CSSA

2024
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

DO NOT REMOVE PAPER FROM EXAMINATION ROOM

Centre Number

Student Number

Mathematics Advanced

Morning Session
Monday, 12 August 2024

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- Use the Multiple-Choice Answer Sheet provided for Section I
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 100

Section I – 10 marks (pages 2–7)
- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8–35)
- Attempt Questions 11–32
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet for Questions 1–10.

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1 Consider the graph of $y = 4x + 1$.

What is the equation of the graph after it has been translated 2 units to the right?

A. $y = 2x - 7$

B. $y = 2x + 9$

C. $y = 4x - 7$

D. $y = 4x + 9$

2 Giana and Maria run a painting business.

Giana paints 1 m^2 of a wall in x minutes and Maria paints 1 m^2 of a wall in y minutes.

Which expression represents the area of the wall they can paint in 1 hour if Giana and Maria worked together?

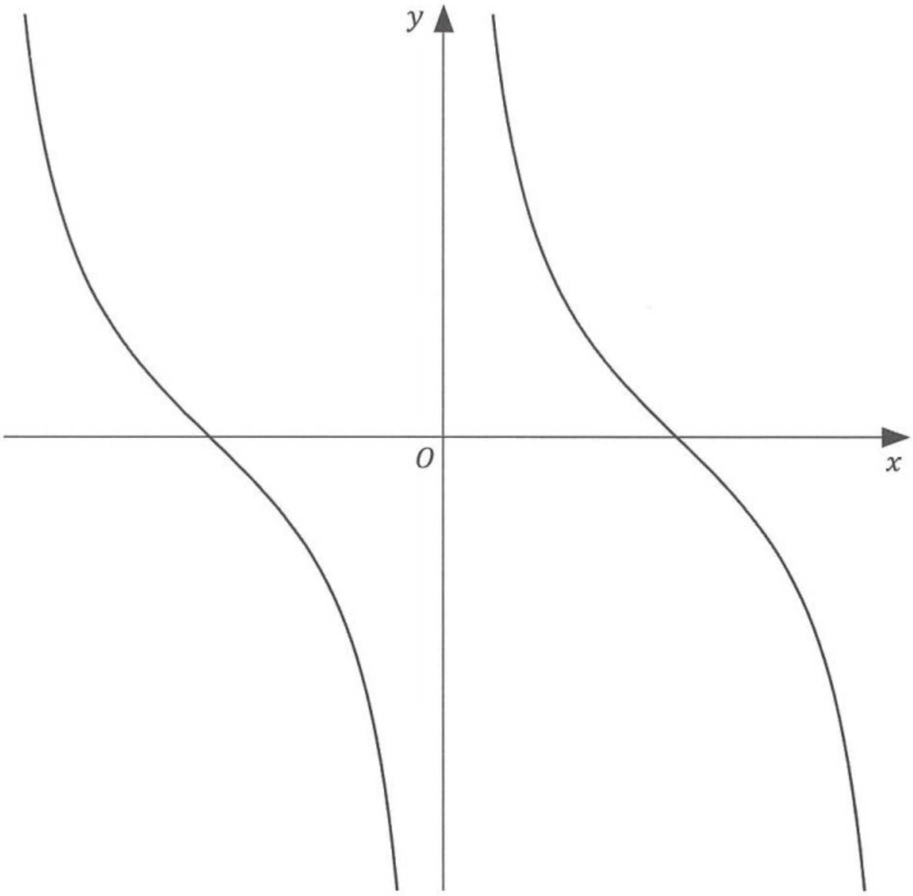
A. $x + y$

B. $\dfrac{x + y}{60}$

C. $\dfrac{x + y}{xy}$

D. $60\left(\dfrac{x + y}{xy}\right)$

3 Consider the following graph of $y = f(x)$ which crosses the horizontal axis at $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.



Which of the following equations could represent $y = f(x)$?

