Neap

Trial Examination 2022

Suggested Solutions

QCE Mathematical Methods Units 1&2

Paper 1 – Technology-free

SECTION 1 – MULTIPLE CHOICE QUESTIONS

	A	B	C	D
1.	○	○	●	○
2.	○	○	●	○
3.	●	○	○	○
4.	●	○	○	○
5.	○	○	●	○
6.	○	○	○	●
7.	○	○	○	●
8.	○	○	●	○
9.	○	●	○	○
10.	○	●	○	○

QUESTION 1 C

$$16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}}$$
$$= \frac{1}{2}$$

QUESTION 2 C

$x = 6.5$ is within the domain of the second function.

$$f(6.5) = 4(6.5) + 1$$
$$= 27$$

QUESTION 3 A

The graph of $y = 2\sin\left(\frac{\pi x}{3}\right) - 1$ has a midline (centre) at $y = -1$ and an amplitude of 2, which gives a peak at $y = 1$ and a trough at $y = -3$. The period is $\frac{2\pi}{b} = \frac{2\pi}{\frac{\pi}{3}} = 6.$ This means that one complete cycle is 6 units. There is no phase shift, meaning that the y-axis will be at the centre of the sinusoidal curve in a positive direction. Therefore, the equation corresponds to graph **A**.

QUESTION 4 A

$$f(x) = \frac{(x-2)^3}{3x}$$
$$f'(x) = \frac{u'v - v'u}{v^2}$$
$$= \frac{3(x-2)^2 - 3(x-2)^3}{(3x)^2}$$
$$= \frac{9x(x-2)^2 - 3(x-2)^3}{9x^2}$$
$$= \frac{3(x-2)^2(3x-(x-2))}{9x^2}$$
$$= \frac{(x-2)^2(2x+2)}{3x^2}$$
$$= \frac{2(x-2)^2(x+1)}{3x^2}$$

QUESTION 5 C

The equation is a hyperbola. An untransformed hyperbola has the equation $y = \frac{1}{x}$ and has asymptotes given by $x = 0$ and $y = 0$. The function $g(x)$ is obtained from a horizontal translation of +3 and no vertical translation. Therefore, $g(x)$ has asymptotes given by $x = 3$ and $y = 0$.

QUESTION 6 D

Rearranging the equation into the standard form for a circle gives:

$$\begin{aligned} y^2 - 4y + x^2 + 8x &= 16 \\ (y^2 - 4y + 4) + (x^2 + 8x + 16) - 4 - 16 &= 16 \\ (y-2)^2 + (x+4)^2 &= 36 \\ (y-2)^2 + (x+4)^2 &= 6^2 \end{aligned}$$

The centre of the circle is at (–4, 2) and it has a radius of 6.

QUESTION 7 D

Probability distribution **A**:

$$\begin{aligned} E(X) &= 0 \times 0.1 + 5 \times 0.4 + 10 \times 0.4 + 15 \times 0.1 \\ &= 7.5 \end{aligned}$$

Probability distribution **B**:

$$\begin{aligned} E(X) &= 3 \times 0.25 + 5 \times 0.25 + 7 \times 0.25 + 9 \times 0.25 \\ &= 6 \end{aligned}$$

Probability distribution **C**:

$$\begin{aligned} E(X) &= 5 \times 0.1 + 6 \times 0.2 + 7 \times 0.6 + 8 \times 0.1 \\ &= 6.7 \end{aligned}$$

Probability distribution **D**:

$$\begin{aligned} E(X) &= 4 \times 0.05 + 6 \times 0.45 + 8 \times 0.45 + 10 \times 0.05 \\ &= 7 \end{aligned}$$

QUESTION 8 C

$$\begin{aligned} \tan(2x) + 1 &= 0 \\ \tan(2x) &= -1 \\ 2x &= \tan^{-1}(-1) \\ 2x &= -\frac{\pi}{4} \text{ or } \frac{3\pi}{4} \\ x &= -\frac{\pi}{8} \text{ or } \frac{3\pi}{8} \end{aligned}$$

QUESTION 9 B

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$
$$\Pr(A \cup B) = 2 \times \Pr(B) + \Pr(B) - \Pr(A \cap B)$$
$$3 \times \Pr(B) = \Pr(A \cup B) + \Pr(A \cap B)$$
$$3 \times \Pr(B) = 0.5 + 0.1$$
$$\Pr(B) = 0.2$$

QUESTION 10 B

Method 1:

The expansion will begin with $2^6 x^0$ as the first term and, for each subsequent term, the power of x will increase by 1.

