Neap

Trial Examination 2023

Suggested Solutions

QCE Mathematical Methods Units 1&2

Paper 1 – Technology-free

SECTION 1 – MULTIPLE CHOICE QUESTIONS

	A	B	C	D
1.		●		
2.		●		
3.	●			
4.				●
5.		●		
6.			●	
7.			●	
8.			●	
9.		●		
10.	●			

QUESTION 1 B

B is correct.

$N(\bar{D} \cup \bar{C}) = 100 - 57 - 64 + 29 = 8$

A is incorrect. This option swaps the values for dogs only and cats only.

C is incorrect. This option incorrectly places the dogs and cats value in the neither category.

D is incorrect. This option does not include the neither category and has incorrect values for dogs only and cats only.

QUESTION 2 B

B is correct. The sequence (3, 6, 12, …) is geometric with a common ratio, r, of 2; therefore, the next term in the sequence is $12 \times 2 = 24$.

A is incorrect. This option is reached by using arithmetic and a second common difference, d, of 6.

C is incorrect. This option is reached by miscalculation.

D is incorrect. This option is the fifth term in the sequence.

QUESTION 3 A

A is correct. The cubic term is $-x^3 = -1 \times x^3$; therefore, its coefficient is –1.

B is incorrect. This option is the constant term.

C is incorrect. This option is the coefficient of the quartic term.

D is incorrect. This option is the coefficient of the quadratic term.

QUESTION 4 D

D is correct.

$$\begin{aligned} T &= \frac{2\pi}{b} \\ &= \frac{2\pi}{\left(-\frac{\pi}{4}\right)} \\ &= -8 \end{aligned}$$

As the period measures a distance per oscillation, it is always a positive value. Therefore, the period is 8.

A is incorrect. This option states the negative period from the calculation.

B is incorrect. This option is the b value rather than the period.

C is incorrect. This option is the c value or phase shift rather than the period.

QUESTION 5 B

B is correct. $\cap$ is the intersection symbol. Therefore, $A \cap B$ is the intersection of *A* and *B*, meaning that the values are found in both *A* and *B*.

A is incorrect. The complement of *A* is A' and *B* is B'.

C is incorrect. The outcome is the possible result of the selection from events *A* and *B*.

D is incorrect. The union of *A* and *B* is $A \cup B$.

QUESTION 6 C

C is correct. The average rate of change between $x = 1$ and $x = 2$ is $\frac{y_2 - y_1}{x_2 - x_1}$.

$y_1 = 3 \times 1^3 + 2 \times 1 - 7 = -2$

$y_2 = 3 \times 2^3 + 2 \times 2 - 7 = 21$

$$\begin{aligned}\text{average rate of change} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{21 - (-2)}{2 - 1} \\ &= 23\end{aligned}$$

A is incorrect. This option calculates the instantaneous rate of change at $x = 1$, or the gradient if the function were quadratic, not cubic.

B is incorrect. This option calculates the instantaneous rate of change at $x = 1.5$.

D is incorrect. This option calculates the instantaneous rate of change at $x = 2$.

QUESTION 7 C

C is correct.

$$TP_x = -\frac{b}{2a}$$

For option **C**:

$$\begin{aligned}TP_x &= -\frac{-6}{2 \times (-1)} \\ &= -3\end{aligned}$$

$$\begin{aligned}y(-3) &= -(-3)^2 - 6 \times (-3) - 5 \\ &= 4\end{aligned}$$

Therefore, the turning point for option **C** is (–3, 4).

A is incorrect. This option has a turning point at (3, 4).

B is incorrect. This option has a turning point at (3, –4).

D is incorrect. This option has a turning point at (–3, –4).

QUESTION 8 C

C is correct. The graph can be determined by finding the *x*- and *y*-intercepts.

x-intercept:

$$y = -2\times 3^{x+1} + 2$$
$$0 = -2\times 3^{x+1} + 2$$
$$1 = 3^{x+1}$$
$$x = -1$$

Therefore, there is an *x*-intercept at (–1, 0).

y-intercept:

$$\begin{aligned} y &= -2\times 3^{x+1} + 2 \\ &= -2\times 3^{0+1} + 2 \\ &= -4 \end{aligned}$$

Therefore, there is a *y*-intercept at (0, –4).

A is incorrect. This graph does not have a horizontal translation of –1.

B is incorrect. This graph has a base of 2 instead of 3.

D is incorrect. This graph does not show a vertical translation of +2.

