Neap

Trial Examination 2022

Suggested Solutions

QCE Mathematical Methods Units 3&4

Paper 2 – Technology-active

SECTION 1 – MULTIPLE CHOICE QUESTIONS

	A	B	C	D
1.		●		
2.			●	
3.			●	
4.		●		
5.			●	
6.				●
7.			●	
8.			●	
9.		●		
10.				●

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QUESTION 1 B

Let $a = 2,\ b = 5,\ C = \frac{\pi}{6}$.

$c^2 = a^2 + b^2 - 2ab\cos(C)$

$c^2 = 2^2 + 5^2 - 2 \times 2 \times 5\cos\left(\frac{\pi}{6}\right)$

$c^2 = 4 + 25 - 20 \times \frac{\sqrt{3}}{2}$

$c \approx 3.42$ units

QUESTION 2 C

$\text{area} = \frac{1}{2}bc\sin(A)$

$= \frac{1}{2} \times 2 \times 2 \times \sin\left(\frac{\pi}{5}\right)$

$\approx 1.18 \text{ units}^2$

QUESTION 3 C

According to the graph, the mean is approximately 2.5. The probability of $x > 3.1$ is 16%. This means that the probability of $2.5 \le x \le 3.1$ is 34% and thus the probability for $1.9 \le x \le 3.1$ is 68%. The area within one standard deviation of the mean is approximately 68%. Thus, the difference between 3.1 and 2.5 is approximately one standard deviation. Thus, the standard deviation is approximately 0.6.

QUESTION 4 B

B is correct. This expression accurately describes the cumulative distribution function for any probability density function.

A is incorrect. The cumulative distribution function is not an indefinite integral.

C is incorrect. The lower bound is not necessarily 0, and the use of x as both the upper bound and the variable being integrated is incorrect.

D is incorrect. The lower bound is not necessarily 0.

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QUESTION 5 C

Let E_{1971} be the energy released by the 1971 earthquake.

$$\begin{aligned} M_{w1971} &= \frac{2}{3}\log_{10}\left(E_{1971}\right) - 10.7 \\ &= 6.4 \end{aligned}$$

Let E_{1975} be the energy released by the 1975 earthquake.

$$\frac{1}{2}E_{1971} = E_{1975}$$

$$\begin{aligned} M_{w1975} &= \frac{2}{3}\log_{10}\left(E_{1975}\right) - 10.7 \\ &= \frac{2}{3}\log_{10}\left(\frac{E_{1971}}{2}\right) - 10.7 \\ &= \frac{2}{3}\left(\log_{10}\left(E_{1971}\right) - \log_{10}(2)\right) - 10.7 \\ &= \frac{2}{3}\log_{10}\left(E_{1971}\right) - 10.7 - \frac{2}{3}\log_{10}(2) \\ &= 6.4 - \frac{2}{3}\log_{10}(2) \\ &\approx 6.2 \end{aligned}$$

QUESTION 6 D

$$\begin{aligned} f(x) &= 8^{3x} \\ &= e^{\ln\left(8^{3x}\right)} \\ &= e^{3\ln(8)x} \end{aligned}$$

$$\begin{aligned} f'(x) &= e^{3\ln(8)x} \times 3\ln(8) \\ &= 3\ln(8)e^{3\ln(8)x} \\ &= 3\ln(8) \times 8^{3x} \end{aligned}$$

QUESTION 7 C

According to the fundamental theorem of calculus:

$$\begin{aligned} \int_0^1 g(x)dx &= F(1) - F(0) \\ &= (3\times 1^3 + 2\times 1 - 7) - (3\times 0^3 + 2\times 0 - 7) \\ &= 5 \end{aligned}$$

QUESTION 8 C

C is correct. Euler's number (e) is the unique number that makes the equation true.

A is incorrect. If $a = 1$, the limit would equal 0.

B is incorrect. While close in value to e, 2.72 is a rational number, not an irrational number like e.