Therefore, the fifth term will be:

$$\binom{6}{4} \times 2^2 \times x^4 = 15 \times 4x^4$$
$$= 60x^4$$

The coefficient of x^4 is 60.

Method 2:

The complete expansion can be found using the binomial theorem and the Pascal's triangle values given in the question.

$$(2+x)^6 = \binom{6}{0} \times 2^6 \times x^0 + \binom{6}{1} \times 2^5 \times x^1 + \ldots + \binom{6}{4} \times 2^2 \times x^4 + \ldots$$
$$= 1 \times 64 + 6 \times 32 \times x + \ldots + 15 \times 4 \times x^4 + \ldots$$
$$= 64 + 192x + 240x^2 + 160x^3 + 60x^4 + 12x^5 + x^6$$

The coefficient of x^4 is 60.

SECTION 2

QUESTION 11 (3 marks)

a) The turning point occurs at $(3, -5)$.

[1 mark]
1 mark for identifying the turning point as a pair of coordinates.

b) The axis of symmetry occurs at $x = 3$.

[1 mark]
1 mark for identifying the axis of symmetry as an equation.

c) As there is a y-intercept at $y = 1$, $(0, 1)$ must exist on the function.

$$y = a(x-3)^2 - 5$$
$$1 = a(0-3)^2 - 5$$
$$6 = a \times 9$$
$$a = \frac{2}{3}$$

[1 mark]
1 mark for identifying the value of a.

QUESTION 12 (5 marks)

Method 1:

$f(x) = x^3 + 2x^2 - 5x - 6$

$f(-3) = 0$

Thus, $(x + 3)$ is a factor.

The factorised form of $f(x)$ is $f(x) = (x + 3)(Ax^2 + Bx + C)$.

$A = 1$, $C = -2$ so $f(x) = (x + 3)(x^2 + Bx - 2)$

$Bx^2 + 3x^2 - 2x^2$, therefore $B = -1$ and so $f(x) = (x + 3)(x^2 - x - 2)$.

Factorising the quadratic term:

$x^2 - x - 2 = (x - 2)(x + 1)$

Therefore, $f(x) = (x + 3)(x - 2)(x + 1)$.

[5 marks]

1 mark for stating that $x + 3$ is a factor. Note: This may be implied by subsequent working.
1 mark for evaluating $A = 1$. Note: Accept polynomial division as a method.
1 mark for fully factorising into a linear term and quadratic term.
1 mark for factorising the quadratic term.
1 mark for reaching the fully factorised $f(x)$.

Method 2:

$f(x) = x^3 + 2x^2 - 5x - 6$

$f(-3) = 0$

Thus, $(x + 3)$ is a factor.

Finding the other quadratic factor by applying polynomial division gives:

$$\begin{array}{r l}
 & x^2 - x - 2 \\
x + 3 \big) & x^3 + 2x - 5x - 6 \\
 & x^3 + 3x^2 \\
 & \quad -x^2 - 5x - 6 \\
 & -x^2 - 3x \\
 & \quad -2x - 6 \\
 & -2x - 6 \\
 & \quad 0 \quad \text{(no remainder)}
\end{array}$$

$f(x) = (x + 3)(x^2 - x - 2)$

$x^2 - x - 2 = (x - 2)(x + 1)$

$\therefore f(x) = (x + 3)(x - 2)(x + 1)$

[5 marks]

1 mark for recognising $(x + 3)$ as a factor. Note: This may be implied by subsequent working.
1 mark for using polynomial division to determine x^2 as the first term in the quadratic.
1 mark for stating the quadratic factor from polynomial division or implied.
1 mark for factorising the quadratic term.
1 mark for the final, decomposed value of $f(x)$.