QUESTION 9 B

B is correct.

$$\begin{aligned} P(-2) = (-2)^3 + 6\times(-2)^2 - 3\times(-2) + k &= 0 \\ -8 + 24 + 6 + k &= 0 \\ k &= -22 \end{aligned}$$

A is incorrect. This option uses *P*(2) rather than *P*(–2).

C is incorrect. This option values *k* as a positive in the final step of the calculation.

D is incorrect. This option uses *P*(2) rather than *P*(–2) and values *k* as a positive in the final step of the calculation.

QUESTION 10 A

A is correct.

$$S_\infty = \frac{t_1}{1-r}$$

$$30 = \frac{18}{1-r}$$

$$1-r = \frac{18}{30}$$

$$r = \frac{2}{5}$$

B is incorrect. This option states $1-r = \frac{18}{30} = \frac{3}{5}$.

C is incorrect. This option rearranges incorrectly to give $r = \frac{30}{18} - 1 = \frac{5}{3} - 1 = \frac{2}{3}$.

D is incorrect. This option applies the reciprocal incorrectly.

$$30r = 12$$

$$r = \frac{30}{12}$$

$$= \frac{5}{2}$$

SECTION 2

QUESTION 11 (4 marks)

a) $d = 3$

[1 mark]
1 mark for stating the common difference.

b) $t_n = t_1 + (n-1)d$

$t_{10} = -5 + (10-1) \times 3$

$= 22$

[1 mark]
1 mark for determining the value of t_{10}.

c) **Method 1:**

$$S_n = \frac{n}{2}\left(2t_1 + (n-1)d\right)$$

$$S_{10} = \frac{10}{2}(2 \times (-5) + (10-1) \times 3)$$

$$= 5 \times (-10 + 27)$$

$$= 85$$

[2 marks]
1 mark for substituting into the appropriate formula.
1 mark for determining the sum.

Method 2:

$$S_n = \frac{n}{2}\left(t_1 + t_n\right)$$

$$S_{10} = \frac{10}{2}(-5 + 22)$$

$$= 5 \times 17$$

$$= 85$$

[2 marks]
1 mark for substituting into the appropriate formula.
1 mark for determining the sum.
*Note: Consequential on answer to **Question 11b)**.*

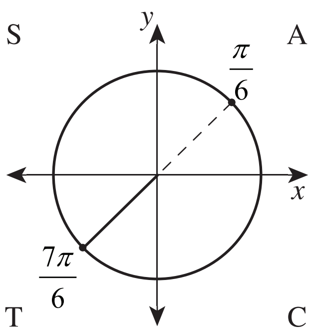
QUESTION 12 (6 marks)

a) i) $\sin(45°) = \frac{1}{\sqrt{2}}$ **OR** $\frac{\sqrt{2}}{2}$

[1 mark]
1 mark for providing the simplified answer in either form.

ii) $\tan\left(\frac{7\pi}{6}\right) = \tan\left(\frac{\pi}{6}\right)$

$= \frac{1}{\sqrt{3}}$ **OR** $\frac{\sqrt{3}}{3}$



[1 mark]
1 mark for providing the simplified answer in either form.
Note: A diagram is not required to obtain this mark.

iii) $\cos\left(\frac{3\pi}{2}\right) = 0$

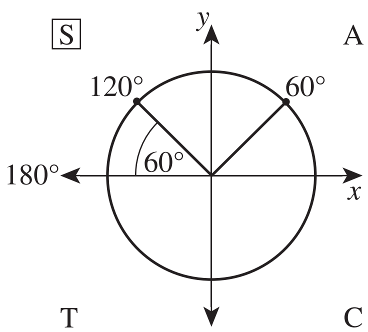
[1 mark]
1 mark for providing the simplified answer.

b) $2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= 60°$ (quadrant 1)



Therefore:

- $x = 120°$ (quadrant 2)
- $x = –300°, –240°$ (one period previous)

The solutions are –300°, –240°, 60°, 120°.

[3 marks]
1 mark for rearranging and solving to find the first solution (60°).
1 mark for determining the second solution (120°).
1 mark for determining the last two solutions (–300° and –240°).
Note: A diagram is not required to obtain full marks.

