Neap

Trial Examination 2022

Suggested Solutions

QCE Mathematical Methods Units 3&4

Paper 1 – Technology-free

SECTION 1 – MULTIPLE CHOICE QUESTIONS

	A	B	C	D
1.	●	○	○	○
2.	●	○	○	○
3.	●	○	○	○
4.	○	○	○	●
5.	○	○	●	○
6.	○	○	●	○
7.	○	○	●	○
8.	○	●	○	○
9.	○	●	○	○
10.	○	○	○	●

QUESTION 1 A

$$P(X=3)=\binom{3}{3}0.5^3 0.5^0$$
$$=\frac{1}{8}$$

QUESTION 2 A

$$\frac{\sin B}{AC}=\frac{\sin C}{AB}$$
$$\frac{\sin B}{1}=\frac{\sin 45°}{\sqrt{2}}$$
$$\sin B=\frac{1}{2}$$

Therefore, $B = 30°$.

This is not an ambiguous case of the sine rule as the side opposite B is shorter than the side opposite C.

QUESTION 3 A

A is correct. The vertical asymptote occurs at $x = 1$. Therefore, there is horizontal translation 1 unit to the right. The vertical translation can be ascertained by letting $x = 2$: $\log_5(2 - 1) + 3 = \log_5(1) + 3 = 3$, implying that $f(2) = 3$, which matches the graph. Lastly, $f(6) = \log_5(6 - 1) + 3 = \log_5(5) + 3 = 4$, which matches the graph as the curve goes through the point (6, 4).

B is incorrect. The vertical translation is incorrect. The graph shows that $f(2) = 3$, not $f(2) = 1$.

C is incorrect. The horizontal translation is incorrect. The graph shows a vertical asymptote at $x = 1$, not $x = -1$.

D is incorrect. When $x = 6$, this function would be greater than 4, not 4 as shown in the graph.

QUESTION 4 D

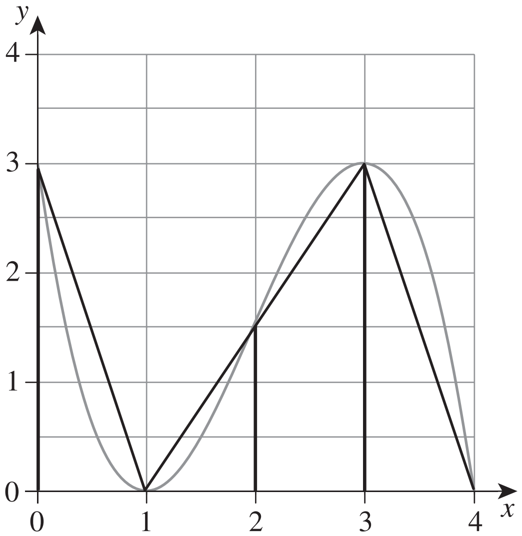
$$f(x)=-\cos(-2x+1)$$

Using the chain rule:

$$f'(x)=--\sin(-2x+1)\times -2$$
$$=-2\sin(-2x+1)$$

QUESTION 5 C

The number of trapezoids required is 4. Therefore, $f(0)$, $f(1)$, $f(2)$ and $f(3)$ can be found on the graph and four trapezoids drawn.



$$\text{area} \approx \sum_{i=1}^{4} A_i$$

$$= \frac{1}{2}\left(f(0) + 2f(1) + 2f(2) + 2f(3) + f(4)\right)$$

$$= \frac{1}{2}(3 + 2 \times 0 + 2 \times 1.5 + 2 \times 3 + 0)$$

$$= \frac{1}{2}(3 + 3 + 6)$$

$$= 6 \text{ units}^2$$

QUESTION 6 C

$$f(x) = \frac{1}{x} + x^2 - 9$$

$$\int f(x)dx = \int \frac{1}{x} + x^2 - 9dx$$

$$= \ln(x) + \frac{x^3}{3} - 9x + c \quad (c \text{ is an arbitrary constant.})$$

QUESTION 7 C

$$\int (6 - 4x)\sin\left(x^2 - 3x + 9\right)e^{-\cos\left(x^2 - 3x + 9\right)}dx$$

$$= -2\int (2x - 3)\sin\left(x^2 - 3x + 9\right)e^{-\cos\left(x^2 - 3x + 9\right)}dx$$

$$= -2e^{-\cos\left(x^2 - 3x + 9\right)} + c \quad (c \text{ is an arbitrary constant.})$$

QUESTION 8 B

Let the unknown side length be b and the opposite angle be B.

Let the known sides be a and c.