D is incorrect. If $a = \pi$, the limit would equal $\ln\pi$.

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QUESTION 9 B

Let $x(t)$ be the displacement function.

$$x(t) = \int v(t)dt$$
$$= \int 5t^2 - 4t + 1dt$$
$$= \frac{5t^3}{3} - 2t^2 + t + c$$

$x(0) = 2$

$$\therefore \frac{5 \times 0^3}{3} - 2 \times 0^2 + 0 + c = 2$$
$$c = 2$$

So, $x(t) = \frac{5t^3}{3} - 2t^2 + t + 2.$

QUESTION 10 D

D is correct. The function $f(x)$ may have three points of inflection because a, b and c are the only zeroes of the function $f''(x)$.

A is incorrect. Since $f''(a) = f''(c) = 0,$ there are potentially up to three points of inflection.

B and **C** are incorrect. Since $f'(a) \neq 0$ and $f'(c) \neq 0,$ they cannot be local extrema.

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SECTION 2

QUESTION 11 (4 marks)

a) $\int 2\sin(x+4)dx = -2\cos(x+4)+c$

[1 mark]
1 mark for determining the antiderivative of the trigonometric term.
Note: The constant of integration is not required to obtain this mark.

b) $\int \frac{1}{2x-9}dx = \frac{1}{2}\ln(2x-9)+c$

[1 mark]
1 mark for determining the antiderivative of the algebraic fraction term.
Note: The constant of integration is not required to obtain this mark.

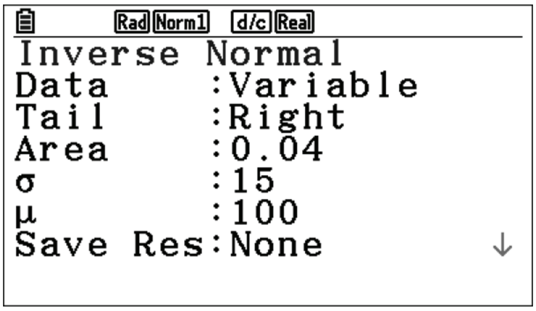
c) $\int \frac{1}{3}e^x dx = \frac{1}{3}e^x + c$

[2 marks]
1 mark for determining the antiderivative of the exponential term.
(Note: The constant of integration is not required to obtain this mark.)
1 mark for including constants of integration for each of parts a, b and c.

QUESTION 12 (4 marks)

a) **Method 1:**

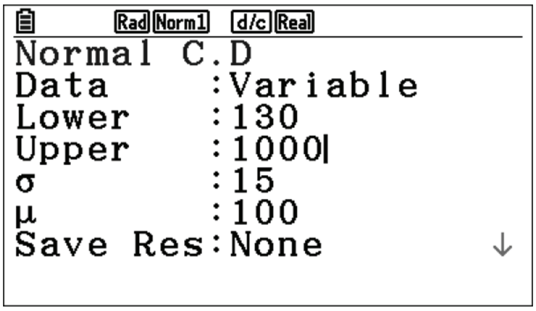
Using a graphics calculator: Statistics, DIST, NORM, InvN.



A score higher than approximately 126.26 would be in the top 4%. Since this is lower than 130, scoring in the top 4% is not sufficient to receive an invitation.

Method 2:

Using a graphics calculator: Statistics, DIST, NORM, Ncd.



A score higher than 130 means that one is in the top 2.28%. Thus, being in the top 4% is not sufficient to receive an invitation.

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Method 3:

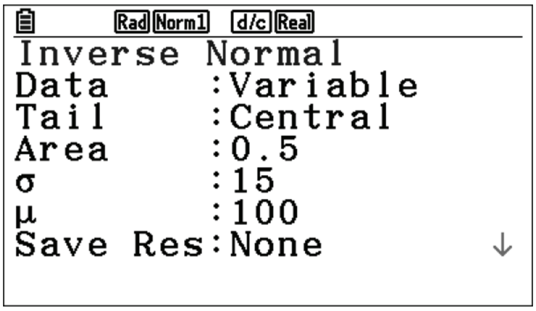
A score of 130 would have a z-score as follows:

$$z = \frac{130 - 100}{15} = 2$$

An individual would need to score two standard deviations above the mean. 95% of the adult population will score within two standard deviations from 100. Therefore, a score higher than 130 is in the top 2.5% of the adult population. Thus, being in the top 4% is not sufficient to receive an invitation.

