Neap

Trial Examination 2023

Question and Response Booklet

QCE Mathematical Methods Units 1&2

Paper 1 — Technology-free

Student's Name: _____

Teacher's Name: _____

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- Calculators are not permitted.
- Formula booklet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (45 marks)

- 10 short response questions

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has 10 questions and is worth 45 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (4 marks)

Consider the following arithmetic sequence.

$$-5, -2, 1, 4$$

a) State the value of the common difference, d. *[1 mark]*

b) Determine the value of the tenth term, t_{10}. *[1 mark]*

c) Determine the sum of the first 10 terms. *[2 marks]*

QUESTION 12 (6 marks)

a) Evaluate the following. Give your answers in exact form.

i) $\sin(45°)$ *[1 mark]*

ii) $\tan\left(\frac{7\pi}{6}\right)$ *[1 mark]*

iii) $\cos\left(\frac{3\pi}{2}\right)$ *[1 mark]*

b) Solve the equation $2\sin x = \sqrt{3}$ for the domain $-360° \leq x \leq 360°$. *[3 marks]*

QUESTION 13 (5 marks)

Determine the derivatives of the following functions.

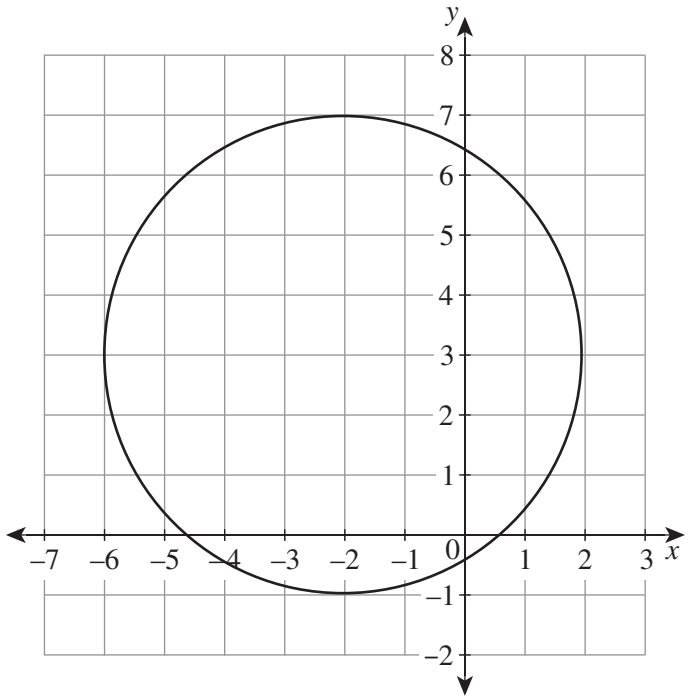
a) $y = 2x^3$ *[1 mark]*

b) $f(x) = (2x^2 - 3)^4$ *[2 marks]*

c) $y = \dfrac{2x}{\sqrt{x+8}}$ *[2 marks]*

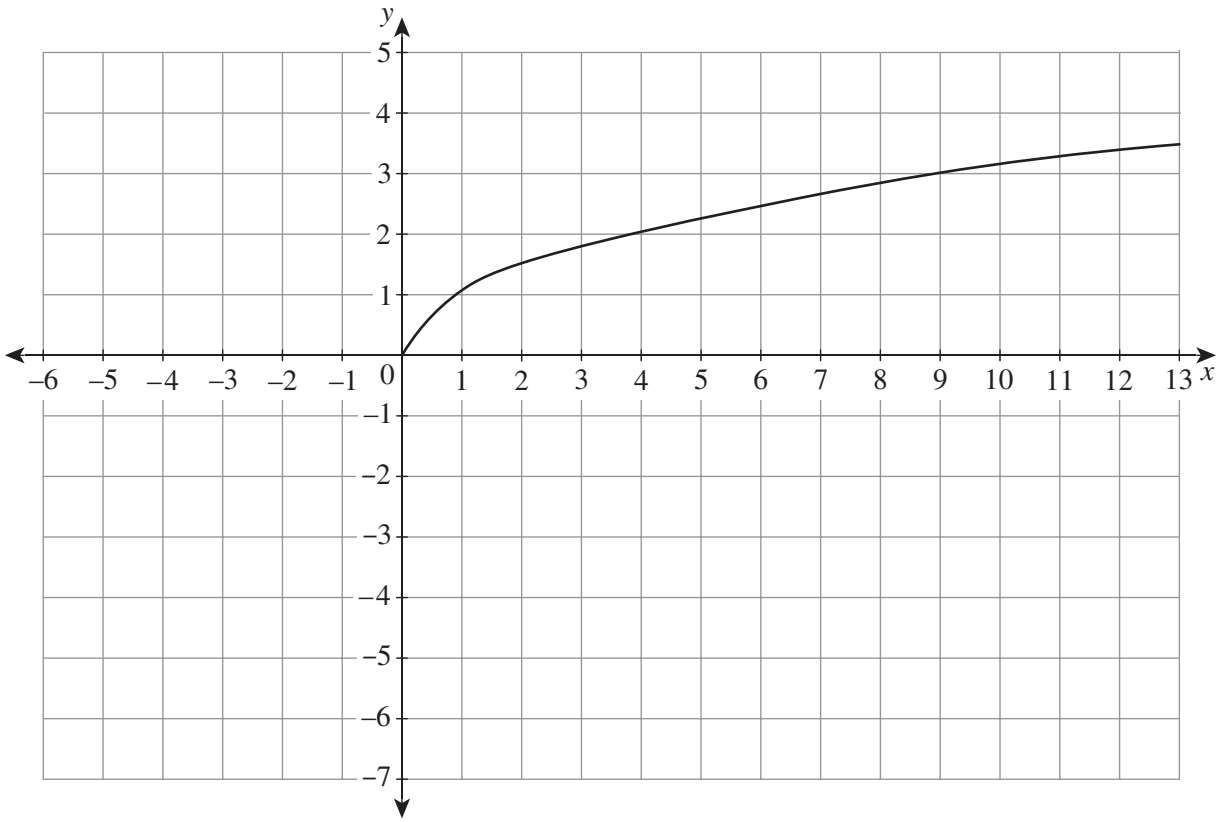
QUESTION 14 (6 marks)

a) Consider the following graph of a circle.



Determine the equation of the circle. *[2 marks]*

b) Consider the square root function $y = \sqrt{x}$ shown below.



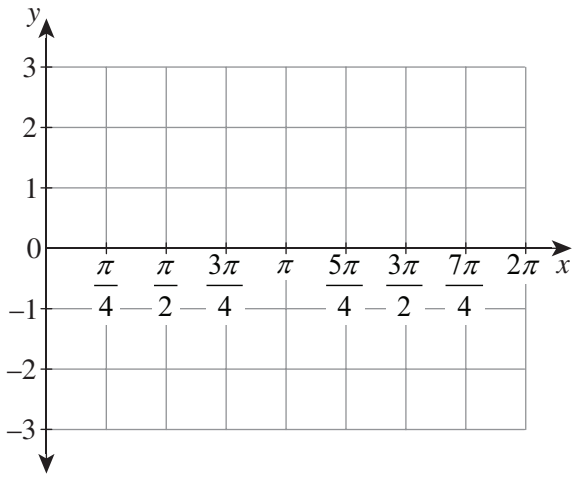
The function has the following transformations applied to it.

- vertical dilation by a factor of 2
- horizontal translation by a factor of –1
- vertical reflection