QUESTION 13 (3 marks)

a) $t_1 = 4 \times 1 - 3$
$= 1$

[1 mark]
1 mark for calculating the first term.

b) $t_2 = 4 \times 2 - 3$
$= 5$

Common difference, $\mathrm{d} = t_2 - t_1$
$= 4$

[1 mark]
1 mark for determining the common difference (d).

c) $t_8 = 4 \times 8 - 3$
$= 29$

[1 mark]
1 mark for calculating the eighth term.

QUESTION 14 (6 marks)

a) $y = 2x^3$
$y' = 3 \times 2x^2$
$= 6x^2$

[1 mark]
1 mark for providing the correct, simplified answer.

b) $y = (3x - 4)^5$

$y' = 5 \times (3x - 4)^4 \times \frac{d}{dx}(3x - 4)$
$= 15(3x - 4)^4$

[2 marks]
1 mark for differentiating the power with coefficient 5.
1 mark for providing the correct, simplified answer.

c) $y = 2x\sqrt{x^2 - 4}$

$= 2x(x^2 - 4)^{\frac{1}{2}}$

Let:

$u = 2x$

$u' = 2$

$v = (x^2 - 4)^{\frac{1}{2}}$

$v' = \frac{1}{2}(x^2 - 4)^{-\frac{1}{2}} \times 2x$

$= x(x^2 - 4)^{-\frac{1}{2}}$

[3 marks]

1 mark for identifying a power of a half.

$y' = u'v + v'u$

$= 2 \times (x^2 - 4)^{\frac{1}{2}} + x(x^2 - 4)^{-\frac{1}{2}} \times 2x$

$= 2(x^2 - 4)^{\frac{1}{2}} + 2x^2(x^2 - 4)^{-\frac{1}{2}}$

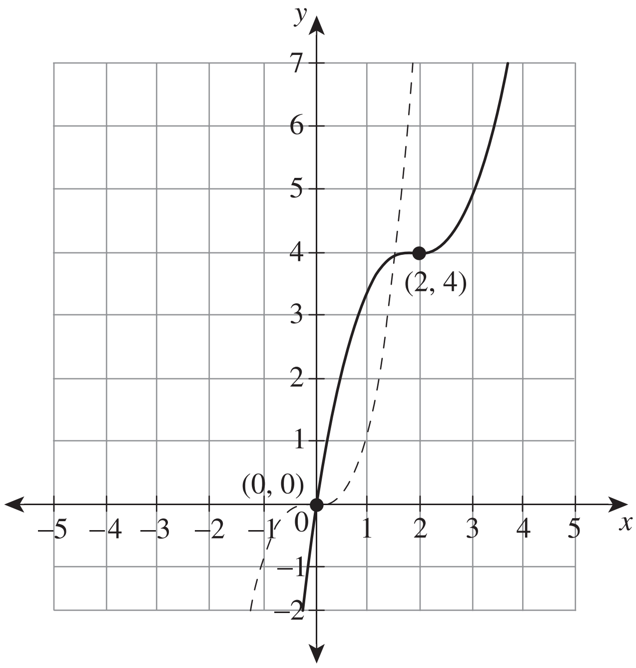
$= \frac{4(x^2 - 2)}{\sqrt{x^2 - 4}}$

1 mark for applying the product rule. Note: Allow follow-through errors.

1 mark for providing the correct, simplified answer.

QUESTION 15 (5 marks)

a)



Key points:

- (0, 0) is the axis intercept.
- (2, 4) is the centre or point of inflection.