[3 marks]
1 mark for using an appropriate method.
1 mark for determining an appropriate and accurate probability or z-score to make a judgement.
1 mark for determining that a score in the top 4% is not sufficient based on sound reasoning.

b) Using a graphics calculator, Statistics, DIST, NORM, InvN.

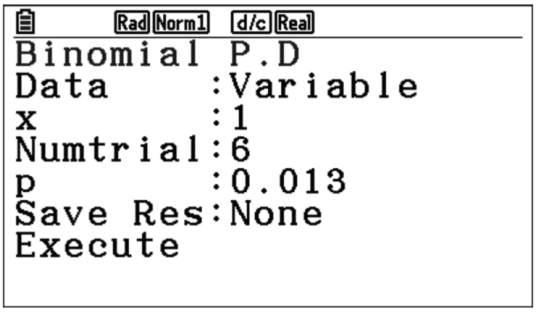


The individual would need to score between approximately 89.88 and approximately 110.12.

[1 mark]
1 mark for determining the score range.
Note: Accept lower bounds in the range 89–90, and accept upper bounds in the range 110–111.

QUESTION 13 (5 marks)

a) Using a graphics calculator: Statistics, BINOMIAL, Bpd.



$P(X = 1) \approx 0.0731$

[1 mark]
1 mark for determining the probability.
Note: Accept values rounded to fewer decimal places, including 0.07.

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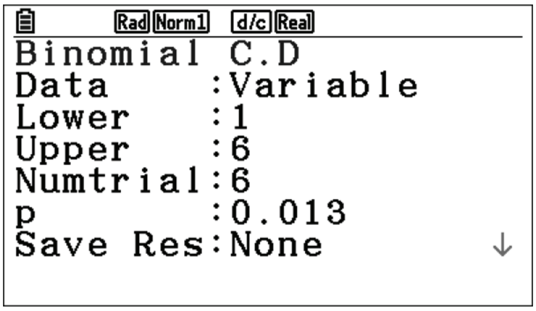
b) Using a graphics calculator: Statistics, BINOMIAL, Bcd.



$P(X \geq 1) \approx 0.0755$

[2 marks]
1 mark for using an appropriate procedure.
Note: This may be implied by the correct answer; from evidence of use of the binomial cumulative distribution; or by writing a formula, such as $P(X \geq 1) = 1 - P(X = 0)$.
1 mark for determining the probability.

c) $$\begin{aligned}\text{expected value} &= np \\ &= 50 \times 0.0755 \\ &\approx 3.8\end{aligned}$$

[2 marks]
1 mark for substituting into the correct formula to find the mean.
1 mark for determining the probability.
Note: Accept follow-through errors. Consequential on answer to ***Question 13b)****.*

QUESTION 14 (4 marks)

$$f'(x) = 15e^{9x-4}$$

$$\begin{aligned} f(x) &= \int f'(x)\,dx \\ &= \int 15e^{9x-4}\,dx \\ &= \frac{15}{9}e^{9x-4} + c \\ &= \frac{5}{3}e^{9x-4} + c \end{aligned}$$

$$\begin{aligned} f(0) &= \frac{5}{3}e^{9\times 0-4} + c \\ &= \frac{5}{3}e^{-4} + c \end{aligned}$$

$$\frac{5}{3}e^{-4} + c = 1$$

$$c = 1 - \frac{5}{3}e^{-4}$$

Thus, $f(x) = \frac{5}{3}e^{9x-4} + 1 - \frac{5}{3}e^{-4}$.