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 2^2 + 3^2 - 2 \times 2 \times 3 \cos 60$

$b^2 = 4 + 9 - 2 \times 2 \times 3 \times \frac{1}{2}$

$b^2 = 7$ units

$b = \sqrt{7}$ (The negative is rejected as b is a length.)

QUESTION 9 B

B is correct. The confidence interval is calculated using the sample size, not the population size.

A is incorrect. The z value comes from the standardised normal distribution, not the binomial distribution.

C is incorrect. The sample proportions should be approximately normally distributed so that the confidence interval is appropriate. This requires that the sample size be large enough.

D is incorrect. The data that is being counted are the outcomes of individual Bernoulli trials. Even if the data is continuous, it needs to be converted into discrete data so that a proportion can be calculated. Calculating confidence intervals for continuous variables (for example, height) do not require the calculation of sample proportions.

QUESTION 10 D

D is correct, and **A** and **B** are incorrect. The margin of error is $E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. Increasing the level of confidence increases the value of z and thus increases the margin of error.

C is incorrect. While increasing the value of n decreased the margin of error, increasing the value of z increases, not decreases, the margin of error.

 TEQ MM34_SS_P1_2022

SECTION 2

QUESTION 11 (3 marks)

Chemistry:

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{81-51}{15} \\ &= \frac{30}{15} \\ &= 2 \end{aligned}$$

Biology:

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{76-58}{12} \\ &= \frac{18}{12} \\ &= 1.5 \end{aligned}$$

Since the z-score for Chemistry is higher, Clara scored better in Chemistry than Biology when compared to the rest of the state.

[3 marks]
1 mark for calculating the z-score for Chemistry.
1 mark for calculating the z-score for Biology.
1 mark for making an appropriate comparison based on the z-scores.
Note: Accept z-scores presented as unsimplified fractions.

QUESTION 12 (7 marks)

a) $\frac{d}{dx}\left(\sin(x)\times\left(2x^2-3x\right)+\cos(x)\right)=\cos(x)\times\left(2x^2-3x\right)+\sin(x)\times(4x-3)-\sin(x)$

$=\cos(x)\times\left(2x^2-3x\right)+\sin(x)\times(4x-4)$

$=\cos(x)\times(2x-3)\times x+4\sin(x)\times(x-1)$

[3 marks]
1 mark for deriving all trigonometric terms.
1 mark for applying the product rule.
1 mark for determining the derivative in factorised form.
Note: Accept any suitably simplified and factorised equivalent.

b) $\frac{d}{dx}\left(\frac{3\ln(x)}{e^x}\right)=\frac{3\times\frac{1}{x}\times e^x-3\ln(x)\times e^x}{\left(e^x\right)^2}$

$=\frac{3e^x\left(\frac{1}{x}-\ln(x)\right)}{\left(e^x\right)^2}$

$=\frac{3\left(\frac{1}{x}-\ln(x)\right)}{e^x}$

[2 marks]
1 mark for applying the quotient or product rule.
1 mark for determining the derivative.
Note: Accept equivalent simplified expressions.

c) $\frac{d}{dx}\left(\cos^3(x)+9\right)=3\cos^2(x)\times-\sin(x)$

$=-3\cos^2(x)\times\sin(x)$

[2 marks]
1 mark for applying the chain rule.
1 mark for determining the derivative.

QUESTION 13 (4 marks)

a) $p=0.80$

mean $=np$

$=0.80$

[1 mark]
1 mark for determining the mean.

b) variance $=p(1-p)$

$=0.80(1-0.80)$

$=0.80\times0.20$

$=0.16$

[1 mark]
1 mark for determining the variance.

TEQ MM34_SS_P1_2022

c) Probability that no Top 10 products will be sold:

$$P(X=0)=\binom{3}{0}0.8^0 0.2^3$$
$$=0.2^3$$

Probability that at least one Top 10 product will be sold:

$$P(X\geq 1)=1-P(X=0)$$
$$=1-0.2^3$$
$$=0.992$$

[2 marks]
1 mark for using an appropriate method to determine the required probability.
1 mark for determining the probability.
Note: Accept 0.992 for the final answer.

QUESTION 14 (5 marks)

a) $f(t)=4t^3-2t^2+3e^{2t}$

$$f'(t)=12t^2-4t+6e^{2t}$$
$$f'(1)=12\times 1-4\times 1+6e^2$$
$$=8+6e^2$$

[2 marks]
1 mark for determining the first derivative.
1 mark for determining the first derivative at t = 1.

b) $f''(t)=24t-4+12e^{2t}$

$$f''(1)=24\times \ln 2-4+12e^{2\ln 2}$$
$$=24\ln 2-4+12\times 4$$
$$=24\ln 2+44$$

[3 marks]
1 mark for determining the second derivative.
1 mark for determining the second derivative at t = 1.
1 mark for simplifying the answer by removing the logarithm from the term.