[2 marks]
1 mark for providing a correct and neatly drawn graph.
1 mark for identifying the two key points, (0, 0) and (2, 4).

b) Transformations applied:

- vertical dilation by a factor of one half (or compression by 2)
- vertical translation by 4 units upwards (or in the positive direction)
- horizontal translation by 2 units to the right (or in the positive direction)

[3 marks]
1 mark for stating that there is a vertical dilation by a factor of one half.
1 mark for stating that there is a vertical translation by 4 units in the positive direction.
1 mark for stating that there is a horizontal translation by 2 units in the positive direction.

QUESTION 16 (4 marks)

a) The initial distance occurs at $t = 0$.

$$x(0) = 4 + 20(0) - 5(0)^2$$
$$= 4 \text{ m}$$

[1 mark]
1 mark for providing the correct initial distance.
Note: Including units and commenting on the positive direction are not required.

b) The velocity function of the car is the derivative of the position function.

$$x'(t) = 20 - 5t \times 2$$
$$= 20 - 10t$$

[1 mark]
1 mark for providing the correct velocity function.

c) **Method 1:**

The maximum distance occurs at $x'(t) = 0$.

$$20 - 10t = 0$$
$$t = 2 \text{ seconds}$$

$$x(2) = 4 + 20(2) - 5(2)^2$$
$$= 24 \text{ m}$$

Method 2:

The maximum distance occurs at the turning point (TP).

$$TP_x = -\frac{b}{2a}$$
$$= -\frac{20}{2 \times -5}$$
$$= 2 \text{ seconds}$$

$$x(2) = 4 + 20(2) - 5(2)^2$$
$$= 24 \text{ m}$$

[2 marks]
1 mark for providing the value of t at the furthest distance. Note: This may be implied by subsequent working.
1 mark for providing the maximum distance in the positive direction.
Note: Units and commenting on the positive direction are not required.

QUESTION 17 (7 marks)

a) $y = 2x^2 - kx + k - 3$

$a = 2;\ b = -k;\ c = (k - 3)$

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (-k)^2 - 4 \times 2 \times (k - 3) \\ &= k^2 - 8k + 24 \\ &= (k - 4)^2 - 16 + 24 \\ &= (k - 4)^2 + 8\end{aligned}$$

This quadratic equation has a minimum turning point at (4, 8). The range is 8. Therefore, the equation is positive for all values of k.

Thus, the equation $y = 2x^2 - kx + k - 3$ has two real solutions for all values of k.

[4 marks]

1 mark for identifying the discriminant as a key value.

1 mark for calculating the discriminant.

1 mark for evaluating the discriminant to establish that it is always positive (consider a range of approaches).

1 mark for drawing the conclusion that this implies k is always positive and, therefore, two solutions always exist.

b) **Method 1:**

Using the discriminant:

If 2 is a solution, then $x = 2$ when $y = 0$.

$0 = 2(2)^2 - k(2) + k - 3$

$0 = 8 - 2k + k - 3$

$0 = 5 - k$

$k = 5$

[3 marks]

1 mark for identifying (2, 0) as an x-axis intercept. Note: This may be implied by subsequent working.

1 mark for substituting (2, 0) into the original equation.

1 mark for providing the correct answer.

Method 2:

Using the quadratic formula:

$$x = \frac{-(-k) \pm \sqrt{k^2 - 8k + 24}}{2 \times 2}$$

$$2 = \frac{k \pm \sqrt{k^2 - 8k + 24}}{4}$$

$$8 = k \pm \sqrt{k^2 - 8k + 24}$$

$$8 - k = \pm\sqrt{k^2 - 8k + 24}$$

$$64 - 16k + k^2 = k^2 - 8k + 24$$

$$40 = 8k$$

$$k = 5$$

[3 marks]

1 mark for identifying $x = 2$ in a quadratic formula involving k.

1 mark for applying a suitable method to simplify the square root component of the equation.

1 mark for providing the correct answer.