[4 marks]
1 mark for integrating f(x).
1 mark for substituting 0 into f(x) and setting up an appropriate equation to solve for the constant.
1 mark for solving for c.
1 mark for stating the full solution of f(x).
Note: Accept follow-through errors, including using an incorrect integral.

QUESTION 15 (5 marks)

a) $\hat{p} = \frac{13}{80} = 0.1625$

$$\begin{aligned} \hat{\sigma} &= \sqrt{\frac{p(1-p)}{n}} \\ &= \sqrt{\frac{0.1625(1-0.1625)}{80}} \\ &\approx 0.041 \end{aligned}$$

[2 marks]
1 mark for calculating the sample proportion.
1 mark for calculating the standard deviation of the sample proportion.

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b) standard deviation $= \sqrt{n\hat{p}(1-\hat{p})}$

$$= \sqrt{80 \times 0.1625(1-0.1625)}$$

$$\approx 3.3 \text{ trees}$$

[1 mark]
1 mark for calculating the standard deviation of the sample count.
*Note: Consequential on working for **Question 15a)**.*

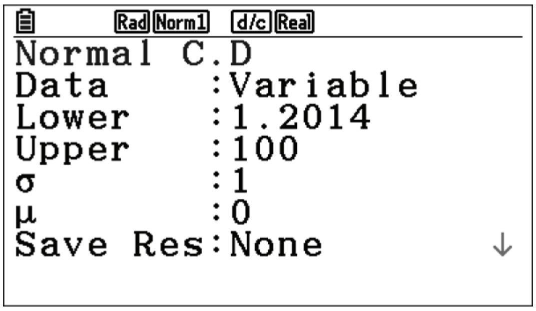
c) $z = \dfrac{\hat{p} - p}{\hat{\sigma}}$

$$= \frac{0.1625 - 0.1130}{0.0412}$$

$$\approx 1.2014$$

With this z-score, standardised normal distribution is used to calculate probability.

Using a graphics calculator: Statistics, DIST, NORM, Ncd.



The probability is 0.115 or 11.5%.

[2 marks]
1 mark for using an appropriate method used to determine the probability.
Note: This may be implied by the correct answer or shown, such as correct use of the formula to calculate the z-score.
1 mark calculating the probability.
*Note: Consequential on working for **Question 15a)**.*

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QUESTION 16 (5 marks)

Since $f(x)$ is a probability density function:

$$\int_0^1 f(x)dx = 1$$

$$\begin{aligned}\int_0^1 f(x)dx &= \int_0^1 \frac{1}{kx+c}dx \\ &= \left[\frac{1}{k}\ln(kx+c)\right]_0^1 \\ &= \frac{1}{k}\ln(k+c) - \frac{1}{k}\ln(c) \\ &= \frac{1}{k}\left(\ln(k+c) - \ln(c)\right) \\ &= \frac{1}{k}\ln\left(\frac{k+c}{c}\right)\end{aligned}$$

$$\begin{aligned}\frac{1}{k}\ln\left(\frac{k+c}{c}\right) &= 1 \\ \ln\left(\frac{k+c}{c}\right) &= k \\ \frac{k+c}{c} &= e^k \\ c &= \frac{k}{e^k - 1}\end{aligned}$$

[5 marks]

1 mark for integrating the function.

1 mark for stating the correct answer after the substitution of bounds.

1 mark for setting up an appropriate equation where the integrated function equals 1.

1 mark for applying an appropriate strategy to solve for the constant.

1 mark for solving for c in terms of k.