A. $y = \cot(x)$

B. $y = \tan(x)$

C. $y = \sec(x)$

D. $y = \operatorname{cosec}(x)$

4 The results of a class quiz are shown in the frequency table below.

Result	Frequency
6	3
7	2
8	8
9	5
10	6

How many students had a result within one standard deviation of the mean?

A. 8

B. 13

C. 15

D. 21

5 Ali decides to invest \$500 at the end of each month into an annuity. The interest rate quoted is 9% per annum, compounded monthly.

Let A_n represent the future value of the annuity after n months.

Which recurrence relation models this financial situation?

A. $A_n = A_{n-1} \times 1.0075 + 500, A_0 = 0$

B. $A_n = A_{n-1} \times 1.0075, A_0 = 500$

C. $A_n = A_{n-1} \times 1.09 + 500, A_0 = 0$

D. $A_n = A_{n-1} \times 1.09, A_0 = 500$

6 The following cumulative frequency table shows heights of students at a school.

Height (cm)	Cumulative frequency
131 – 140	70
141 – 150	210
151 – 160	420
161 – 170	680
171 – 180	800

What is the height of a student in the 7th decile?

A. 131 – 140 cm

B. 151 – 160 cm

C. 161 – 170 cm

D. 171 – 180 cm

7 The strength of an earthquake is calculated using Richter's formula

$$M = \log_{10}(I),$$

where M is the magnitude of the earthquake and I is the intensity.

An earthquake with a magnitude of 5.4 was detected in the desert.

A second earthquake occurred in the same location a few years later and was measured to be twice as intense as the first earthquake.

What was the magnitude of the second earthquake?

A. $2 \times 10^{5.4}$

B. $10^{5.4}$

C. 10.8

D. 5.7

8 Two independent events A and B are given such that $P(A) = m$ and $P(B) = m + 0.4$.

It is known that $P(A \cap B) = 0.21$.

What is the value of $P(A \cup B)$?

A. 1

B. 0.79

C. 0.3

D. 0.09

9 Let $f(x)$ be a continuous and differentiable function for all real values of x.

The following facts are known.

- $f'(-x) = f'(x)$
- $f'(x) \neq 0$
- $f(0) = f''(0) = 0$

Which of the following statements is always true?

A. $(0, 0)$ is a point of inflection.

B. $f(x)$ is an odd function.

C. $f(x)$ is an even function.

D. $f(x)$ is neither even nor odd.

10 Let $f(x)$ be an even function. It is given that $\int_{-3}^{-1} f(x)\,dx = \frac{25}{6}$.

Which of the following equals $\int_{3}^{5} \big(f(2-x) + f(x-2)\big)\,dx$?

A. $-\frac{25}{3}$

B. 0

C. $\frac{25}{3}$

D. $\frac{50}{3}$

Section II

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 36–39. If you use this space, clearly indicate which question you are answering.
- Extra writing booklets are available.

Section II begins on page 9

Question 11 (2 marks)

Solve the equation $5 - x = \frac{2x}{3}$. **2**

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Question 12 (2 marks)

Let $f(x) = 3x^2 + 5x$. **2**

Use the definition $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of $f(x)$.

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Question 13 (2 marks)

Trish is planning a function. The cost per person to attend the function varies inversely with the number of people attending. **2**

The cost per person is \$70 when 50 people attend.

What is the cost per person if 125 people attend the function?

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Question 14 (3 marks)

Consider the functions $f(x) = \frac{1}{x} + 3$ and $g(x) = x - 2$. **3**

Let $h(x) = f(g(x))$.

Find the domain and range of $h(x)$.