QUESTION 18 (5 marks)

Equation (1):

$$4^{2x-1} = 8^{x+y}$$

$$2^{2(2x-1)} = 2^{3(x+y)}$$

Equation (2):

$$27^{3y} = 3^{2x-7}$$

$$3^{3(3y)} = 3^{2x-7}$$

Equation (3):

$$2(2x-1) = 3(x+y)$$

$$4x-2 = 3x+3y$$

$$x = 3y+2$$

Equation (4):

$$3(3y) = 2x-7$$

Substituting (3) into (4) gives:

$$(4)\ 3(3y) = 2x-7$$

$$9y = 2(3y+2)-7$$

$$9y = 6y+4-7$$

$$3y = -3$$

$$y = -1$$

Substituting $y = -1$ into (3) gives:

$$x = 3y+2$$

$$x = 3(-1)+2$$

$$= -1$$

$x = -1$ and $y = -1$

[5 marks]

1 mark for simplifying the bases to give equations 1 and 2.
1 mark for giving equations 3 and 4 without indices.
1 mark for identifying x or y.
1 mark for calculating the second value.
1 mark for showing logical organisation and communication of key steps up to equation 4.

QUESTION 19 (6 marks)

The two gradients may be equal at two different values for x; therefore, y_1, x_1, y_2 and x_2 are introduced.

$$y_1 = x_1^2$$
$$y_1' = 2x_1$$

$$y_2 = x_2^2 + 4x_2 - 7$$
$$y_2' = 2x_2 + 4$$

When $y_1' = y_2'$ (the two gradients are equal), the relationship between the x values is

$$2x_1 = 2x_2 + 4$$
$$x_1 = x_2 + 2$$

$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{((x_2 + 2) - x_2)^2 + ((x_1)^2 - (x_2^2 + 4x_2 - 7))^2} \\ &= \sqrt{(2)^2 + ((x_2 + 2)^2 - x_2^2 - 4x_2 + 7)^2} \\ &= \sqrt{4 + (x_2^2 + 4x_2 + 4 - x_2^2 - 4x_2 + 7)^2} \\ &= \sqrt{4 + (11)^2} \\ &= \sqrt{125} \\ &= 5\sqrt{5} \end{aligned}$$

Therefore, the distance between a point on each function with equal gradients is a constant value and, thus, it is independent of the actual gradients at the point or the points themselves.

[6 marks]

1 mark for calculating both gradient (derivative) functions.

1 mark for providing the relationship between the x values when the gradients are equal.

1 mark for using the distance formula, given a value correct or otherwise.

1 mark for simplifying the distance formula in terms of one variable.

1 mark for showing that the distance between any two points meeting the criteria is $5\sqrt{5}$.

1 mark for providing a statement regarding the independence of this distance from the two points due to the numerical value of the distance.

QUESTION 20 (6 marks)

The common ratio can be found by dividing consecutive terms.

$r = \frac{2b}{3b-5}$ and $r = \frac{b+6}{2b}$

$$\frac{2b}{3b-5} = \frac{b+6}{2b}$$

$$2b \times 2b = (3b-5)(b+6)$$

$$4b^2 = 3b^2 - 5b + 18b - 30$$

$$b^2 - 13b + 30 = 0$$

$$(b-10)(b-3) = 0$$

$$b = 10 \text{ and } b = 3$$

If $b = 10$, the sequence is 25, 20, 16 and $r = \frac{2(10)}{3(10)-5} = \frac{4}{5}$.

$$\begin{aligned} S_\infty &= \frac{a}{1-r} \\ &= \frac{25}{1-\frac{4}{5}} \\ &= 125 \end{aligned}$$

If $b = 3$, the sequence is 4, 6, 9 and $r = \frac{2(3)}{3(3)-5} = \frac{6}{4} > 1$. Therefore, S_∞ does not exist.

One value for S_∞ is 125.

[6 marks]

1 mark for recognising one of the two ratios for the common difference.

1 mark for providing the initial correct equation in terms of b.

1 mark for finding that $b = 10$ and $b = 3$.

1 mark for discounting $b = 3$ as a solution.

1 mark for finding that $r = \frac{4}{5}$ when $b = 10$.

1 mark for identifying the single sum to infinity as 125.