Neap

Trial Examination 2021

Question and response booklet

QCE Mathematical Methods Units 3&4

Paper 1 – Technology-free

Student's Name: _____

Teacher's Name: _____

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- Calculators are not permitted.
- Formula sheet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2021 QCE Mathematical Methods Units 3&4 Written Examination.

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has 9 questions and is worth 50 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (5 marks)

Determine the value of x in the following equations.

a) $\log_3 5 + \log_3 (9x) = 2$ *[2 marks]*

b) $\log_{11}(2x+1) - \log_{11}(x-2) = \log_{11}(3)$ *[3 marks]*

QUESTION 12 (5 marks)

Determine the derivative of the following functions.

a) $f(x) = 2e^{3x^2+x+1}$ *[1 mark]*

b) $f(x) = \ln(5x-2)$ *[1 mark]*

c) $f(x) = \log_5(3x^7)$

Express your answer in simplified form. *[3 marks]*

QUESTION 13 (5 marks)

Calculate the area bounded by the function $y = \frac{3}{2}x - x^2$ and the x-axis.

QUESTION 14 (5 marks)

Find all solutions for $22\sin(2x) = -11$, where $0 < x < 2\pi$.

Express your answer in exact value form.

QUESTION 15 (5 marks)

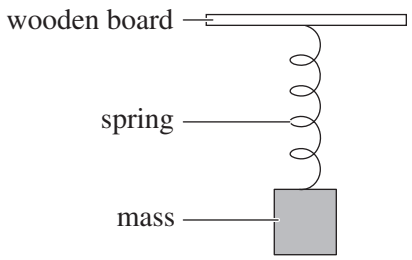
An environmental scientist has modelled the population of the central rock-rat (a type of rodent) in a small region where considerable effort has been put into conservation. The rate of change is given by the formula $P'(t) = \frac{6}{5}e^{\frac{t}{5}}$, where P represents the estimated population and t represents the time in months.

a) Determine the equation for $P(t)$ given that $P(0) = 36$. *[3 marks]*

b) Given that $5.5 \approx \ln(3^5)$, use your answer for 15a) to predict the population at 5.5 months. *[2 marks]*

QUESTION 16 (5 marks)

The diagram below shows a mass attached to a spring and suspended from a wooden board.



The mass is stretched beyond its resting position. After the mass is released at $t = 0$, it is allowed to oscillate. The displacement of the mass is modelled by the following function.

$$x(t) = 9\sin\left(\frac{2t+1}{3}\right)$$

a) Determine the velocity function. *[2 marks]*

b) Determine the acceleration function. *[1 mark]*

c) Determine the first instance that the spring reaches its maximum acceleration, where $(t > 0)$. *[2 marks]*

QUESTION 17 (7 marks)

A caterpillar falls from a tree and crawls to a nearby bush. The following function describes the caterpillar's velocity for the first 20 minutes after it reaches the ground, where t is the time in minutes.

$$y' = \frac{\pi}{8}\cos\left(\frac{\pi}{8}\left(\frac{t}{5} - 8\right)\right)$$

Calculate the distance between the caterpillar's positions at $t = \frac{20}{3}$ and $t = 10$.

Express your answer in exact value form in metres.

QUESTION 18 (5 marks)

X is normally distributed with a mean of $2a^2$ and a standard deviation of a. Assume that Z is a standard normal distribution and that $P(Z < -a \text{ or } Z > a) = b$.

Determine the following probabilities in terms of the variable b.

a) $P(X > a^2)$ *[3 marks]*

b) $P(2a^2 < X < 3a^2)$ *[1 mark]*

c) $P(2a^2 < X < 3a^2 \mid X > a^2)$ *[1 mark]*

QUESTION 19 (8 marks)

The following function has two points of inflection and a stationary point.

$$f(x) = \frac{x^4}{4} - \frac{(a+b)x^3}{2} + \frac{3abx^2}{2} + 4$$

The first point of inflection is at $x = 1$ and the stationary point is at $x = 3$.

Given that $(1 - y)(1 - z) = (y + z) + yz$, find the values of a and b, where $a < b$. Determine the x-coordinate of the second point of inflection.

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Formula sheet

QCE Mathematical Methods Units 3&4

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}\left(2t_1 + (n-1)d\right) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r-1)}$ $S_\infty = \frac{t_1}{(1-r)}, \ \lvert r \rvert < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n\log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Calculus		
$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$		$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$		$\int \frac{1}{x} dx = \ln(x) + c$
$\frac{d}{dx}\sin(x) = \cos(x)$		$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$		$\int \cos(x) dx = \sin(x) + c$
chain rule	If $h(x) = f\left(g(x)\right)$ then $h'(x) = f'\left(g(x)\right)g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	$\text{area} = \frac{1}{2}bc\sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
binomial probability	$P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable *X*	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
continuous random variable *X*	mean	$E(X) = \mu = \int_{-\infty}^{\infty} xp(x)dx$
	variance	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for *p*	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$	
general addition rule for probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
probability of independent events	$P(A \cap B) = P(A) \times P(B)$	
conditional probability	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$	