QUESTION 15 (6 marks)

a) $\ln(x)+\ln(x+1)=\ln(2)$

$\Rightarrow \ln\big(x(x+1)\big)-\ln(2)$

$\Rightarrow x(x+1)=2$

$\Rightarrow x^2+x-2=0$

$\Rightarrow (x+2)(x-1)=0$

Therefore, $x = -2$ or $x = 1$.

$x = -2$ is rejected as, according to the original equation, x cannot be negative.

Therefore, $x = 1$.

[3 marks]
1 mark for accurately applying logarithmic laws to simplify the equation.
1 mark for determining the possible values of x. (Note: accept legitimate responses that avoid the negative value of x.)
1 mark for solving for the value of x (x = 1). (Note: Do not award this mark if the final answer also states x = –2.)

b) $2^{5x+1}+2^{5x+5}=17$

$\Rightarrow 2^{5x+1}+2^4\times 2^{5x+1}=17$

$\Rightarrow \left(1+2^4\right)2^{5x+1}=17$

$\Rightarrow (17)2^{5x+1}=17$

$\Rightarrow 2^{5x+1}=1$

$\Rightarrow 2^{5x+1}=2^0$

$\Rightarrow 5x+1=0$

$\therefore x=-\frac{1}{5}$

[3 marks]
1 mark for accurately using at least one index law.
1 mark for showing logical organisation that communicates key steps up to the development of an equation containing a single power.
1 mark for solving for the value of x.

QUESTION 16 (8 marks)

a) $f(x)=\ln\left(\cos(3x+\pi)\right)$

$$f'(x)=-\frac{3\sin(3x+\pi)}{\cos(3x+\pi)}$$

$$f'(0)=-\frac{3\sin(3\times 0+\pi)}{\cos(3\times 0+\pi)}$$

$$=-\frac{3\sin(\pi)}{\cos(\pi)}$$

$$=0$$

[2 marks]
1 mark for applying the chain rule (may be implied).
1 mark for determining the value of $f'(0)$.

b) $g(x)=\cos(3x+\pi)\times\ln(x+1)$

$$g'(x)=-3\sin(3x+\pi)\times\ln(x+1)+\frac{\cos(3x+\pi)}{x+1}$$

$$g'(0)=-3\sin(3\times 0+\pi)\times\ln(0+1)+\frac{\cos(3\times 0+\pi)}{0+1}$$

$$=-3\sin(\pi)\times\ln(1)+\frac{\cos(\pi)}{1}$$

$$=0-1$$

$$=-1$$

[2 marks]
1 mark for applying the product rule.
1 mark for determining the value of $g'(0)$.

c) $h(x)=\dfrac{\ln(x+1)}{\cos(3x+\pi)}$

$$h'(x)=\frac{\frac{\cos(3x+\pi)}{x+1}-\ln(x+1)\times -3\sin(3x+\pi)}{\cos^2(3x+\pi)}$$

$$h'(x)=\frac{\frac{\cos(3x+\pi)}{x+1}+3\ln(x+1)\times\sin(3x+\pi)}{\cos^2(3x+\pi)}$$

$$h'(0)=\frac{\frac{\cos(3\times 0+\pi)}{0+1}+3\ln(0+1)\times\sin(3\times 0+\pi)}{\cos^2(3\times 0+\pi)}$$

$$=\frac{\frac{\cos(\pi)}{1}+3\ln(1)\times\sin(\pi)}{\cos^2(\pi)}$$

$$=\frac{-1+0}{1}$$

$$=-1$$

[3 marks]
1 mark for applying the quotient rule or product rule to obtain the correct numerator.
1 mark for determining $h'(x)$. Note: $h'(x)$ does not need to be simplified.
1 mark for determining the value of $h'(0)$.

d) $$\begin{aligned} j'(0) &= f'(0) - g'(0) + h'(0) \\ &= 0 - -1 - 1 \\ &= 0 \end{aligned}$$

[1 mark]
1 mark for determining the value of $j'(0)$ using the correct process.
*Note: Consequential on answers to **Questions 16a)**, **16b)** and **16c)**.*
Responses that make errors with double negatives should not be awarded the mark.

