Neap

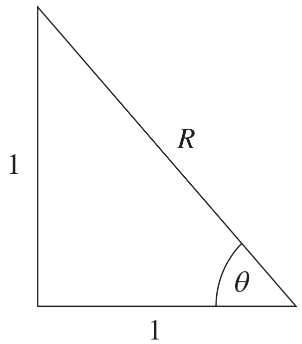
Trial Examination 2023

HSC Year 12 Mathematics Extension 1

Solutions and Marking Guidelines

SECTION I

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 1 **D** **D** is correct. The domain changes from [–1, 1] to [2, 4], which is a translation of 3 to the right. The range changes from $[0, \pi]$ to $[0, 8\pi]$, which is a vertical dilation of scale factor 8. **A**, **B** and **C** are incorrect. These options do not give the correct values of *a* and *b*.	ME–T1 Inverse Trigonometric Functions ME11–1 Band E2
Question 2 **C** $N(t) = 155 + Ae^{kt}$ $\frac{dN}{dt} = kAe^{kt}$ $= k(N - 155)$	ME–C1 Rates of Change ME11–4 Band E2
Question 3 **D** $\cos\theta = \frac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{\lvert\underset{\sim}{u}\rvert\lvert\underset{\sim}{v}\rvert}$ $= \frac{(2)(-6)+(-4)(2)}{\sqrt{(2)^2+(-4)^2} \times \sqrt{(-6)^2+(2)^2}}$ $= \frac{-20}{\sqrt{20} \times \sqrt{40}}$ $= -\frac{1}{\sqrt{2}}$ $\theta = 135°$	ME–V1 Introduction to Vectors ME12–2 Bands E2–E3
Question 4 **D** Letting $n = k + 1$ gives: $(k+1)(2(k+1)-1)(2(k+1)+1) = (k+1)(2k+1)(2k+3)$	ME–P1 Proof by Mathematical Induction ME12–1 Band E2–E3
Question 5 **A** Using a process of elimination: When $x = 1$ and $y = 1$, the gradient is $1^2 - 1^2 = 0$. A gradient of 0 eliminates options **C** and **D**. When $x = 1$ and $y = 0$, the gradient is $0^2 - 1^2 = -1$. A negative gradient eliminates option **B**. Therefore, **A** is the correct response.	ME–C3 Applications of Calculus ME12–4 Bands E2–E3

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 6 B Let $\cos x - \sin x = R\cos(x+\theta)$ $\cos x - \sin x = R\cos x\cos\theta - R\sin x\sin\theta$ Equating $\cos x$ and $\sin x$: $1 = R\cos\theta$ $\cos\theta = \frac{1}{R}$ $1 = R\sin\theta$ $\sin\theta = \frac{1}{R}$  Using Pythagoras' theorem: $R^2 = 1^2 + 1^2$ $R = \sqrt{2}$ $\tan\theta = \frac{1}{1}$ $\theta = \frac{\pi}{4}$ $\therefore \cos x - \sin x = \sqrt{2}\cos\left(x + \frac{\pi}{4}\right)$	ME–T3 Trigonometric Equations ME12–3 Bands E2–E3

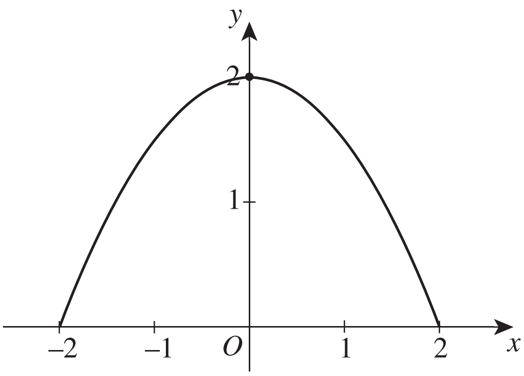
Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 7 **B** $\cos 2x = 1 - 2\sin^2 x$ $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ $\int_0^{\frac{\pi}{12}} \sin^2 x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{12}} (1 - \cos 2x)\,dx$ $= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{12}}$ $= \frac{1}{2}\left(\frac{\pi}{12} - \frac{\sin 2\left(\frac{\pi}{12}\right)}{2}\right) - (0 - 0)$ $= \frac{\pi}{24} - \frac{\sin\frac{\pi}{6}}{4}$ $= \frac{\pi}{24} - \frac{1}{8}$ $= \frac{\pi - 3}{24}$	ME–C2 Further Calculus Skills ME12–1 Bands E2–E3
Question 8 **C** $\underset{\sim}{c} - \underset{\sim}{b} = \frac{1}{2}\left(\underset{\sim}{b} - \underset{\sim}{a}\right)$ $\underset{\sim}{c} - \underset{\sim}{b} = \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$ $\underset{\sim}{c} = \frac{3}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$	ME–V1 Introduction to Vectors ME12–2 Bands E2–E3