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Question 15 (3 marks)

(a) Find the derivative of the function $y = \tan(x^2)$. **1**

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(b) By using an appropriate trigonometric identity, find $\int x \tan^2(x^2)\, dx$. **2**

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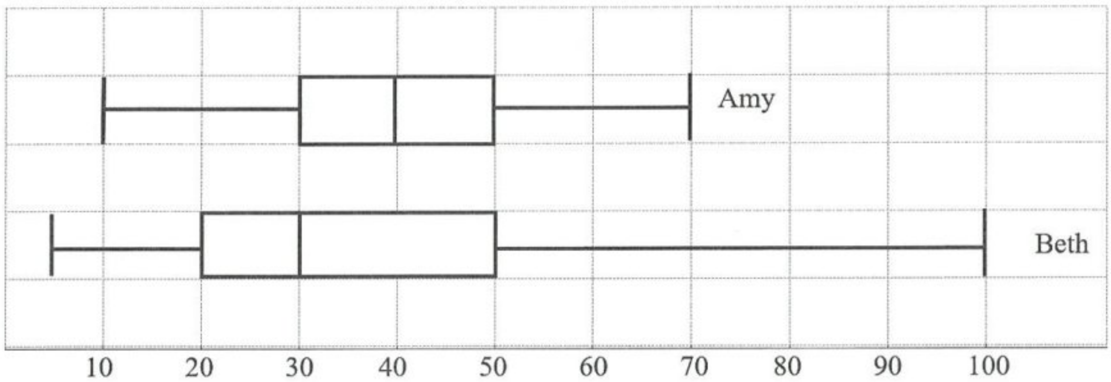
Question 16 (3 marks)

Amy and Beth are members of a cricket team. **3**

The parallel box-plot below summarises their batting scores for the season.

Compare the sets of scores by interpreting the summary statistics and the shape of the distributions.



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Question 17 (3 marks)

The weight of boxes of cereal that are produced in a factory are normally distributed with a mean of 650 grams and a standard deviation of 5 grams. **3**

Boxes of cereal are fit for sale if they weigh more than 640 grams.

If the factory produces one million boxes, how many would you expect to be fit for sale?

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Question 18 (4 marks)

The table below shows the monthly repayments on loans of various amounts over a number of different loan periods.

Principal	Monthly repayment amount		
	Loan period (years)		
	2	5	7
\$10 000	\$480	\$228	\$187
\$20 000	\$960	\$455	\$374
\$30 000	\$1439	\$683	\$561

(a) Spiro is borrowing \$30 000, to be repaid with monthly repayments over 5 years. **2**

How much interest will be paid on the loan?

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(b) Calculate the percentage flat rate of interest per annum on this loan. **2**

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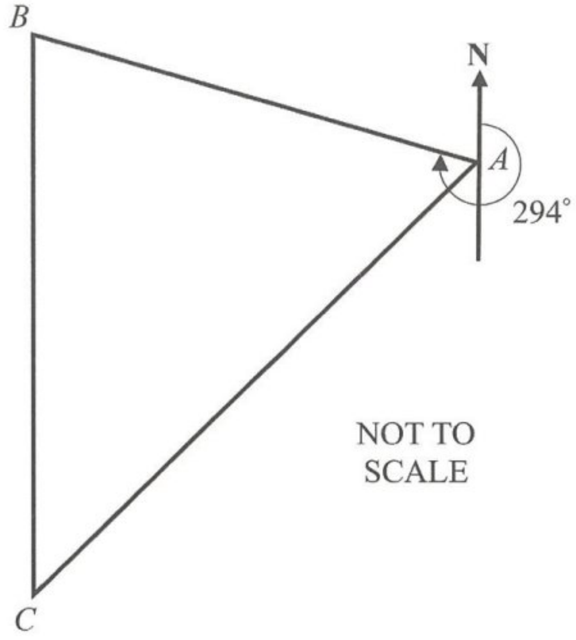
Question 19 (5 marks)

A yacht race takes a triangular course. Yachts leave from *A* and travel on a bearing of 294° for 18 km to a buoy at *B*. **5**

From *B* the yachts travel due south to another buoy at *C*.

The final leg of the race is on a bearing of 030° from *C* back to finish at *A*.



What is the total length of the race, in kilometres, correct to three significant figures?