QUESTION 17 (5 marks)

$$\begin{aligned} f'(x) &= \int f''(x)dx \\ &= \int -\frac{4}{x^2}dx \\ &= \frac{4}{x} + c \end{aligned}$$

$$f'(1) = \frac{4}{1} + c = 5$$
$$c = 1$$

Therefore, $f'(x) = \frac{4}{x} + 1$.

$$\begin{aligned} f(x) &= \int f(x)dx \\ &= \int \frac{4}{x} + 1dx \\ &= 4\ln(x) + x + d \end{aligned}$$

$$f(e) = 4\ln(e) + e + d = e$$
$$d = -4$$
$$f(x) = 4\ln(x) + x - 4$$

[5 marks]
1 mark for integrating $f''(x)$ to obtain $\frac{4}{x} + c$.
1 mark for solving for the value of c.
1 mark for integrating $f'(x)$ to obtain $4\ln(x) + x + d$.
1 mark for solving for the value of d.
1 mark for showing logical organisation that communicates key steps.
Note: Accept follow-through errors.

 TEQ MM34_SS_P1_2022

QUESTION 18 (5 marks)

a) $G(x) = 1.3x\left(1 - \frac{x}{k}\right)$

$G'(x) = 1.3\left(1 - \frac{x}{k}\right) - 1.3x \times -\frac{1}{k}$

$= 1.3\left[1 - \frac{2x}{k}\right]$

Let $G'(x) = 0$.

$1 - \frac{2x}{k} = 0$

$k = 2x$

Since $x = 4$, $k = 8$.

[3 marks]
1 mark for determining the first derivative.
1 mark for setting $G'(x) = 0$ and using this equation to solve for k.
1 mark for determining the value of k.

b) $G''(x) = -\frac{2.6}{k} = -\frac{2.6}{8}$

$-\frac{2.6}{8} < 0$

Therefore, by the second derivative test, $x = 4$ maximises $G(x)$.

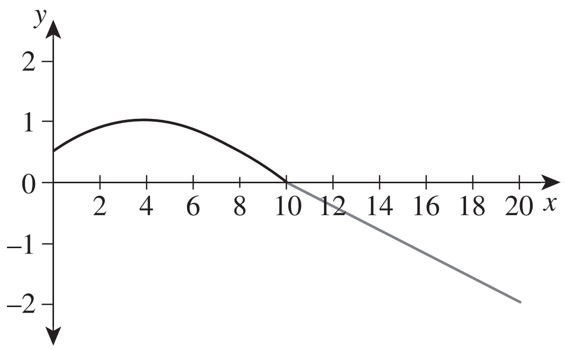
[2 marks]
1 mark for using an appropriate method to establish the nature of the critical point (first derivative test, second derivative test or parabola sketching).
1 mark for determining that the critical point is a maximum.
Note: Consequential on answer to ***Question 18a).***

QUESTION 19 (7 marks)

Checking that $v(t)$ is integrable over $0 \le x \le 20$ gives:

$$\cos\left(\frac{\pi(10-4)}{12}\right) = \cos\left(\frac{\pi}{2}\right) = 0$$

$$-\frac{1}{5} \times 10 + 2 = 0$$



$$\begin{aligned}
\text{displacement} &= \int_0^{20} v(t)dt \\
&= \int_0^{10} \cos\left(\frac{\pi(t-4)}{12}\right)dt + \int_{10}^{20} -\frac{1}{5}t + 2dt \\
&= \left[\frac{12}{\pi}\sin\left(\frac{\pi(t-4)}{12}\right)\right]_0^{10} + \left[-\frac{1}{10}t^2 + 2t\right]_{10}^{20} \\
&= \frac{12}{\pi}\left(\sin\left(\frac{\pi(10-4)}{12}\right) - \sin\left(\frac{\pi(0-4)}{12}\right)\right) + \left(-\frac{1}{10}20^2 + 2\times 20\right) - \left(-\frac{1}{10}10^2 + 2\times 10\right) \\
&= \frac{12}{\pi}\left(\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{-\pi}{3}\right)\right) + 0 - 10 \\
&= \frac{12}{\pi}\left(1 + \frac{\sqrt{3}}{2}\right) + 0 - 10 \\
&= \frac{12}{\pi}\left(1 + \frac{\sqrt{3}}{2}\right) - 10
\end{aligned}$$

The displacement is $\frac{12}{\pi}\left(1+\frac{\sqrt{3}}{2}\right)-10$ m.

[7 marks]

1 mark for checking that the piecewise function is integrable by checking that $\cos\left(\frac{\pi(10-4)}{12}\right)=-\frac{1}{5}\times10+2$. (Note: Responses should ensure that the two parts of the function connect when t = 10; for example, a sketch of v(t) showing that the graph is continuous.)

1 mark for identifying the need to integrate v(t), associating the correct bounds to the different piecewise sections.

1 mark for integrating the cosine function term.

1 mark for integrating the linear term.

1 mark for substituting into the definite integral.

1 mark for determining the displacement.

1 mark for expressing the displacement using exact values.