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QUESTION 17 (5 marks)

Method 1 (integrating without technology):

$$\text{area} = \int_0^1 x^3 - 2x^2 - x + 2dx - \int_1^2 x^3 - 2x^2 - x + 2dx$$

$$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{x^2}{2} + 2x\right]_{-1}^{1} - \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{x^2}{2} + 2x\right]_{1}^{2}$$

$$= \left(\left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right) - \left(\frac{1}{4} - \frac{-2}{3} - \frac{1}{2} - 2\right)\right) - \left(\left(\frac{16}{4} - \frac{16}{3} - \frac{4}{2} + 4\right) - \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right)\right)$$

$$= \frac{8}{3} - \left(\frac{2}{3} - \frac{13}{12}\right)$$

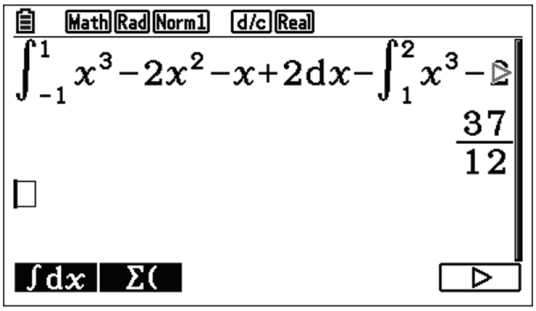
$$= \frac{37}{12} \text{ units}^2$$

$$\approx 3.08 \text{ units}^2$$

[5 marks]
1 mark for recognising that f(x) intersects with the x-axis at –1, 1 and 2.
1 mark for recognising the need to integrate the two areas separately.
1 mark for recognising the need to multiply the second integral by –1.
1 mark for integrating the polynomial terms.
1 mark for determining that the area is $\frac{37}{12}$ *or 3.08 units*^{2}.

Method 2 (using a graphics calculator):

Calculating the definite integrals using a graphics calculator gives: Run-Matrix, MATH, F5, F1.



[5 marks]
1 mark for recognising that f(x) intersects with the x-axis at –1, 1 and 2.
1 mark for recognising the need to integrate the two areas separately.
1 mark for recognising the need to multiply the second integral by –1.
1 mark for showing logical organisation and communicating key steps.
1 mark for determining that the area is $\frac{37}{12}$ *or 3.08 units*^{2}.

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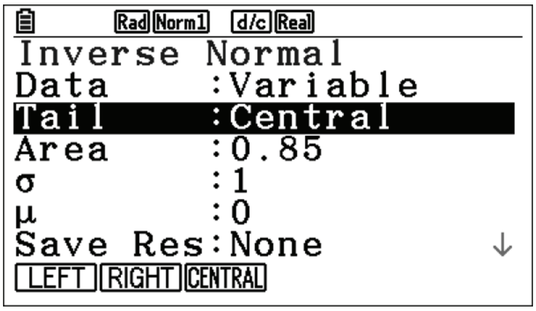
QUESTION 18 (5 marks)

a) number of people with an accurate test = 18 + 347 = 365

$$\hat{p} = \frac{18+347}{400}$$
$$= 0.9125$$

Using a graphics calculator to determine the z-value: Statistics, DIST, NORM, InvN



$z \approx 1.4395$

confidence interval:

$$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$
$$= \left(0.9125 - 1.4395\sqrt{\frac{0.9125(1-0.9125)}{400}},\ 0.9125 + 1.4395\sqrt{\frac{0.9125(1-0.9125)}{400}}\right)$$
$$= (0.892, 0.933)$$

[3 marks]
1 mark for calculating the value of $\hat{p}$.
1 mark for determining the z-value. (Note: Allow for rounding differences.)
1 mark for calculating the confidence interval.

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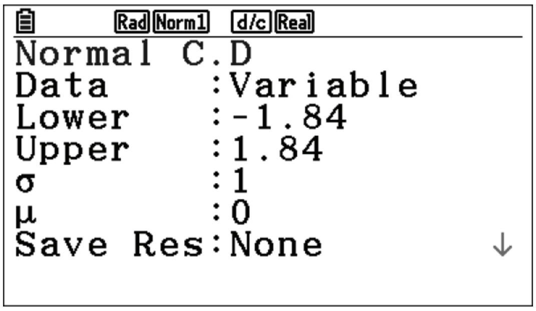
b) $$E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.026 = z\sqrt{\frac{0.9125(1-0.9125)}{400}}$$

$$z = \frac{0.026}{\sqrt{\frac{0.9125(1-0.9125)}{400}}}$$

$$\approx 1.84$$

Using a graphics calculator: Statistics, DIST, NORM, Ncd.