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Question 20 (3 marks)

Find the equation of the normal to the curve $y = \dfrac{\ln(x-1)}{x-1}$ at the point $(2, 0)$. **3**

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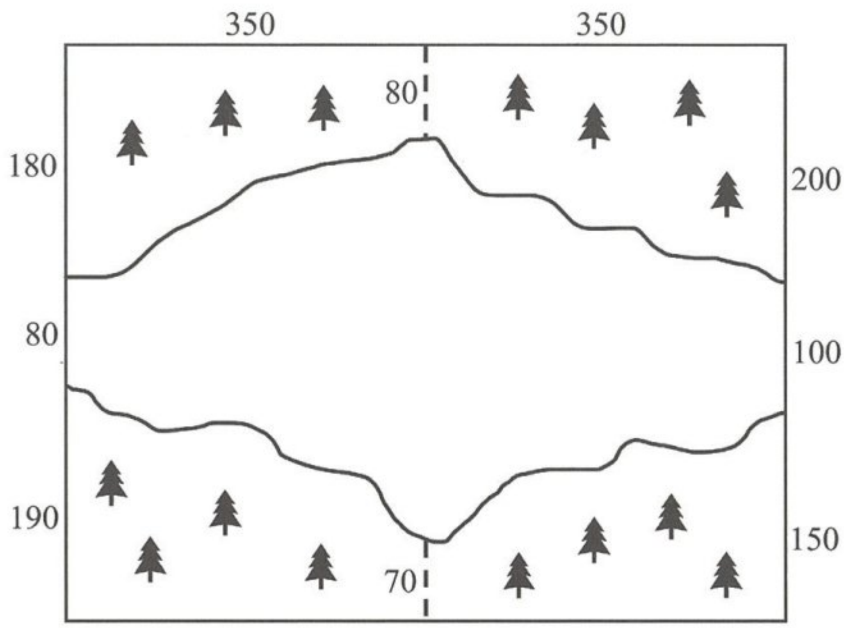
Question 21 (4 marks)

A plane is flying through clouds to drop a water quality monitoring system onto a lake. **4**

The lake is surrounded by forest as shown below, with all measurements in metres.



NOT TO SCALE

Estimate the probability that the plane will drop the monitoring system onto the lake.

Use the Trapezoidal rule in your calculations.

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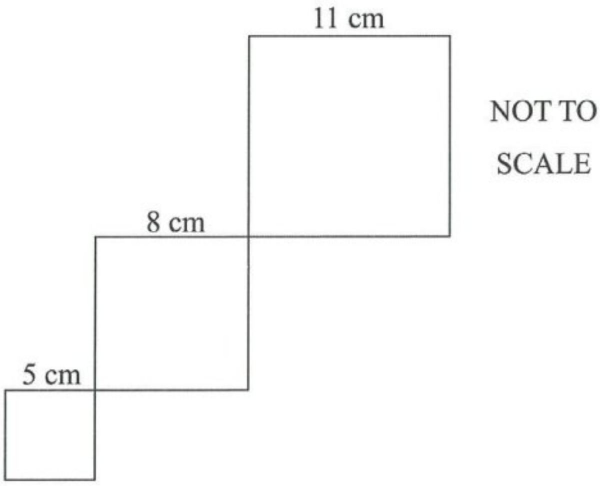
Question 22 (5 marks).

A robot is programmed to draw a pattern consisting of squares in increasing sizes with no overlapping lines, as shown.



The side length of each square follows an arithmetic sequence. The first three squares have side lengths 5 cm, 8 cm and 11 cm respectively.

(a) What is the perimeter of the 24th square in this sequence? **2**

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(b) Find the total perimeter of all the squares after the robot draws the 24th square. **1**

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Question 22 continues on page 19

Question 22 (continued)

(c) The amount of ink in the robot's pen can only draw up to a maximum of 12 000 cm. **2**
How many complete squares can the robot draw using the one pen?