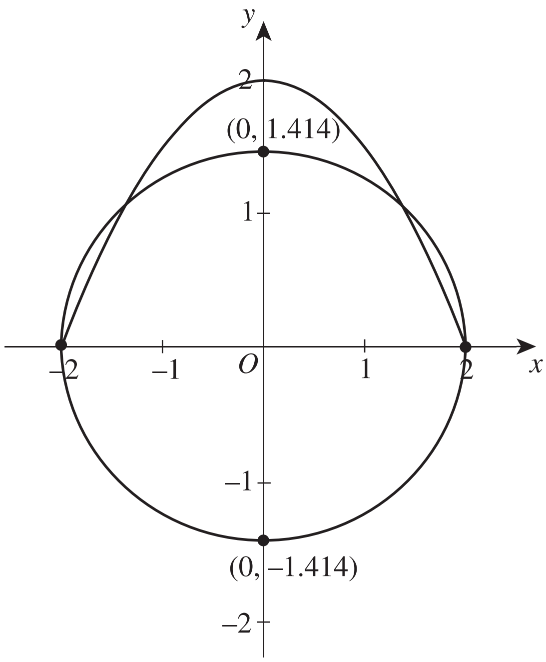
Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 9 **A** $\int \frac{dy}{(y-1)^2} = \int \frac{1}{4} dx$ $\int (y-1)^{-2} = \frac{1}{4}x + c$ $\frac{(y-1)^{-1}}{-1} = \frac{1}{4}x + c$ $-\frac{1}{y-1} = \frac{1}{4}x + c$ When $x = 0$, $y = 0$. $1 = 0 + c$ $c = 1$ $-\frac{1}{y-1} = \frac{1}{4}x + 1$ When $y = 2$: $-\frac{1}{2-1} = \frac{1}{4}x + 1$ $\frac{1}{4}x = -2$ $x = -8$	ME–C3 Applications of Calculus ME12–4 Bands E3
Question 10 **C** $\frac{dV}{dh} = \pi(10h + 225)$ $= 5\pi(2h + 45)$ $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $= \frac{1}{5\pi(2h+45)} \times \frac{-5\sqrt{h}}{2h+45}$ $= -\frac{\sqrt{h}}{\pi(2h+45)^2}$ cm min^{-1}	ME–C1 Rates of Change ME11–4 Bands E3–E4

SECTION II

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 11	
(a) $P(x)=2x^4-15x^3+2x^2+ax+b$ $P(5)=2(5)^4-15(5)^3+2(5)^2+5a+b$ $=0$ $-575+5a+b=0$ $5a+b=575 \quad (1)$ $P'(x)=8x^3-45x^2+4x+a$ $P'(5)=8(5)^3-45(5)^2+4(5)+a$ $=0$ $-105+a=0$ $a=105 \quad (2)$ Substituting (2) into (1) gives: $5(105)+b=575$ $b=50$	ME–F2 Polynomials ME11–1 Bands E2–E3 • Provides the correct solution . . . . . .3 ——— • Substitutes $x = 5$ into $P(x)$. AND • Differentiates $P(x)$ . . . . .2 ——— • Substitutes $x = 5$ into $P(x)$. OR • Differentiates $P(x)$ 1
(b) $u=\cos x$ $\frac{du}{dx}=-\sin x$ When $x=0,\ u=1$. When $x=\frac{\pi}{2},\ u=0$. $\int_0^{\frac{\pi}{2}}\frac{\sin^3 x}{\sqrt{\cos x}}dx=\int_0^{\frac{\pi}{2}}\frac{\sin x\left(1-\cos^2 x\right)}{\sqrt{\cos x}}dx$ $=\int_1^0\frac{1-u^2}{\sqrt{u}}\times(-du)$ $=\int_0^1 u^{-\frac{1}{2}}-u^{\frac{3}{2}}du$ $=\left[2u^{\frac{1}{2}}-\frac{2}{5}u^{\frac{5}{2}}\right]_0^1$ $=\left(2(1)^{\frac{1}{2}}-\frac{2}{5}(1)^{\frac{5}{2}}\right)-\left(2(0)^{\frac{1}{2}}-\frac{2}{5}(0)^{\frac{5}{2}}\right)$ $=\frac{8}{5}$	ME–C2 Further Calculus Skills ME12–1 Bands E2–E3 • Provides the correct solution . . . . . .3 ——— • Finds the complete integrand in terms of u. . . . . .2 ——— • Finds $\frac{du}{dx}$. OR • Changes the limits 1

	Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c)	**Method 1:** The total number of ways for eight people to line up in a queue is 8! The number of ways that Lily can be in front of Ben is equal to the number of ways that Ben can be in front of Lily. $\therefore \frac{8!}{2} = 20\ 160$ ways **Method 2:** Option 1: If Lily is first in line, then Ben can be in seven other positions, and the other six people can arrange themselves in 6! ways. Option 2: If Lily is second in line, then Ben can be in six other positions, and the other six people can arrange themselves in 6! ways. Therefore, the number of ways is: $(7 + 6 + 5 + 4 + 3 + 2 + 1) \times 6! = 20\ 160$	ME–A1 Working with Combinatorics ME11–5 Bands E2–E3 • Provides the correct solution . . . . . . 2 ——— • Finds the total number of ways that the eight people can line up. OR • Equivalent merit 1
(d)	Let $t = \tan\frac{\theta}{2}$. $\frac{1+t^2}{1-t^2} - 2\left(\frac{2t}{1-t^2}\right) = 1$ $1+t^2-4t = 1-t^2$ $2t^2-4t=0$ $2t(t-2)=0$ $t = 0, 2$ $\tan\frac{\theta}{2} = 0,\ 2$ for $0 \le \theta \le \pi$ $\frac{\theta}{2} = 0,\ \pi,\ 1.107\ldots$ $\theta = 0,\ 2\pi,\ 2.214$ Test $\theta = \pi : \sec\pi - 2\tan\pi = -1$ $\therefore \theta = 0,\ 2\pi,\ 2.214$	ME–T3 Trigonometric Equations ME12–3 Bands E2–E3 • Provides the correct solution . . . . . . 3 ——— • Solves and finds the values of *t*. OR • Equivalent merit . . . . 2 ——— • Finds an expression in terms of *t*. OR • Equivalent merit 1

	Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(e) (i)	$\frac{4-x^2}{2} \geq 0$ $4-x^2 \geq 0$ $(2-x)(2+x) \geq 0$ $\therefore -2 \leq x \leq 2$	ME–F1 Further Work with Functions ME11–1 Bands E2–E3 • Provides the correct solution 1
(ii)	Sketching the graph of $y = f(x)$ for $-2 \leq x \leq 2$ gives:  When $f(x) = 0$, $\sqrt{f(x)} = 0$ and when $f(x) = 1$, $\sqrt{f(x)} = 1$. Hence, the two curves meet at these points. When $f(x) > 1$, $\sqrt{f(x)} < 1$. Hence, $\sqrt{f(x)}$ is below $f(x)$. When $0 < f(x) < 1$, $\sqrt{f(x)}$ is larger than $f(x)$. Hence, $\sqrt{f(x)}$ is above $f(x)$. (continues on next page)	ME–F1 Further Work with Functions ME11–1 Bands E3–E4 • Provides the correct solution . . . . . . 3 • Sketches the graph of $\sqrt{f(x)}$ without intercepts. OR • Equivalent merit . . . . 2 • Finds the x-intercepts. OR • Equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(continued) x-intercepts are $x = -2$ and 2. y-intercept is (0, 2) on $y = f(x)$; so it is $(0, \sqrt{2})$ or (0, 1.414) on $y = \sqrt{f(x)}$. $y = -\sqrt{f(x)}$ is the reflection of $y = \sqrt{f(x)}$ about the x-axis.  *Note: Consequential on answer to* ***Question 11(e)(i).***	