$p = 0.934$

Therefore, the confidence interval was 93.4%.

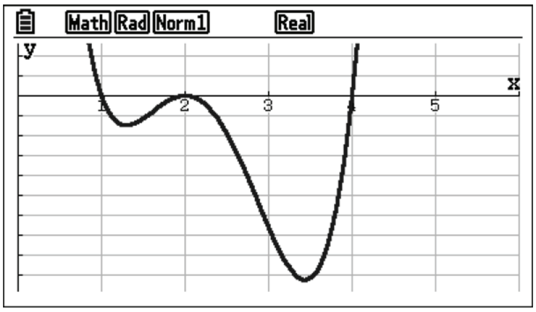
[2 marks]
1 mark for determining the z-value.
1 mark for determining the percentage of the confidence interval.

QUESTION 19 (7 marks)

a) The domain is $(0, \infty)$.

Stationary points occur when $f'(x) = 0$. Thus, $x = 1$, $x = 2$ and $x = 4$.

Using a graphics calculator: Graph option, plot $f'(x)$.



Based on the graph, the features of $f(x)$ can be identified.

At $x = 1$, there is a maximum using the first derivative test since $f'(x) > 0$ for values of x just below 1 and $f'(x) < 0$ for values of x just above 1.

At $x = 2$, there is an inflection point since $f''(x) = 0$ because at $x = 2$ $f'(x)$ is a stationary point.

At $x = 4$, there is a minimum using the first derivative test since $f'(x) < 0$ for values of x just below 4 and $f'(x) > 0$ for values of x just above 4.

[4 marks]
1 mark for identifying the three stationary points of x = 1, x = 2 and x = 4.
3 marks for classifying each stationary point and providing some justification for each (1 mark for each point).

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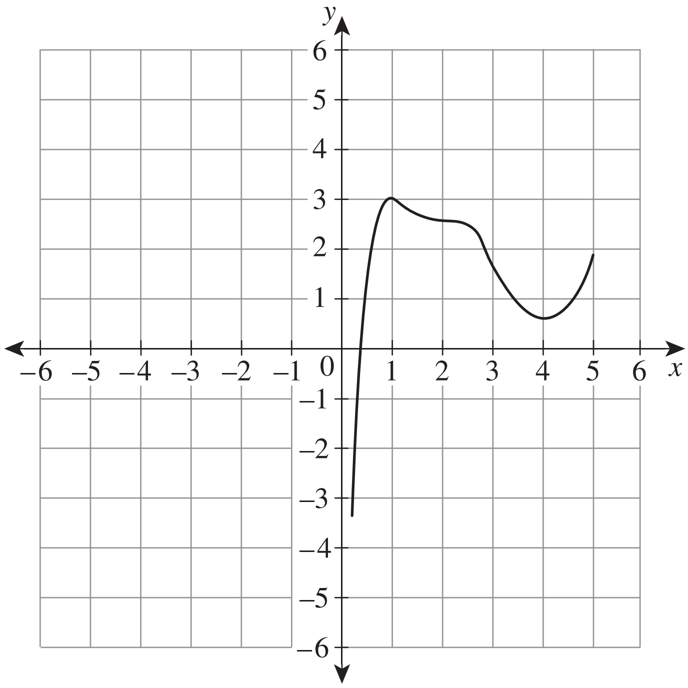
b)



[3 marks]

1 mark for showing an asymptote at $x = 0$. (Note: The function should not cut through the y-axis.)

1 mark for showing $f(1) = 3$.

1 mark for showing a maximum at $x = 1$, inflection point at $x = 2$ and minimum at $x = 4$.