QUESTION 13 (5 marks)

a) $y' = 6x^2$

[1 mark]
1 mark for deriving the function.

b) $f'(x) = 4\left(2x^2 - 3\right)^3 \times \frac{d}{dx}\left(2x^2 - 3\right)$

$= 4\left(2x^2 - 3\right)^3 \times 4x$

$= 16x\left(2x^3 - 3\right)^3$

[2 marks]
1 mark for showing evidence of the chain rule.
1 mark for deriving the function.

c) $y=\dfrac{2x}{\sqrt{x+8}}=\dfrac{u}{v}$

$u=2x,\ u'=2$

$v=\sqrt{x+8}=(x+8)^{\frac{1}{2}},\ v'=\dfrac{1}{2}(x+8)^{-\frac{1}{2}}$

$$y'=\frac{u'v-v'u}{v^2}$$

$$=\frac{2\times(x+8)^{\frac{1}{2}}-\frac{1}{2}(x+8)^{-\frac{1}{2}}\times 2x}{x+8}$$

$$=\frac{2(x+8)^{\frac{1}{2}}-x(x+8)^{-\frac{1}{2}}}{x+8}$$

$$=\frac{(x+8)^{-\frac{1}{2}}\left(2(x+8)-x\right)}{x+8}$$

$$=\frac{(x+8)^{-\frac{1}{2}}(2x+16-x)}{x+8}$$

$$=\frac{x+16}{(x+8)^{\frac{3}{2}}}$$

[2 marks]
1 mark for identifying and deriving u and v.
1 mark for deriving the function using the quotient rule.
Note: The answer does not need to be simplified to obtain full marks.

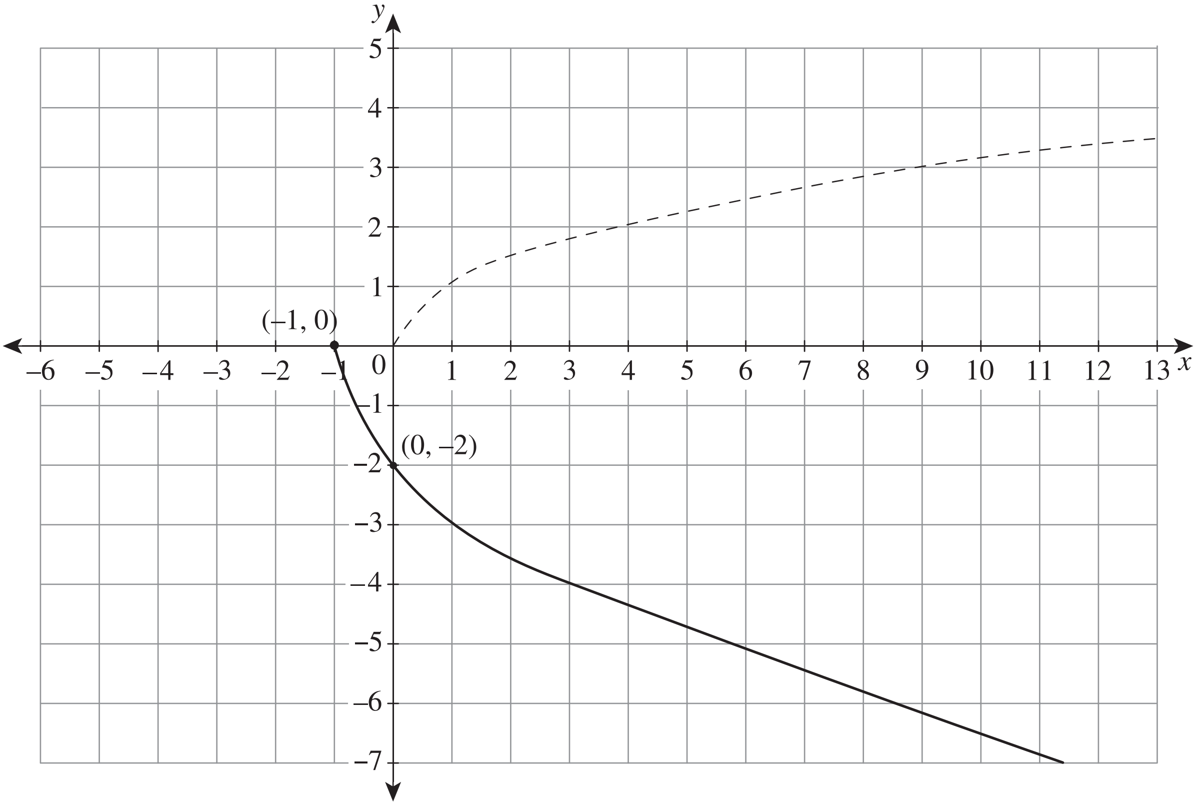
QUESTION 14 (6 marks)

a) The centre is at (–2, 3) and the radius is 4.
Therefore, $(y-3)^2+(x+2)^2=16$.