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 12	
(a) $V=\pi\int_0^{\frac{\pi}{2}}(2\cos 3y+4)^2\,dy$ $=\pi\int_0^{\frac{\pi}{2}}\left(4\cos^2 3y+16\cos 3y+16\right)dy$ $4\cos^2 3y=2(1+\cos 6y)$ $V=\pi\int_0^{\frac{\pi}{2}}\left(2(1+\cos 6y)+16\cos 3y+16\right)dy$ $=\pi\int_0^{\frac{\pi}{2}}(2\cos 6y+16\cos 3y+18)dy$ $=\pi\left[\frac{2\sin 6y}{6}+\frac{16\sin 3y}{3}+18y\right]_0^{\frac{\pi}{2}}$ $=\pi\left(\frac{\sin 6\left(\frac{\pi}{2}\right)}{3}+\frac{16\sin 3\left(\frac{\pi}{2}\right)}{3}+18\left(\frac{\pi}{2}\right)-(0+0+0)\right)$ $=\pi\left(0-\frac{16}{3}+9\pi\right)$ $=9\pi^2-\frac{16}{3}\pi$	ME–C3 Applications of Calculus ME12–4 Bands E2–E4 • Provides the correct solution . . . . . . 3 • Uses the double angle formula to rewrite the expression . . . . 2 • Finds an expression for the volume of the solid of revolution 1
(b) (i) $\text{RHS}=\frac{1}{P}+\frac{1}{20\,000-P}$ $=\frac{20\,000-P}{P}+\frac{P}{20\,000-P}$ $=\frac{20\,000}{P(20\,000-P)}$ $=\text{LHS}$ $\therefore\ \text{RHS}=\text{LHS}$	ME–C3 Applications of Calculus ME12–4 Bands E2–E3 • Provides the correct solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) $\frac{dP}{st} = 0.1P\left(1 - \frac{P}{20\ 000}\right)$ $= 0.1P\left(\frac{20\ 000 - P}{20\ 000}\right)$ $= \frac{0.1P(20\ 000 - P)}{20\ 000}$ $\int \frac{20\ 000}{P(20\ 000 - P)} dP = \int 0.1 dt$ $\int \frac{1}{P} + \frac{1}{20\ 000 - P} dP = 0.1t + c$ $\ln P - \ln(20\ 000 - P) = 0.1t + c$ $\ln\left(\frac{P}{20\ 000 - P}\right) = 0.1t + c$ When $t = 0$, $P = 1000$. $\ln\left(\frac{1000}{20000 - 1000}\right) = c$ $\therefore c = \ln\frac{1}{19}$ $\ln\left(\frac{P}{20\ 000 - P}\right) = 0.1t + \ln\frac{1}{19}$ $\frac{P}{20\ 000 - P} = e^{0.1t + \ln\frac{1}{19}}$ $\frac{P}{20\ 000 - P} = e^{0.1t} \times e^{\ln\frac{1}{19}}$ $\frac{P}{20\ 000 - P} = \frac{e^{0.1t}}{19}$ $19P = 20\ 000e^{0.1t} - Pe^{0.1t}$ $P(19 + e^{0.1t}) = 20\ 000e^{0.1t}$ $P = \frac{20\ 000e^{0.1t}}{19 + e^{0.1t}}$ (continues on next page)	ME–C3 Applications of Calculus ME12–4 Bands E2–E4 • Provides the correct solution . . . . . . 4 • Uses the given information to find the constant AND makes some progress towards rearranging the equation to find *P*. OR • Equivalent merit . . . . 3 • Integrates both sides. OR • Equivalent merit . . . . 2 • Makes some progress separating the variables in the differential equation. OR • Equivalent merit 1

	Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
	(continued) When $t = 7$: $P = \dfrac{20\,000e^{0.1\times 7}}{19 + e^{0.1\times 7}}$ $= 1916.604$ $= 1917$ Therefore, the population of rabbits after seven months is 1917.	
(c) (i)	$P = \dfrac{3}{8+k}$	ME–S1 The Binomial Distribution ME12–5 Bands E2–E3 • Provides the correct solution 1
(ii)	$1 - p = 1 - \dfrac{3}{8+k}$ $= \dfrac{5+k}{8+k}$ $\text{Var}(X) = np(1-p)$ $= 4\left(\dfrac{3}{8+k}\right)\left(\dfrac{5+k}{8+k}\right)$ $4\left(\dfrac{3}{8+k}\right)\left(\dfrac{5+k}{8+k}\right) < 0.8$ $\dfrac{12(5+k)}{(8+k)^2} < 0.8$ $60 + 12k < 0.8(8+k)^2$ $75 + 15k < 64 + 16k + k^2$ $k^2 + k - 11 > 0$ $k < -3.85,\ k > 2.85$ $\therefore k = 3$	ME–S1 The Binomial Distribution ME12–5 Bands E3–E4 • Provides the correct solution . . . . . . 3 • Attempts to solve for $\text{Var}(X) < 0.8$. OR • Equivalent merit . . . . 2 • Finds an expression for $(1-p)$. OR • Equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(d) $y = \sin^{-1}(x+1) + p\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$ The range of $y = \sin^{-1}(x+1)$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Finding the minimum value gives: $-\frac{\pi}{2} + p\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) \geq 0$ $p\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) \geq \frac{\pi}{2}$ $p \geq \frac{4\pi}{2\left(\sqrt{6}-\sqrt{2}\right)}$ $\therefore p = \frac{2\pi}{\sqrt{6}-\sqrt{2}}$	ME–T1 Inverse Trigonometric Identities ME11–4 Bands E3–E4 • Provides the correct solution . . . . . .3 • Recognises that the minimum value of $\sin^{-1}(x+1)$ occurs at $-\frac{\pi}{2}$ . . . . . . .2 • Creates an inequality that matches the range restrictions in the question. 1
Question 13	
(a) (i) $\lvert \underset{\sim}{p} \rvert = \sqrt{15^2 + (-8)^2}$ $= 17$ The minimum range is 20 – 17 = 3. The maximum range is 20 + 17 = 37.	ME–V1 Introduction to Vectors ME12–2 Bands E2–E3 • Provides the correct solution 1
(ii) Unit vector of $\underset{\sim}{p}$: $\hat{p} = \frac{1}{17}\begin{pmatrix}15\\-8\end{pmatrix}$ $\underset{\sim}{a}$ is in the opposite direction to $\underset{\sim}{p}$ with magnitude 3. $\therefore \underset{\sim}{a} = -\frac{3}{17}\begin{pmatrix}15\\-8\end{pmatrix}$	ME–V1 Introduction to Vectors ME12–2 Bands E3–E4 • Provides the correct solution . . . . . .2 • Finds the unit vector $\underset{\sim}{p}$. OR • Equivalent merit 1

	Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
	(iii) $\underset{\sim}{b}$ is perpendicular to $\underset{\sim}{p}$, so has the direction vector $\begin{pmatrix} 8 \\ 15 \end{pmatrix}$. $\underset{\sim}{b} = k\begin{pmatrix} 8 \\ 15 \end{pmatrix}$ $\lvert\underset{\sim}{b}\rvert = \sqrt{(8k)^2 + (15k)^2}$ $= \sqrt{64k^2 + 225k^2}$ $= 17k$ $17k = 20$ $k = \frac{20}{17}$ $\therefore \underset{\sim}{b} = \frac{20}{17}\begin{pmatrix} 8 \\ 15 \end{pmatrix}$	ME–V1 Introduction to Vectors ME12–2 Bands E3–E4 • Provides the correct solution . . . . . .2 • Finds the direction vector that is perpendicular to $\underset{\sim}{p}$. OR • Equivalent merit 1
(b)	(i) $f(x) = \sin^{-1}(\cos x)$ $f'(x) = -\frac{\sin x}{\sqrt{1-\cos^2 x}}$ $= -\frac{\sin x}{\sqrt{\sin^2 x}}$ $= -\frac{\sin x}{\sin x}$ for $0 \le x < \pi$ $= -1$ Therefore, the function has a constant gradient and so is a linear function.	ME–T2 Further Calculus Skills ME12–1 Bands E2–E3 • Provides the correct solution . . . . . .2 • Attempts to differentiate the function 1
	(ii) $f(x) = \sin^{-1}(\cos x)$ $= \sin^{-1}\left(\sin\left(\frac{\pi}{2} - x\right)\right)$ (using complementary angles) $\therefore f(x) = -x + \frac{\pi}{2}$	ME–C2 Further Calculus Skills ME12–1 Bands E3–E4 • Provides the correct solution . . . . . .2 • Identifies the gradient as –1 OR uses the incorrect gradient from part (b)(i). 1

		Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c)	(i)	Horizontal component: $\ddot{x} = 0$ $\dot{x} = c_1$ When $t = 0$, $\dot{x} = -8$. $\therefore \dot{x} = -8$ $x = -8t + c_2$ When $t = 0$, $x = 60$. $\therefore\ x = -8t + 60$ Vertical component: $\ddot{y} = -9.8$ $\dot{y} = -9.8t + c_3$ When $t = 0$, $\dot{y} = 20$. $\therefore \dot{y} = -9.8t + 20$ $y = -4.9t^2 + 20t + c_4$ When $t = 0$, $y = 25$. $\therefore y = -4.9t^2 + 20t + 25$ Therefore, the position vector of particle B is $(-8t + 60)\underset{\sim}{i} + (-4.9t^2 + 20t + 25)\underset{\sim}{j}$.	ME–V1 Introduction to Vectors ME12–2 Bands E2–E3 • Provides the correct solution . . . . . . 3 • Finds the equations of motion in the horizontal OR vertical direction. AND • Makes some progress in the other direction . . . . 2 • Makes some progress toward deriving the horizontal OR vertical equations of motion 1
	(ii)	For the time at which the two particles collide, letting $16t = -8t + 60$ (from Question 13(c)(i)) gives: $16t = -8t + 60$ $24t = 60$ $t = 2.5$ secs For the point of intersection, substituting $t = 2.5$ into displacement equations for particle A gives: $x = 16(2.5) = 40$ $y = -4.9(2.5)^2 + 30(2.5)$ $= 44.375$ $\therefore (40, 44.375)$ *Note: Consequential on answer to **Question 13(c)(i)**.*	ME–V1 Introduction to Vectors ME12–2 Bands E2–E3 • Provides the correct solution . . . . . . 3 • Finds the time at which the two particles collide. . . . . 2 • Equates the displacement equations 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 14	
(a) (i) $\text{LHS} = \sin(A+B) - \sin(A-B)$ $= \sin A \cos B + \cos A \sin B$ $- (\sin A \cos B - \cos A \sin B)$ $= \sin A \cos B + \cos A \sin B$ $- \sin A \cos B + \cos A \sin B$ $= 2\cos A \sin B$ $= \text{RHS}$	ME–T2 Further Trigonometric Identities ME11–3 Bands E2–E3 • Provides the correct solution 1
(ii) Step 1: Proving the statement is true for $n = 1$ gives: $\text{LHS} = \cos(2(1)-1)\theta$ $= \cos\theta$ $\text{RHS} = \dfrac{\sin 2(1)\theta}{2\sin\theta}$ $= \dfrac{\sin 2\theta}{2\sin\theta}$ $= \dfrac{2\sin\theta\cos\theta}{2\sin\theta}$ $= \cos\theta$ $\therefore \text{LHS} = \text{RHS}$ Therefore, the statement is true for $n = 1$. Step 2: Assuming the statement is true for $n = k$ gives: $\cos\theta + \cos 3\theta + \cos 5\theta + \ldots$ $+ \cos(2k-1)\theta = \dfrac{\sin 2k\theta}{2\sin\theta}$ (continues on next page)	ME–P1 Proof by Mathematical Induction ME12–1 Bands E3–E4 • Provides the correct proof for all steps. .3 • Provides the correct proof for step 2. AND • Makes some progress using the assumption for step 3 . . . . .2 • Provides the correct proof for step 1 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(continued) Step 3: Proving the statement is true for $n = k + 1$ requires proving: $\cos\theta + \cos 3\theta + \cos 5\theta + \ldots$ $+\cos(2k-1)\theta + \cos = (2k+1)\theta$ $\frac{\sin 2(k+1)\theta}{2\sin\theta}$ $\text{LHS} = \frac{\sin 2k\theta}{2\sin\theta} + \cos(2k+1)\theta$ (by assumption) $= \frac{\sin 2k\theta}{2\sin\theta} + \frac{\cos(2k\theta+\theta)\times 