*Note: Consequential on answer to **Question 19a)**.*

QUESTION 20 (6 marks)

a) Model for the salp population in terms of time:

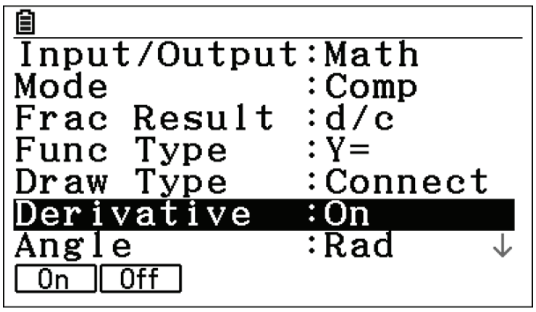
$$S(t) = S(C(t))$$

$$= -39\cos\left(\frac{\pi}{50} \times -\frac{1}{7}(t-100)\ln(t+30)\right) + 40$$

$$= -39\cos\left(-\frac{\pi}{350}(t-100)\ln(t+30)\right) + 40$$

Method 1:

Inputting the above function into a graphics calculator: SET UP, Derivative: On.



Graph, Trace, then typing 30 making sure the window is an appropriate size.

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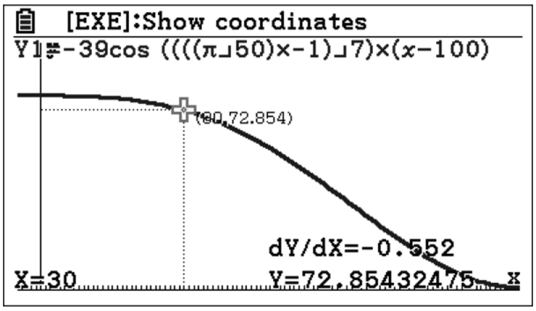


The rate of change is approximately –0.552 individuals m^{-2} t^{-1}.

Method 2:

$$S(t) = -39\cos\left(-\frac{\pi}{350}(t-100)\ln(t+30)\right) + 40$$

$$S'(t) = 39\sin\left(-\frac{\pi}{350}(t-100)\ln(t+30)\right) \times -\frac{\pi}{350}\left(\ln(t+30) + \frac{t-100}{t+30}\right)$$

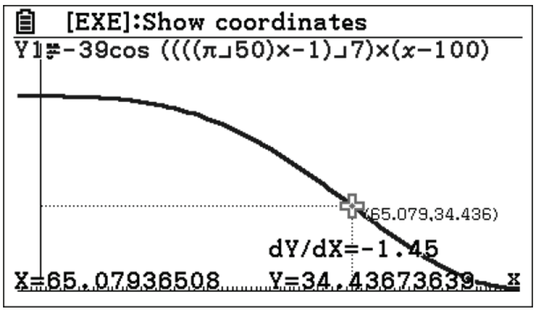
$$= -\frac{39\pi}{350}\sin\left(-\frac{\pi}{350}(t-100)\ln(t+30)\right) \times \left(\ln\left(t+30\right) + \frac{t-100}{t+30}\right)$$

$$S'(30) = -\frac{39\pi}{350}\sin\left(-\frac{\pi}{350}(30-100)\ln(30+30)\right) \times \left(\ln(30+30) + \frac{30-100}{30+30}\right)$$

$$\approx -0.522 \text{ individuals m}^{-2}\ \text{t}^{-1}$$

[4 marks]
1 mark for attempting to create the composition of the functions in the correct order. (Note: This may be implied.)
1 mark for composition of the functions. (Note: This does not need to be simplified and may be implied.)
1 mark for communicating a suitable method to solve.
1 mark for determining the rate of change.
Note: Units are not required to obtain full marks.

b) Using a graphics calculator's trace function, the maximum decrease will occur around the year 2065 with a rate of change of –1.45 individuals m^{-2} t^{-1}.



[2 marks]
1 mark for identifying the year of the highest rate of decrease. (Note: Accept years in the range 2063–2068.)
1 mark for identifying the rate of change. (Note: Accept values from –1.45 to –1.46).