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End of Question 22

Question 23 (4 marks)

Steve produces videos on remote control cars and shares them on a YouTube channel. **4**

The actual view times, in hours, on the YouTube channel over a 5-year period was recorded.

Steve suspects that the bivariate relationship between x and y is exponential. He decides to transform the data to make the relationship appear linear by finding the natural logarithm of the view times, and recording those values in the table shown.

Year	1	2	3	4	5
Transformed view time	4.01	5.01	6.31	7.29	8.49

Let n represent the year and $\ln(t)$ represent the transformed view time.

By finding the equation of the least-squares regression line of the transformed data, predict the actual view time on the YouTube channel in the 7th year. Give your answer correct to the nearest hour.

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Question 24 (9 marks)

Consider the gradient function $f'(x) = 3(x+3)(x-1)$.

The graph of $y = f(x)$ passes through the point $(2, -8)$.

(a) Show that $f(x) = x^3 + 3x^2 - 9x - 10$. **3**

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(b) Find the minimum and maximum values of $f(x)$ in the interval $-4 \leq x \leq 4$. **2**

You do not need to determine the nature of the stationary points.

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Question 24 continues on page 22

Question 24 (continued)

(c) Show that a point of inflection exists at $(-1, 1)$. **2**

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(d) Sketch the graph of $y = f(x)$ in the interval $-4 \leq x \leq 4$, showing the locations of the endpoints, the stationary points and the point of inflection. **2**

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End of Question 24

Question 25 (4 marks)

The discrete random variable X has the probability distribution given in the table below.

x	2	3	4	5
$P(X = x)$	0.3	a	b	0.1

(a) Find the values of a and b, given $\mu = 3.1$. **2**

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(b) Using the formula $\text{Var}(X) = \text{E}((X - \mu)^2)$, find the variance of X. **2**

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Question 26 (6 marks)

An electric vehicle with an empty battery is being recharged. The capacity, $C\%$, of the battery while charging at time t minutes may be modelled by the equation

$$C = 100(1 - 2^{-kt}).$$

The battery is charged to 35% capacity after 50 minutes.

(a) Show that the value of k is 0.01243, correct to 4 significant figures. **2**

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(b) To prolong the service life of the battery, the charger may be set to switch off when the battery reaches 90% capacity. **2**

For how long will the battery be on charge until it reaches 90% capacity?

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Question 26 continues on page 25

Question 26 (continued)

(c) By considering the first and second derivatives, describe the behaviour of C for $t \geq 0$ if the battery is left on charge indefinitely. **2**

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End of Question 26

Question 27 (4 marks)

A continuous random variable, X, has the following probability density function.

$$f(x) = \begin{cases} \dfrac{A}{\sqrt{3x+1}} & 1 \le x \le 5 \text{ (where } A > 0) \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the value of A is $\frac{3}{4}$. **2**

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(b) Given that $P(X < c) = 3P(X > c)$, find the value of c. **2**

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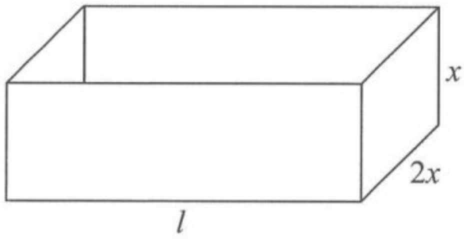
Question 28 (4 marks)

A factory produces boxes in the shape of rectangular prisms as shown below.



NOT TO SCALE

The length, height and width of the box are l cm, x cm and $2x$ cm respectively.

The factory uses k cm^2 of sheet metal to make the box, where k is a constant.