2\sin\theta}{2\sin\theta}$ $= \frac{\sin 2k\theta + 2\cos(2k\theta+\theta)\sin\theta}{2\sin\theta}$ $= \frac{\sin 2k\theta + \sin(2k\theta+\theta+\theta) - \sin(2k\theta+\theta-\theta)}{2\sin\theta}$ (using product to sums identity) $= \frac{\sin 2k\theta + \sin(2k\theta+2\theta) - \sin(2k\theta)}{2\sin\theta}$ $= \frac{\sin 2k\theta + \sin(2k\theta+2\theta) - \sin(2k\theta)}{2\sin\theta}$ $= \frac{\sin 2(k+1)\theta}{2\sin\theta}$ $= \text{RHS}$ If $n = k$ is true, then $n = k + 1$ is true. Therefore, by mathematical induction, the statement is true for $n \geq 1$.	
(iii) $\cos\theta + \cos 3\theta = \frac{\sin 4\theta}{2\sin\theta}$ $\frac{\sin 4\theta}{2\sin\theta} = \frac{\operatorname{cosec}\theta}{2} - \sin\theta$ $\frac{\sin 4\theta}{2\sin\theta} = \frac{1}{2\sin\theta} - \sin\theta$ $\sin 4\theta = 1 - 2\sin^2\theta$ $2\sin 2\theta\cos 2\theta = \cos 2\theta$ $2\sin 2\theta\cos 2\theta - \cos 2\theta = 0$ $\cos 2\theta(2\sin 2\theta - 1) = 0$ $\cos 2\theta = 0,\ \sin 2\theta = \frac{1}{2}$ for $0 < 2\theta < 2\pi$ $2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ or $2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\theta = \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{3\pi}{4}$	ME–T3 Trigonometric Equations ME12–1 Bands E3–E4 • Provides the correct solution . . . . . .3 • Uses the double angle formula and attempts to solve the equation . . . . .2 • Uses the identity proven in part (a)(ii) to rewrite $\cos\theta + \cos 3\theta = \frac{\sin 4\theta}{2\sin\theta}$ 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) (i) $P(X < 255) = 1 - 0.16$ $= 0.84$ Therefore, the probability that a flight is not late is 0.84.	ME–S1 The Binomial Distribution ME12–5 Bands E2–E3 • Provides the correct solution 1
(ii) $P(X < 255) - P(X < p) = 0.91$ $0.88 - P(X < p) = 0.91$ $P(X < p) = 0.07$ This gives a z-score of $z = -1.48$. $-1.48 = \frac{p - 240}{15}$ $p = 217.8$ ≈ 218 mins	ME–S1 The Binomial Distribution ME12–5 Bands E3–E4 • Provides the correct solution . . . . . . 3 • Finds the z-score. OR • Equivalent merit . . . . 2 • Finds that $P(X < p) = 0.07$. OR • Equivalent merit 1
(iii) $P(X = 12) + P(X = 13) + P(X = 14) + P(X = 15)$ $= \binom{15}{12} \times (0.91)^{12} \times (0.09)^3 + \binom{15}{13} \times (0.91)^{13} \times (0.09)^2 + \binom{15}{14} \times (0.91)^{14} \times (0.09) + \binom{15}{15} \times (0.91)^{15}$ $= 0.9601$ ≈ 0.96 Therefore, the probability that at least 12 of the flights are on time is 0.96.	ME–A1 Working with Combinatorics ME11–5 Bands E3–E4 • Provides the correct solution . . . . . . 2 • Finds $P(X = 12)$. OR • Equivalent merit 1
(iv) $P(X = 14) = \binom{15}{14} \times (0.91)^{14} \times (0.09)$ $= 0.3605\ldots$ $P(X = 14 \mid X \geq 12) = \frac{P(X = 14 \cap X \geq 12)}{P(X \geq 12)}$ $= \frac{0.3605}{0.9600}$ $= 0.3755$ ≈ 0.38 Therefore, the probability that exactly 14 flights are on time is 0.38.	ME–A1 Working with Combinatorics ME11–5 Bands E3–E4 • Provides the correct solution . . . . . . 2 • Finds $P(X = 14)$. OR • Equivalent merit 1