[2 marks]
1 mark for stating the location of the centre and the radius. Note: This may be implied by subsequent working.
1 mark for determining the equation.

b)

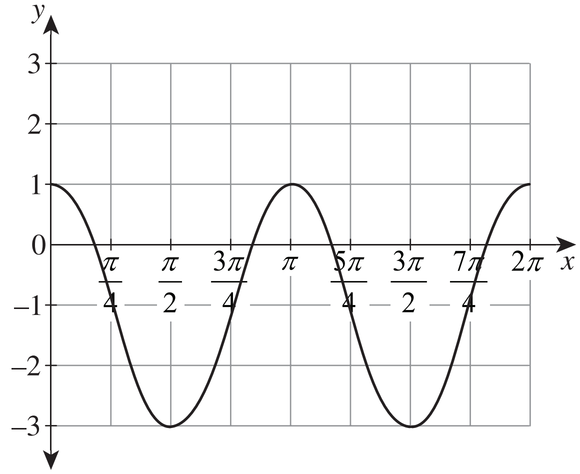
Transformation	Function $\left(y = \sqrt{x}\right)$
vertical dilation by a factor of 2	$y = 2\sqrt{x}$
horizontal translation by a factor of –1	$y = 2\sqrt{x+1}$
vertical reflection	$y = -2\sqrt{x+1}$



[4 marks]
1 mark for showing 2 to denote the dilation.
1 mark for showing (x + 1) to denote the translation.
1 mark for showing –2 to denote the reflection.
1 mark for sketching the function and labelling all three key points.

QUESTION 15 (4 marks)

a)



[2 marks]
1 mark for the phase shift (the function begins at a peak) and the correct period of π (the function repeats twice across the domain of the graph).
1 mark for the amplitude and midline (the function oscillates between y = 1 and y = –3).

b) peak = 3, trough = –2

Thus:

$$A = \frac{3-(-2)}{2} = 2.5$$

$$D = \frac{3+(-2)}{2} = 0.5$$

The graph starts at 0.5, which is D; thus, the phase shift, C, is 0.

The period is $T = 10 - 2 = 8$ (locations of the first two peaks).

Thus, $B = \frac{2\pi}{8} = \frac{\pi}{4}$.

Therefore, the equation is $y = 2.5\sin\left(\frac{\pi}{4}x\right) + 0.5$.

[2 marks]
1 mark for determining the values of A and D.
1 mark for determining the values of B and C and the equation.

QUESTION 16 (4 marks)

If $x = -1$ is a solution, then $(x + 1)$ is a factor.

$2x^3 + 7x^2 + 2x - 3 = (x+1)(ax^2 + bx + c)$

By considering the expansion, $a = 2$ and $c = -3$. The coefficient of x^2 is $(b + a) = 7$; therefore, $b = 5$.

Alternatively:

$$
\begin{array}{r|l}
 & 2x^2 + 5x - 3 \\
\hline
x + 1 & 2x^3 + 7x^2 + 2x - 3 \\
 & \underline{2x^3 + 2x^2} \\
 & 5x^2 + 2x - 3 \\
 & \underline{5x^2 + 5x} \\
 & -3x - 3 \\
 & 0
\end{array}
$$

$$
\begin{aligned}
2x^3 + 7x^2 + 2x - 3 &= (x+1)(2x^2 + 5x - 3) \\
&= (x+1)(2x^2 + 6x - x - 3) \\
&= (x+1)(2x(x+3) - 1(x+3)) \\
&= (x+1)(2x-1)(x+3)
\end{aligned}
$$

Therefore:

$2x^3 + 7x^2 + 2x - 3 = 0$

$(x+1)(2x-1)(x+3) = 0$

$\therefore x = \frac{1}{2}, -3 \text{ or } -1$

[4 marks]
1 mark for recognising the factor of (x + 1).
1 mark for determining the quadratic expression in the factorisation.
1 mark for factorising completely.
1 mark for stating the other two values for x.

QUESTION 17 (4 marks)

a) The ball will land on the ground when $h(t) = 0$.

$$
\begin{aligned}
-5t^2 + 20t + 5 &= 0 \\
t^2 - 4t - 1 &= 0 \\
t &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-1)}}{2 \times 1} \\
&= \frac{4 \pm \sqrt{20}}{2} \\
&= 2 \pm \sqrt{5} \text{ s}
\end{aligned}
$$

Time must be positive in this context; therefore, the ball will land when $t = 2 + \sqrt{5}$ s.