(a) Show that $l = \dfrac{k - 4x^2}{4x}$, given that the box is open at the top. **1**

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Question 28 continues on page 28

Question 28 (continued)

(b) Hence find the maximum volume of the box that can be produced from a sheet of metal with an area of 1200 cm^2. **3**

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End of Question 28

Question 29 (6 marks)

An astronomer is attempting to predict the height of a satellite above the ground as it orbits around the Earth. Using precise measurements, the height of the satellite can be modelled with the function

$$h(t) = 6\sin\left(\frac{\pi}{6}(t-15)\right) + 8,$$

where $h(t)$ is the height of the satellite in thousands of kilometres at time t hours since midnight on any day of the year.

(a) Find the minimum and maximum heights reached by the satellite. **1**

(b) On the grid below, sketch the graph of $h(t)$ for $0 \le t \le 24$. **2**

Question 29 continues on page 30

Question 29 (continued)

(c) The satellite is out of range if it is higher than 11 000 km above the ground. **3**

Use algebraic techniques to find how many hours in the day the satellite is out of range.

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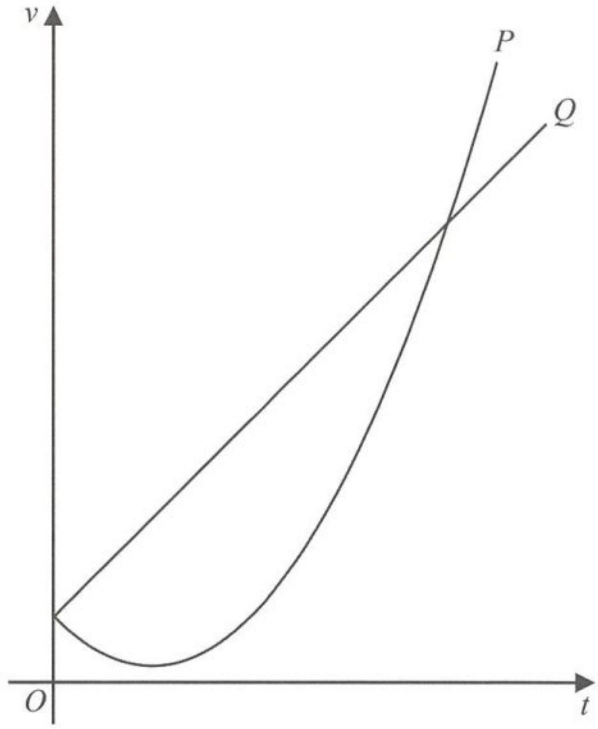
End of Question 29

Question 30 (5 marks)

The velocity of two racing cars, P and Q, are shown on the graph below.



NOT TO SCALE

The velocity of car P at time t seconds is given by the function

$$v_P = 3t^2 - 6t + 4,$$

where v_P is the velocity in metres per second.

The velocity of car Q is accelerating at a constant rate. Both cars start at the same point and have the same velocity at times $t = 0$ and $t = 4$

(a) Show that the equation for the velocity, v_Q, of car Q is given by $v_Q = 6t + 4$. **2**

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Question 30 continues on page 32

Question 30 (continued)

(b) Both cars start the race from the same point. **3**

Find the earliest time when car P will pass car Q after the race starts.

End of Question 30

Question 31 (4 marks)

Let $f(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{3} + \cdots$, where $0 < x < 1$. **4**

By considering $f'(x)$, show that $f(a) - f(b) = \ln\left(\frac{1+b}{1+a}\right)$.

Question 32 (5 marks)

Let C be the circle with equation $(x-1)^2 + (y-3)^2 = 9$, and ℓ the line given by the equation $y = x + d$ for some real constant d.

(a) Show that if $2 - 3\sqrt{2} < d < 2 + 3\sqrt{2}$, then ℓ intersects C at exactly two points. **3**

Question 32 continues on page 35

Question 32 (continued)

(b) Let A and B be the two points of intersection in part (a). **2**

It is given that the distance between A and B is $2\sqrt{3}$ units.

By determining the coordinates of A and B, find a possible value of d, leaving your answer in exact form.

End of Examination

Section II extra writing space

If you use this space, clearly indicate which question you are answering by writing the question number before beginning the response.

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EXAMINERS

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