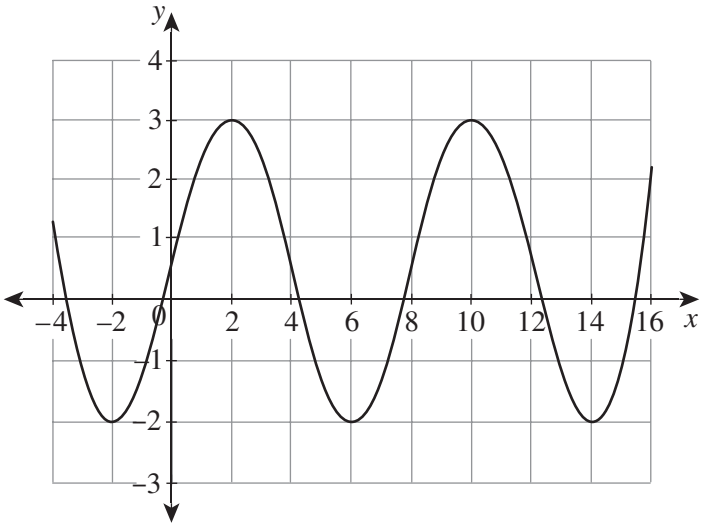
Determine the equation of the transformed function and sketch the transformed function on the axes above. Label all axis intercepts. *[4 marks]*

QUESTION 15 (4 marks)

a) On the axes below, sketch the graph of $y = 2\cos(2x) - 1$. *[2 marks]*



b) Determine the equation of the periodic function shown below in the form $y = A\sin\big(B(x - C)\big) + D$. *[2 marks]*



QUESTION 16 (4 marks)

Consider the following cubic equation.

$$2x^3 + 7x^2 + 2x - 3 = 0$$

Given that $x = -1$ is a solution to the equation, use an algebraic method to determine the other two solutions.

QUESTION 17 (4 marks)

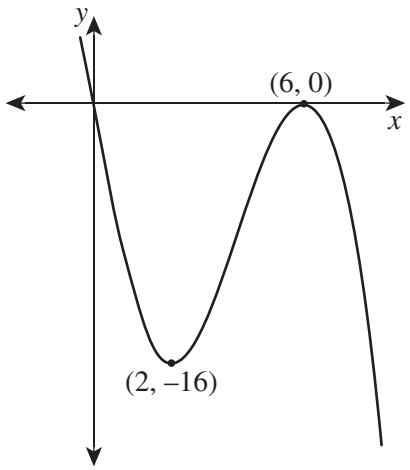
A local sports field has raised, tiered seating for spectators to sit on. A spectator is watching a ball game at the sports field when a player accidentally kicks the ball into the seating area. The spectator kicks the ball back towards the players on the field. The ball travels through the air such that its height above the ground, h metres, after t seconds is given by the function $h(t) = -5t^2 + 20t + 5$.

a) Determine when the ball will land on the ground. *[2 marks]*

b) Determine the height, h, of the ball at its peak. *[2 marks]*

QUESTION 18 (4 marks)

Consider the following graph of a cubic function.



Determine the gradient of the function at (0, 0).

QUESTION 19 (4 marks)

Consider the following quadratic function, where k is an unknown constant.

$$y = x^2 + 4kx + (3 + 11k)$$

Determine the values of k for which the quadratic does not have x-intercepts.

QUESTION 20 (4 marks)

Exposure index (*EI*) measures the amount of light that reaches the image receptor in a camera lens. It is determined using the equation

$$EI = \log_2\left(\frac{f^2}{t}\right),$$

where f is the f-stop setting on the camera and t is the exposure length in seconds.

A photographer is taking a series of photographs using a consistent f-stop setting.

If the photograph triples the exposure length, determine the effect that this will have on the *EI*.

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Formula Booklet

QCE Mathematical Methods Units 1&2

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$ü = \frac{1}{2}$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}\left(2t_1 + (n-1)d\right) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r-1)}$ $S_\infty = \frac{t_1}{(1-r)},\ \lvert r\rvert < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n\log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Calculus		
$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$		$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$		$\int \frac{1}{x} dx = \ln(x) + c$
$\frac{d}{dx}\sin(x) = \cos(x)$		$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$		$\int \cos(x) dx = \sin(x) + c$
chain rule	If $h(x) = f\left(g(x)\right)$ then $h'(x) = f'\left(g(x)\right)g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	$\text{area} = \frac{1}{2}bc\sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
binomial probability	$P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable *X*	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
continuous random variable *X*	mean	$E(X) = \mu = \int_{-\infty}^{\infty} xp(x)dx$
	variance	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for *p*	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$	
general addition rule for probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
probability of independent events	$P(A \cap B) = P(A) \times P(B)$	
conditional probability	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$	