[2 marks]
1 mark for recognising $h(t) = 0$.
1 mark for determining the positive solution and rejecting the negative solution.

b) The peak occurs when the velocity is 0.

$h(t) = -5t^2 + 20t + 5$

$v(t) = h'(t) = -10t + 20$

$-10t + 20 = 0$

$t = 2$

At $t = 2$:

$h(2) = -5\times 2^2 + 20\times 2 + 5$

$= -20 + 40 + 5$

$= 25$ m

The height of the ball at its peak is 25 m.

[2 marks]

1 mark for deriving h(t) to obtain v(t).

1 mark for solving h'(t) = 0 to find the value t = 2 and consequently the peak height.

QUESTION 18 (4 marks)

The function is a cubic in the form $y = ax(x - 6)^2$. It has a single intercept at $x = 0$ and a double root at $x = 6$. It also passes through (2, –16).

$-16 = a\times 2(2-6)^2$

$-16 = a\times 32$

$a = -\frac{1}{2}$

$y = -\frac{1}{2}x(x-6)^2$

$y = -\frac{1}{2}x\left(x^2 - 12x + 36\right)$

$y = -\frac{1}{2}x^3 + 6x^2 - 18x$

$y' = -\frac{3}{2}x^2 + 12x - 18$

Let $x = 0$:

$y' = -\frac{3}{2}\times 0^2 + 12\times 0 - 18$

$= -18$

The gradient of the function at the origin (0, 0) is –18.

[4 marks]

1 mark for using the axis intercepts, or otherwise, to write the function as a cubic with one unknown (a).

1 mark for substituting (2, –16) into the function to find a.

1 mark for deriving the function.

1 mark for determining the gradient by substituting x = 0 into the derivative.

QUESTION 19 (4 marks)

$y = x^2 + 4kx + (3 + 11k)$

$\Delta = b^2 - 4ac < 0$

Determining the critical values gives:

$$\begin{aligned}(4k)^2 - 4\times1\times(3+11k) &= 0\\ 16k^2 - 12 - 44k &= 0\\ 4k^2 - 11k - 3 &= 0\\ 4k^2 - 12k + k - 3 &= 0\\ 4k(k-3) + 1(k-3) &= 0\\ (4k+1)(k-3) &= 0\end{aligned}$$

$\therefore k = -\frac{1}{4}$ or 3

Considering inequality:

$16k^2 - 44k - 12 < 0$

$(4k+1)(k-3) < 0$

Testing $k = 0$:

$16 \times 0^2 - 44 \times 0 - 12 = -12 < 0$

Therefore, the boundary required is between the solutions for k.

The quadratic does not have x-intercepts where $-\frac{1}{4} < k < 3$.

[4 marks]

1 mark for recognising the use of the determinant to solve the problem.

1 mark for substituting the values into the determinant to set up the quadratic equation or inequality.

1 mark for solving the resultant quadratic equation or inequality to determine both boundary values of k.

1 mark for determining the possible set of k values in any clear format.

QUESTION 20 (4 marks)

For example:

Let the original values be EI_1 and t_1 and the second set of values be EI_2 and t_2. f remains constant, hence so does f^2.

Photograph 1:

$$EI_1 = \log_2\left(\frac{f^2}{t_2}\right)$$

$$2^{EI_1} = \frac{f^2}{t_1}$$

$$t_1 = \frac{f^2}{2^{EI_1}}$$

Photograph 2 follows the same pattern and results in the following.

$$t_2 = \frac{f^2}{2^{EI_2}}$$

It is known that $t_2 = 3t_1$; therefore:

$$\frac{f^2}{2^{EI_2}} = 3 \times \frac{f^2}{2^{EI_1}}$$

$$\frac{1}{2^{EI_2}} = 3 \times \frac{1}{2^{EI_1}}$$

$$\frac{2^{EI_1}}{2^{EI_2}} = 3$$

$$2^{(EI_1 - EI_2)} = 3$$

$$EI_1 - EI_2 = \log_2 3$$

$$EI_2 = EI_1 - \log_2 3$$

If the exposure length is tripled, the EI will be reduced by $\log_2 3$.

[4 marks]

1 mark for using the EI equation to set up two relationships between EI and time in either logarithmic or exponential form.

1 mark for determining a relationship between the original and new EIs.

1 mark for simplifying the exponential equation to a single power or simplifying a solution in an equivalent manner.

1 mark for determining the final difference and providing a statement about the effect.