{3}$$
$$b = 0.9190$$

Therefore, $P = 3000 \times 0.9190^t$.

Step 2: Plan A

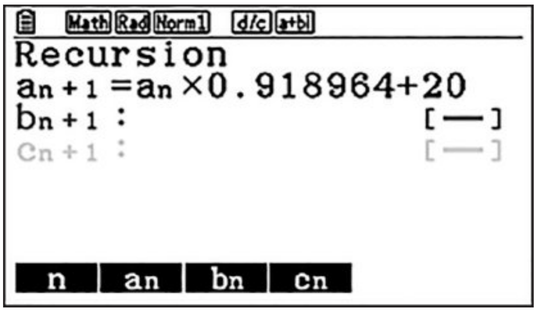
200 trout are added, so there is initially 1200 trout.

Each year, 20 trout are added:

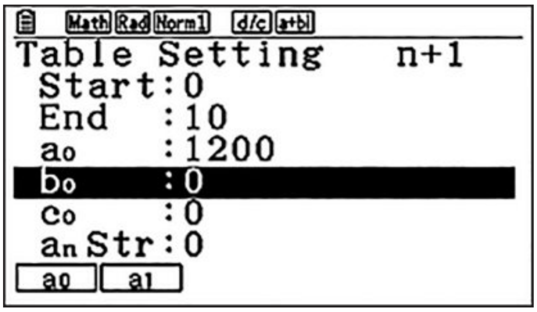
$P_{2024} = 1200 \times 0.9190 + 20 = 1122.7 \ldots$

$P_{2025} = 1122.7 \times 0.9190 + 20 = 1051.7 \ldots$

Using a graphics calculator: Recursion, enter recursion equation.



Using a graphics calculator: SET, set appropriate boundaries.

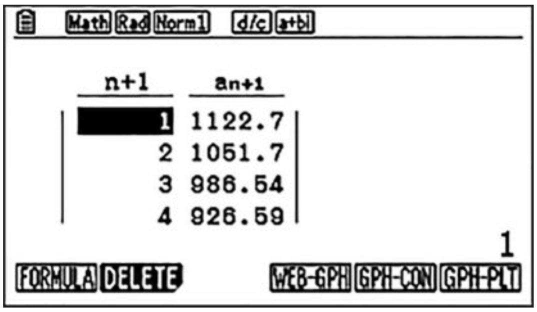


(continues on next page)

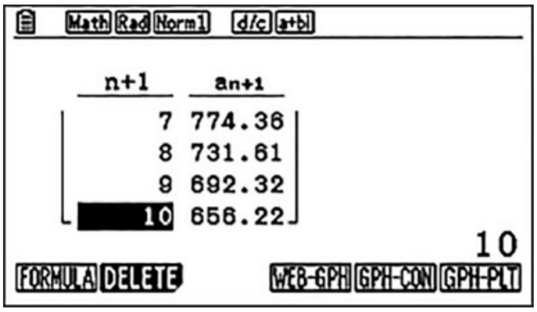
 QCE_MM12_P2_SS_2023

(continued)

Using a graphics calculator: exit, Table.



Using a graphics calculator: exit, Table, scroll down to $n = 10$.



Population in 2033:

$$\begin{aligned} P &= 656.22 \\ &= 656 \text{ trout} \end{aligned}$$

Step 3: Plan B

The current growth rate is 0.9190. The rate of decline is 1 – 0.9190 = 0.0810.

A 30% reduction is 0.0810 × (1 – 0.3) = 0.0567.

The new growth rate is 1 – 0.0567 = 0.9433.

The new model is $P = 1000 \times 0.9433^t$.

In 2033, the population will be:

$$\begin{aligned} P &= 1000 \times 0.9433^{10} \\ &= 557.6765 \\ &= 558 \text{ trout} \end{aligned}$$

(continues on next page)

(continued)

In summary, by the year 2033:

- Plan A: $P = 656$ trout
- Plan B: $P = 558$ trout

Either plan will result in a decline in the trout population, but it is clear that adding to the population regularly (plan A) is the better option.

This conclusion is reasonable due to the:

- reasonableness of the models based on context
- minimal amount of variation that is likely to occur compared to the large difference in outcomes (approximately 100 trout).

This conclusion is unreasonable due to the fact that:

- significant variation may occur between 2023 and 2033 in environment, food and predator cycles, and natural events, which are not accounted for
- improving water quality may have better long-term outcomes
- the amount of data provided was insubstantial for modelling and the predictions are significant extrapolations.

[6 marks]

1 mark for determining the values of A and B in the original model.
1 mark for determining the new model for plan A.
1 mark for calculating the number of trout in the lake in 2033 for plan A.
Note: Accept any whole number answer in the range 646–666 due to rounding error. Also accept any whole number answer in the range 626–642 based on adding 20 trout at the end of each year, provided that there is evidence that this is the student's interpretation.
1 mark for determining the new model for plan B.
1 mark for calculating the number of trout in the lake in 2033 for plan B.
1 mark for determining that plan A is the better option and discussing the reasonableness of the conclusion. Note: Accept any conclusion that is consistent with the response given and justified sufficiently.