Neap

Trial Examination 2021

Question and response booklet

QCE Mathematical Methods Units 3&4

Paper 2 – Technology-active

Student's Name: _____

Teacher's Name: _____

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- QCAA-approved calculator permitted.
- Formula sheet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 10 short response questions

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2021 QCE Mathematical Methods Units 3&4 Written Examination.

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has 10 questions and is worth 50 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (4 marks)

Determine the following derivatives.

a) $\frac{d}{dx}\left(\sin\left(\ln\left(x\right)\right)\right)$ *[2 marks]*

b) $\frac{d}{dx}\left(e^{\pi x}\times\left(3x^2+4x+8\right)\right)$ *[2 marks]*

QUESTION 12 (4 marks)

Determine the following integrals.

a) $\int 8x^3-\frac{1}{x^3}dx$ *[2 marks]*

b) $\int \sin\left(3-5x\right)dx$ *[2 marks]*

QUESTION 13 (4 marks)

During a school camp, a student sees a single cocoon of a rare species of butterfly hanging on a tree branch. The probability that this species emerges from its cocoon is p.

a) If the standard deviation of this distribution is roughly 0.14 and p is less than 0.5, determine the value of p. *[3 marks]*

b) Use your answer for 13a) to determine the expected value of this butterfly emerging from its cocoon. *[1 mark]*

QUESTION 14 (4 marks)

The probability that a pencil produced at a particular factory is defective is 4 out of 1000. The factory manufactures approximately 25 000 pencils each day.

a) Calculate the mean and the standard deviation for the number of defective pencils produced by the factory in one day. *[3 marks]*

b) Determine the probability that in one day, fewer than 101 pencils will be defective. *[1 mark]*

QUESTION 15 (5 marks)

The acceleration of a particle is represented by the following function.

$$x''(t) = 18.8e^{2t} - \frac{2.1}{(t+1)^2}$$

a) Determine the equation for $x'(t)$, if $x'(0) = 11.5$. *[2 marks]*

b) Determine the equation for $x(t)$, if $x(0) = 6$. *[3 marks]*

QUESTION 16 (5 marks)

The amount of sleep that adult cats get each day is a continuous variable measured in hours. It is approximately normally distributed with a mean of approximately 15.2 and a standard deviation of approximately 3.9.

Determine the probabilities that an adult cat sleeps the following number of hours each day.

a) exactly 12 hours

Briefly explain your answer. *[2 marks]*

b) more than 15.2 hours *[1 mark]*

c) within two standard deviations from 15.2 hours *[1 mark]*

d) between 5 and 10 hours *[1 mark]*

QUESTION 17 (5 marks)

A study was conducted to determine how many people frequently fed native Australian wildlife in the past year. A random selection of 200 people were surveyed, and it was found that 47 individuals had frequently fed native wildlife in the past year.

a) Determine the sample proportion. You do not need to simplify. *[1 mark]*

b) Determine an estimate of the population standard deviation. *[1 mark]*

c) Determine a 95% confidence interval for the true population proportion. *[3 marks]*

QUESTION 18 (5 marks)

An entrepreneur founded the social media platform Connexting in 2008. The growth in the platform's total users can be modelled using the following logarithmic function, where x represents the year (such as 2008) and y represents the natural logarithm of the number of users in billions (that is, if N represents the number of users in billions, then $y = \ln N$).

$$y = 2.68\ln\left(0.058x - 116\right)$$

a) Determine the number of users that Connexting is expected to reach by 2030. *[2 marks]*

b) Determine the year that Connexting reached one billion users. *[1 mark]*

A rival social media platform, OPzest, was established in 2015 and experienced tremendous growth. The following function describes OPzest's growth, where x represents the year and y is the natural logarithm of number of users in billions.

$$y = 4.78\ln\left(0.055\left(x - 2000\right)\right)$$

c) Determine the year in which OPzest and Connexting had the same number of users. *[1 mark]*

d) Determine the number of users each platform had when they had the same number of users. *[1 mark]*

QUESTION 19 (6 marks)

A recent study tested how familiar Australian primary school students are with Australia's location on a world map. For the study, researchers randomly selected 50 primary school students, and each student was given 30 seconds to locate Australia on a world map. A student was considered successful if they managed to correctly identify Australia on the world map within the time constraint.

Calculating a 95% confidence interval, the researchers determined the upper bound of the confidence interval as being 0.844.

Determine the lower bound of the confidence interval.

QUESTION 20 (8 marks)

The following function is a probability density function.

$$f(x) = \begin{cases} 2bx^2, & c < x < 1 \\ bx, & 0 < x < c \\ 0, & \text{otherwise} \end{cases}$$

The probability density curve is continuous on its domain, meaning that the density curve is an unbroken line. The different segments of the function meet at $x = c$.

Use the probability density function to determine the expected value of the distribution.

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Formula sheet

QCE Mathematical Methods Units 3&4

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}\left(2t_1 + (n-1)d\right) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r-1)}$ $S_\infty = \frac{t_1}{(1-r)}, \ \lvert r \rvert < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n\log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Calculus		
$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$		$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$		$\int \frac{1}{x} dx = \ln(x) + c$
$\frac{d}{dx}\sin(x) = \cos(x)$		$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$		$\int \cos(x) dx = \sin(x) + c$
chain rule	If $h(x) = f\left(g(x)\right)$ then $h'(x) = f'\left(g(x)\right)g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	$\text{area} = \frac{1}{2}bc\sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
binomial probability	$P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable *X*	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
continuous random variable *X*	mean	$E(X) = \mu = \int_{-\infty}^{\infty} xp(x)dx$
	variance	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for *p*	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$	
general addition rule for probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
probability of independent events	$P(A \cap B) = P(A) \times P(B)$	
conditional probability	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$	